

**THE EVOLUTION OF DAMAGE AT CYCLIC LOADING**

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In this contribution crack initiation and propagation on sharp notched specimens made of steel METASAFE 900 at cyclic loading are examined. The description uses straining parameters determined under the assumptions of continuum mechanics. The influence of overloads on the life time (number of cycles of defined incipient crack) is studied. Overloads leave residual stresses and influence the crack closure and endurance limit by changed mean stresses.

**INTRODUCTION**

The life time at cyclic loading of constructions consists of

- a crack initiation phase (initiation and propagation of micro cracks)
- and a crack propagation phase (propagation of small and macro cracks)

The life time of specimens and constructions with sharp notches consists essentially of a crack propagation phase. With straining parameters we want to classify the crack size. The straining parameters can be determined for different materials laws from local mean equivalent straining  $B_v$  resp.  $\Delta B_v$ :

$$\Delta B_{vm} = \frac{1}{v^*} \int_{(v^*)} \Delta B_v dv = \frac{1}{d^*} \int_{(d^*)} \Delta B_v(r) dr \quad (1)$$

Here are:

- $v^*$  resp.  $d^*$  characteristic volume resp. characteristic length (Neuber: "Ersatzstrukturlänge") dependent on the micro structure of the material,
- $\Delta B_v$  an equivalent cyclic stress  $\Delta\sigma_v$  or an equivalent cyclic strain  $\Delta\varepsilon_v$  or some suitable combination of stresses and strains.
- From eq.(1) we get for linear-elastic material law for a "big" specimen with cracks under Mode I-loading (according to the normal stress hypothesis - NSH):

$$\Delta B_v = \Delta\sigma_1 = \Delta\sigma_n \frac{r+a}{\sqrt{2ra+a^2}}; \Delta B_{vm} = \Delta\sigma_{1m} = \Delta K \sqrt{\frac{2}{\pi d^*}} \sqrt{1 + \frac{d^*}{2a}} \quad (2)$$

Here are  $\Delta K = \Delta\sigma_n \sqrt{\pi a}$ , resp.  $\Delta K = \Delta\sigma_n \sqrt{\pi a} Y(a)$ , resp  $\Delta K = \Delta\varepsilon_n E \sqrt{\pi a} Y(a)$ .  $Y(a)$  depends on the geometry of specimens.

The generalized straining parameter  $\Delta B_{vm}$  is qualified according to crack size classification dependent on  $d^*$  (see table 1)

*Table 1: Crack size classification and special suitable straining parameters*

crack size	straining parameter	
macro cracks: $a \gg d^*$	$\Delta B_{vm} \sim \Delta K$	stress intensity factor (SIF)
micro cracks: $a \ll d^*$	$\Delta B_{vm} \sim \Delta\sigma_n$ resp. $\sim \Delta\varepsilon_n$	nominal stress nominal strain
small cracks: $a \approx d^*$	$\Delta B_{vm} \sim \Delta K \sqrt{1 + \frac{d^*}{2a}}$	modified SIF

The parameters in table 1 are suitable to describe the strength phenomena in the field of endurance limit [2]. The endurance limit is given by  $\Delta B_{vm}^D = \text{const}$  for small and macro cracks. With that we get

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$$\frac{\Delta K_{th}^{Kl}}{\Delta K_{th}} = \frac{1}{\sqrt{1 + \frac{d^*}{2a}}} \quad (3)$$

Formaly this coherence corresponds with the following equation (see [3]):

$$\frac{\Delta K_{th}^{Kl}}{\Delta K_{th}} = \frac{1}{\sqrt{1 + \frac{a_c}{a}}} \quad (3.1)$$

With  $d^* = 2a_c = \frac{2}{\pi} \left( \frac{\Delta K_{th}}{\Delta \sigma_D Y} \right)^2$ . Here are  $\Delta K_{th}$ - threshold value of macro cracks,  $\Delta K_{th}^{Kl}$ - threshold value of small cracks,  $a_c$ - critical crack length.

The value  $d^*$  in eq.(3) can also be understood as a free parameter to describe threshold values of small cracks. In this case  $d^*$  is influenced by the microstructure of the material and by different crack closure of small cracks compared to macrocracks.

