THE INFLUENCE OF KINETIC PROCESSES AND MEDIUM DEGASSING ON CRACK GROWTH

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A number of materials used in modern engineering exhibit specific properties of gas emission in bulk under certain mechanical and/or physical influence or aging. Gas emission due to aging is typical for a number of polymers. Some metals and alloys applied in nuclear-power engineering become gas emissionable under radiation [1].

Gas emission in bulk can frequently cause crack or crack-like defects initiation and their kinetic propagation.

In this case, crack kinetics analysis implies simultaneous consideration of gas diffusion into the crack and slow crack growth due to the action of inner gas pressure and other mechanical loads .

We suggest a numerical method for solving the 3D problem for a medium with cracks occupying a plane region. Problems of the gas diffusion into the crack and crack propagation are solved by reducing to integro-differential equations in the crack domain. The kinetics calculation is performed by step wise procedure. We used numerical methods and algorithms based on earlier results [2,3].

In model calculations, the crack velocity v at each point of the crack contour is assumed to be dependent on the stress intensity factor K at this point. The more general situation when v is a functional of the gas concentration near the crack tip and stress intensity factor can be studied by a similar way. These results will be published elsewhere.

1. STATEMENT OF THE PROBLEM

We consider the slow quasisteady growth of a tensile crack initiated at t=0 and occupying a domain G in the plane $x_3=0$. The velocity v at each crack contour point is assumed to be dependent on the stress intensity factor K (as is adopted in kinetic crack theories) and specified by a curve v(K) which is the material function. The crack is growing under the action of a gas produced by gas emission sources distributed in bulk. The crack is modeled by an ideal sink (far from equilibrium state). The crack velocity is assumed to be small as compared to the transient period. Under this assumption, the flow into the crack can be found from the solution of the stationary diffusion problem for each t. Suppose that initially there are two diffusion sources of the intensity W placed inside the body on the x_3-axis symmetrically at a distance ξ_3 from the crack .In view of the symmetry with respect to the crack plane, we can consider the problem in the half-space $x_3 \geq 0$.

The boundary value problem for the gas concentration $c(x_1, x_2)$ is the following one:

$$\Delta c = -\frac{W}{D}\delta(x_1)\delta(x_2)\left[\delta(x_3-\xi_3)+\delta(x_3+\xi_3)\right],$$

$$c\Big|_{x_3=0} = 0, \quad (x_1,x_2) \in G; \quad \frac{\partial c}{\partial x_3}\Big|_{x_3=0} = 0, (x_1,x_2) \notin G; \; c\Big|_{x_3=\infty} = 0, \tag{1.1}$$

where D is the coefficient of the gas diffusion in the medium.

The diffusion flow density $q(x_1,x_2) = \partial c / \partial x_3\big|_{x_3=0}$ for $(x_1,x_2) \in G$ is the unknown function in the problem. As usually, to construct an integral equation for the function q, first we consider the gas diffusion problem with sources in a medium without a crack:

$$\Delta c_o = -\frac{W}{D}\delta(x_1)\delta(x_2)[\delta(x_3-\xi_3)+\delta(x_3+\xi_3)], \tag{1.2}$$

$$\frac{\partial c_o}{\partial x_3}\Big|_{x_3=o} = 0, \quad c_o\Big|_{x_3=\infty} = 0$$

The solution to problem (1.2) is the function

$$c_o(x_1,x_2,x_3) = \frac{W}{4\pi D}\left(\frac{1}{R_1}+\frac{1}{R_2}\right),$$

$$R_{1,2} = \sqrt{x_1^2+x_2^2+(x_3\mp\xi_3)^2} \tag{1.3}$$

and the gas concentration in the crack plane is given by

$$c_o(x_1,x_2,0) = \frac{W}{2\pi D}\frac{1}{\sqrt{x_1^2+x_2^2+\xi_3^2}}. \tag{1.4}$$

Let us now write out the solution to the diffusion problem without sources but with the gas concentration inside the crack to be equal in magnitude and opposite in sign to that in the first problem:

$$\Delta c = 0, \quad c\big|_{x_3=o} = -\frac{W}{2\pi D}\frac{1}{\sqrt{x_1^2 + x_2^2 + \xi_3^2}}, (x_1,x_2) \in G; \tag{1.5}$$

$$\frac{\partial c}{\partial x_3}\bigg|_{x_3=o} = 0, \quad (x_1,x_2) \notin G; \quad c\big|_{x_3=\infty} = 0.$$

The following integral equation is obtained for the diffusion flow density q from Eq.(1.5):

$$\frac{1}{2\pi}\iint_G \frac{q(\xi_1,\xi_2)d\xi_1\, d\xi_2}{\sqrt{(x_1-\xi_1)^2 + (x_2-\xi_2)^2}} = -\frac{W}{2\pi D}\frac{1}{\sqrt{x_1^2 + x_2^2 + \xi_3^2}}. \tag{1.6}$$