• For a prognosis of life time, knowledge is necessary about propagation of small cracks (see [4]). The straining parameter  $\Delta B_{vm}$  according to eq.(2) is qualified to give a life time prognosis in the phase of propagation of small cracks (see [5]):

$$\frac{da}{dN} = C \left[ \Delta K \sqrt{1 + \frac{d_0^*}{2a}} - \Delta K_{th} \right]^m \quad \text{resp.} \quad \frac{da}{dN} = C_1 \left[ \left( \Delta K \sqrt{1 + \frac{d_0^*}{2a}} \right)^m - (\Delta K_{th})^m \right] \quad (4)$$

Here are - C,  $C_1$ , m material constants obtained from propagation of macrocracks -  $d_0^*$  free parameter to describe propagation of small cracks.  $d_0^*$  is influenced by

- the microstructure of the material
- different crack closure behaviour of small cracks compared to macrocracks
- plastic corrections compared with linear-elastic material law.

### CRACK INITIATION ON SHARP NOTCHES

From investigations carried out up to now on steel with  $R_e = (300-800)$ MPa it is known:  $d^* \approx (50...100)\mu m$ ;  $d_0^* \approx (50...250)\mu m$  according to NSH.

In accordance with that we define an incipient crack by  $a_A \approx d^*/10 \approx (5...10)\mu m$ . For cracks with  $a < (5...10)\mu m$  we expect, according to the straining parameter  $\Delta B_{vm}$ , a crack propagation velocity  $da/dN$  fairly independent of the crack length.

(Because these cracks may be smaller than the size of constituents (e.g. grain size) the crack propagation velocity may decrease with increasing crack length. Barriers (e.g. grain boundaries) are the reason for this phenomenon /4/. A description of the local propagation of such microcracks is possible with

$$\frac{da}{dN} = C_2 \Delta \varepsilon_n^\alpha (d - a)^\beta \quad (5)$$

Here are  $C_2, \alpha, \beta$  material constants, d size of constituent (e.g. grain diameter). The propagation of these very small cracks is here not examined in detail).

In this contribution it is shown that the initiation of very small cracks at sharp notches can be described with the parameter corresponding to eq.(1)  $\Delta B_{vm} = \Delta \varepsilon_{vm}$ :

$$(\Delta \varepsilon_{vm} - \Delta \varepsilon_D)^n N_A = K_1 \left( 1 - \frac{\varepsilon_{vm} - \varepsilon_D}{\varepsilon_{krit} - \varepsilon_D} \right) \quad (6)$$

Here are  $K_1, n$  - material constants from single-stage straining,  $\Delta \varepsilon_D = 2\varepsilon_D$  - endurance strain limit,  $\varepsilon_{krit}$  - critical strain at quasistatic load,  $N_A$  - number of cycles of defined incipient crack,  $\Delta \varepsilon_{vm}$  - straining parameter determined from local strains at elastic-plastic material law and  $\varepsilon_{vm}$  - straining parameter for quasistatic loading.  $d^*$  is examined at strainings leading to a defined crack and the same  $N_A$  at different notched specimens: f.i.  $\Delta \varepsilon_{vm}(\varrho_1, d^*) N_A = \Delta \varepsilon_{vm}(\varrho_2, d^*) N_A$ ; (see [7])  
For incipient cracks  $a_A \ll d^*$  the crack length does not influence the straining parameter.

In the contribution it is examined with priority the influence of overloads before crack initiation to the crack initiation and crack propagation. Overloads produce a quasistatic damage (see  $\varepsilon_{vm}$  in eq.6) and residual stresses.

Near notch root there arise compressive stresses after tension overloads and tensile stresses after pressure overloads.

The endurance limit can be influenced by mean stresses:

$$\Delta\varepsilon_D = \frac{\Delta\sigma_D}{E} = \frac{2\sigma_{-1}}{E} \sqrt{1 - \frac{\sigma_{1m}}{R_m}}; \quad \text{with} \quad \sigma_{1m} = \frac{1}{d^*} \int \sigma_1(r) dr \quad (7)$$

Here are  $\sigma_{-1}$  - endurance limit at  $R = -1$  and  $R_m$  - ultimate tensile strength

The life time is importantly influenced at strainings  $\Delta\varepsilon_{vm}$  near the endurance limit  $\Delta\varepsilon_D$ .

### CRACK PROPAGATION

The crack propagation behaviour can be calculated on principle with eq.(6) and by knowlegde of a damage accumulation hypothesis. The Miner hypothesis gives:

$$\int_0^1 dS = \int_0^{N_A} \frac{dN}{N_A(\Delta\varepsilon_{vm})} = 1$$

If we assume an elastic strain distribution on macro cracks (this is only a rude approach) then we get with  $\Delta\varepsilon_{vm}(r) = \Delta\varepsilon_{1m}(r)$ :

$$\Delta\varepsilon_{1m}(r) = \frac{2(1-\nu)\Delta K}{E\sqrt{2\pi}d^*} \left[ \sqrt{r+d^*} - \sqrt{r} \right] \quad \text{for plane stress}$$

If we furthermore assume that damages only occur in the cyclic plastic zone  $\Delta r_{pl}$ ,

then we obtain with  $\Delta r_{pl} \approx \left(\frac{\Delta K}{R_e}\right)^2 \frac{1}{4\pi}$  for plane stress and with  $n=2$  in eq.(6) a mean crack propagation velocity over  $l_0 = \Delta r_{pl}$ :

$$\left(\frac{\Delta a}{\Delta N}\right)_{l_0} = \frac{da}{dN} = C (\Delta K)^{5/2} \left[ 1 - \left(\frac{\Delta K_{th}}{\Delta K}\right)^{1/2} \right] \quad (8)$$

Here are  $C = f(E, \nu, K_1, d^*, R_e, n = 2, R)$ .  $C$  and  $\Delta K_{th}$  are influenced by crack closure and by residual stress in front of the crack tip.

With knowlegde about  $\Delta\sigma_D$  (f.i. from eq.(7)) it is possible to estimate the thresholds without crack closure:  $\Delta K_{th} = \sqrt{\frac{\pi d^*}{2}} \Delta\sigma_D$ . A realistic life time prognosis requires an experimental determination of  $C$ .

The number of parameters is smaller with use of effective cyclic SIF  $\Delta K_{eff}$ . Koherences between  $\Delta K$  and  $\Delta K_{eff}$  for macro cracks are known in following form (see /9/ and /10/):

$$\begin{aligned} \Delta K_{eff} &= \Delta K \cdot F(R) \\ \text{resp.} \quad \Delta K_{eff,th} &= \Delta K_{th} \cdot F_1(R) \end{aligned} \quad (9)$$

The propagation of small cracks can be approximatly calculated with a parameter

$$\Delta B_{vm} \sim \Delta K^{Kl} \sqrt{1 + \frac{d_0^*}{2a}}$$

With that we assume for the effective cyclic SIF of small cracks

$$\Delta K_{eff}^{Kl} = F(R) \cdot \Delta K^{Kl} \sqrt{1 + \frac{d_0^*}{2a}} \quad (10)$$

with  $d_0^* \approx d^* + d_{schl}^*$ .  $d_{schl}^*$  is intended to describe different crack closure of small cracks in contrast to macro cracks. For this end the crack opening and partial crack closure are calculated with a FE program.

Fig. 1 shows the crack opening under loading and unloading without crack propagation by loading and Fig. 2 shows the crack opening with a crack growth at maximum load under the assumption of a smooth crack area.

Table 2 shows the effective cyclic SIF  $\Delta K_{eff}^{Kl} = K_{max} - K_{cl}$  for  $R=-1$  according to model which leads to Fig. 1 and the approximation by following equation:

$$\Delta K_{eff}^{Kl} = F(R) \cdot \Delta K^{Kl} \sqrt{1 + \frac{d_{schl}^*}{2a}} \quad (10.1)$$

F(R) can be determined by experiments. In Table 2, F(R) is ascertained from  $\Delta K_{eff}$  for macro cracks  $F(R = -1) = \frac{\Delta K_{eff}(a=5mm)}{K_{max} - K_{min}}$ . It is recognised the possibility of description of  $\Delta K_{eff}^{Kl}$  with eq.(10.1), but we also see the dependence of  $d_{schl}^*$  resp.  $K_{cl}$  on loading ( $K_{max}$ ). So we can understand that  $d_0^*$  for description of crack propagation is bigger than  $d^*$  to calculation threshold values of small cracks:  $d_0^* = d^* + d_{schl}^* \approx d^*$  because  $d_{schl}^* \ll d^*$  at strainings near the threshold values.