Similarly, if a nonzero gas concentration $c^o : c\big|_{x_3=o} = c^o, (x_1,x_2) \in G$ is given inside the crack, then we obtain the integral equation

$$\frac{1}{2\pi}\iint_G \frac{q(\xi_1,\xi_2)d\xi_1\, d\xi_2}{\sqrt{(x_1-\xi_1)^2 + (x_2-\xi_2)^2}} = \frac{W}{2\pi D}\frac{1}{\sqrt{x_1^2 + x_2^2 + \xi_3^2}} - c^o(x_1,x_2). \tag{1.7}$$

The following integral equation is valid if two sources symmetric with respect to the crack plane are placed at arbitrary points (a,b,ξ_3) and $(a,b,-\xi_3)$ in bulk:

$$\frac{1}{2\pi}\iint_G \frac{q(\xi_1,\xi_2)d\xi_1\, d\xi_2}{\sqrt{(x_1-\xi_1)^2 + (x_2-\xi_2)^2}} = \frac{W}{2\pi D}\frac{1}{\sqrt{(x_1-a)^2 + (x_2-a)^2 + \xi_3^2}}. \tag{1.8}$$

Using the superposition principle, we obtain the following equations for several point sources of gas diffusion inside the body or for those distributed with density $W(x_1,x_2,x_3)$:

$$\iint_G \frac{q(\xi_1,\xi_2)d\xi_1\, d\xi_2}{\sqrt{(x_1-\xi_1)^2 + (x_2-\xi_2)^2}} = \frac{1}{D}\sum_i \frac{W_i}{\sqrt{(x_1-a_i)^2 + (x_2-a_i)^2 + \xi_{3i}^2}}, \tag{1.9}$$

$$\iint_G \frac{q(\xi_1,\xi_2)d\xi_1\, d\xi_2}{\sqrt{(x_1-\xi_1)^2 + (x_2-\xi_2)^2}} = \frac{1}{D}\iiint_T \frac{W(y_1,y_2,y_3)dy_1\, dy_2\, dy_3}{\sqrt{(x_1-y_1)^2 + (x_2-y_2)^2 + y_3^2}}, \tag{1.10}$$

where W_i are the intensities of the sources at the points $(a_i, b_i, \pm\xi_{3i})$ and T is the region of diffusion sources distribution with density $W(x_1,x_2,x_3)$.

To search for the elastic fields induced by the gas diffusion into the crack, we consider the problem on a normal tensile crack with load p applied to its surfaces, where p is the gas pressure, which depends on the crack volume and mass of gas entered. The gas is assumed to be ideal, then the crack volume, V, the mass of the gas, M, and the pressure, p, are related by the Clapeyron equation $pV=MRT/\mu$. Here μ,R, and T are the molar mass of the gas, the gas constant per mole, and absolute temperature, respectively.

Reducing the elasticity problem to boundary integral equations, we obtain the system

$$\frac{1}{2\pi}\iint_G \frac{q(\xi_1,\xi_2)d\xi_1\, d\xi_2}{\sqrt{(x_1-\xi_1)^2+(x_2-\xi_2)^2}} = -\frac{W}{2\pi D}\frac{1}{\sqrt{x_1^2+x_2^2+\xi_3^2}}. \qquad (1.11)$$

$$p(t) = -\frac{E}{4\pi(1-\nu^2)}\Delta_{x_1x_2}\iint_{G(t)} \frac{u(\xi_1,\xi_2)d\xi_1\, d\xi_2}{\sqrt{(x_1-\xi_1)^2+(x_2-\xi_2)^2}}, \qquad (1.12)$$

$$pV = nRT, \qquad (1.13)$$

$$(t) = \iint_{G(t)} u(\xi_1,\xi_2)d\xi_1\, d\xi_2, \qquad (1.14)$$

$$Q = -\iint_{G(t)} q(\xi_1,\xi_2)d\xi_1\, d\xi_2, \qquad (1.15)$$

$$n(t+\Delta t) = n(t) + Q\Delta t, \qquad (1.16)$$

$$u(\xi,s,t) = \frac{4(1-\nu^2)}{E}N(s,t)\sqrt{\xi}, \qquad (1.17)$$

$$v(s,t) = f(N(s,t)), \qquad (1.18)$$

$$R(t+\Delta t,s) = R(t,s), \qquad (1.19)$$

where integral equation (1.11) for the function $q(x_1,x_2)$ can be replaced by Eqs. (1.7)-(1.10) depending on the number of sources and their distribution. Equation (1.12) is the integro-differential equation for the crack surfaces displacement $u(x_1,x_2)$. Further, n,$Q=\partial n/\partial t$, E, and ν are the number of gas moles in the crack, gas flow rate through the crack, Young's modulus, and Poisson's ratio of the medium, respectively. Eqs. (1.17)-(1.19) provide the calculation of the stress intensity factor N and new crack contour.

The solution is performed by a stepwise procedure. The main computational difficulties of the first t-step are related to solving integro-differential Eqs. (1.11)-(1.12) and searching for a new crack contour via the calculated velocities v at the previous contour points (see Eq.(1.18)). The last computation is a separate computational problem. A procedure for solving the elasticity problem for a normal tensile crack (Eq. (1.12)) has also been developed. For this reason, we focus on a numerical method for solving the diffusion equation (1.11). In case of a circular crack region, we obtain an analytic solution.

2. NUMERICAL METHOD FOR SOLVING THE DIFFUSION EQUATION

Our method for solving the integro-differential equation is based on the variational-difference method [2]. Namely, after discretization, the values of q at the grid points are searched for as an expansion through a system of coordinate functions ψ_{p1p2},

$$q(x_1,x_2)=\sum_{p1\,p2} c_{p1\,p2}\,\psi_{p1\,p2}(x_1,x_2,h), \tag{2.1}$$

where ψ_{p1p2} is a bilinear spline function with a support in the four grid cells adjacent to the point (p_1h,p_2h) of the grid with the step h.

The coefficients c_{p1p2} coincide with the values of $q(x_1,x_2)$ at the grid points and are found by minimizing the corresponding quadratic functional:

$$\min\left\{ I(h)=\sum_{p_1p_2}\sum_{q_1q_2} a_{p_1-q_1,\,p_2-q_2}\,c_{p_1p_2}\,c_{q_1q_2}+2\sum_{p_1p_2} c_{p_1p_2}\,b_{p_1p_2}\right\}, \tag{2.2}$$

$$a_{p_1p_2q_1q_2}=a_{|p_1-q_1|,|p_2-q_2|}=\frac{1}{(2\pi)^2}\iint_{-\infty}^{\infty}\frac{1}{|\xi|}\,\psi_{p_1p_2}(\xi,h)\overline{\psi_{q_1q_2}(\xi,h)}\,d\xi, \tag{2.3}$$

$$\psi_{p_1p_2}(\xi,h)=h^2e^{ih(p_1\xi_1+p_2\xi_2)}\,\frac{\sin^2\left(\frac{1}{2}h\xi_1\right)}{\left(\frac{1}{2}h\xi_1\right)^2}\,\frac{\sin^2\left(\frac{1}{2}h\xi_2\right)}{\left(\frac{1}{2}h\xi_2\right)^2}, \tag{2.4}$$

$$b_{p_1p_2}=\iint_{g^{2h}_{p_1p_2}} p(x_1,x_2)\,\psi(x_1,x_2)\,dx_1dx_2 \tag{2.5}$$

$$p(x_1,x_2)=\frac{W}{D}\frac{1}{\sqrt{x_1^2+x_2^2+\xi_3^2}} \tag{2.6}$$

The minimization is carried out by the gradient projection method with an automatic step choice according to the relation between the linear and the actual functional increments.

3. CRACK PROPAGATION DUE TO THE GAS DIFFUSION FROM THE BULK SOURCES

Software is developed to calculate the crack propagation time and evolution of the crack shape and sizes under the action of the gas diffusion from the unit source or sources distributed in bulk with a given density. This program is based on the discribed methods for solving the integro-differential equations (1.11) of the diffusion problem and Eqs.(1.12) of the elasticity problem. A quasisteady statement of the problem is used. System (1.11)-(1.19) is solved at each time step. The incubation period, t_i, before the crack growth start is calculated. The maximum stress intensity factor along the crack contour becomes greater than the fracture toughness threshold value K_{scc} at the end of the incubation period.

Calculation of the growth time t_m is performed by the following scheme:

1) the gas pressure is calculated in the current crack region $G(t)$: $p^2(t)=n(t)RT/V_1(t)$, where $V_1(t)$ is the volume of the crack occupying the new region $G(t)$ for unit load p=1, the gas mass n(t) being found at the previous step.

2) the stress intensity factor along the crack contour is calculated and used for calculation of the crack velocity v_1.

3) crack extension is determined by a distances $\Delta_i=v_i\Delta t$ calculated along the outer normal to the discrete set of the crack contour points. Time interval Δt is calculated so that the maximum crack increment at the step Δt didn't exceed certain value Δ chosen by numerical experiments.

4) a new contour shape is defined using a smoothing procedure over propagating and stationary points coordinates.

5) the diffusion problem is solved; the diffusion flow density $q(x_1,x_2)$ through the crack surface, the total gas flow rate Q, and the new gas amount in the crack $n(t+\Delta t)=n(t)+Q\Delta t$ are defined.

6) the above procedure is repeated starting from step 1.

Model calculations were performed for a circular plane crack. Its kinetics was studied in the case of the gas diffusion from a unit bulk source. The incubation period, t_i, and time of the crack growth from the initial to double radius, t_m, were calculated. The dependence of the values t_i and t_m on the distance, h, from the diffusion source to the crack plane was studied. On diminishing the distance h, the gas flux to the crack increases and the gas pressure becomes greater, thus leading to the stress intensity factor growth along the contour. As a result, the velocity tends to its stationary value (on kinetic diagram), propagation and incubation times being practically independent of h. One more series of calculation was performed to study the dependence of the lifetime on the diffusion source intensity w. The greater the source intensity, the less is the lifetime.

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