Table 2: Description of effective SIF of small cracks by crack closure with the  $d^*$ -concept

a / mm	0,075	0,10	0,50	1,0	5,0	F(R), $d_{schl}^*$ (**)
$K_{max}/MPa\sqrt{mm}$	126,5	126,5	126,5	126,5	126,5	$F(R) = 0, 50$ $d_{schl}^* = 4, 8\mu m$
$K_{cl}/MPa\sqrt{mm}$	-3,1	-1,3	-0,7	-0,7	0	
$\Delta K_{eff}^{Kl} = K_{max} - K_{cl}$	129,6	127,8	127,2	127,2	126,5	
$\Delta K_{eff}^{Kl}$ eq.(10.1)	128,5	128	126,8	126,5	126,5	
$K_{max}/MPa\sqrt{mm}$	253	253	253	253	253	$F(R) = 0, 508$ $d_{schl}^* = 46, 5\mu m$
$K_{cl}/MPa\sqrt{mm}$	-37,3	-35,1	-5,5	-3,6	-4,2	
$\Delta K_{eff}^{Kl} = K_{max} - K_{cl}$	290,7	288,1	258,5	256,3	257,3	
$\Delta K_{eff}^{Kl}$ eq.(10.1)	294,2	285,4	263,0	260,0	257,6	
$K_{max}/MPa\sqrt{mm}$	300	300	300	300	300	$F(R) = 0, 50$ $d_{schl}^* = 76, 7\mu m$
$K_{cl}/MPa\sqrt{mm}$	-75,8	-45,9	-8,9	-5,4	0	
$\Delta K_{eff}^{Kl} = K_{max} - K_{cl}$	375,8	345,9	308,9	305,4	300	
$\Delta K_{eff}^{Kl}$ eq.(10.1)	368,8	352,9	311,3	305,7	301,7	

(\*\*)  $F(R) = \frac{\Delta K_{eff}(a=5mm)}{\Delta K}$ ;  $d_{schl}^* = \frac{1}{n} \sum_{i=1}^n 2a_i \left[ \left( \frac{\Delta K_{eff}^*}{\Delta K \cdot F(R)} \right)_i^2 - 1 \right]$ ; (R=-1,  $\Delta K = 2K_{max}$ )

### EXPERIMENTS

• Material

The experiments were performed on steel METASAFE 900 with a modified thermo-mechanical treatment.

Tensile properties: Yield stress  $R_{\rho 0,2}=636$  MPa, ultimate tensile strength  $R_m=845$  MPa, uniform elongation  $A_g=7,9$  %, fracture strain  $A_5=19,1$  %.

Chemical composition (m - %):

C	Si	Mn	S	P	Cr	Ni	Mo	Cu	Al	V
0,290	0,120	1,780	0,037	0,012	0,290	0,070	0,060	0,190	0,041	0,100

• Specimens

The experiments were performed on single edge notched specimens of rectangular cross section (Fig 3). The fatigue tests were conducted in cyclic cantilever bending under load control with a load ratio  $R \approx 0$  using a resonance machine DYNACOMP /11/. The experimental results under one step loading without and with overload are represented in table 3.

Table 3: Influence of overloads on life time at sharp notched bending specimens applying a load ratio  $R=0$

experiment	1	2	3	4	5	6	7	8
$g/mm$	0,50	0,50	0,50	0,50	0,50	0,50	0,50	0,50
$\Delta M/Nm$	50	50	50	50	50	61	61	61
$\Delta M_{ov}/Nm$	0	0	150	-150	-150	0	0	152,5
$N_{A(10\mu m)}$	$3,3 \cdot 10^4$	$4,6 \cdot 10^4$	(*)	$2,5 \cdot 10^4$	$2,3 \cdot 10^4$	$2,2 \cdot 10^4$	$2,7 \cdot 10^4$	$2,4 \cdot 10^4$
$N_{A(500\mu m)}$	$7,1 \cdot 10^4$	$8,8 \cdot 10^4$	(*)	$4,8 \cdot 10^4$	$4,6 \cdot 10^4$	$4,9 \cdot 10^4$	$5,7 \cdot 10^4$	$2,5 \cdot 10^5$

(\*) -  $a_A < 10\mu m$ , experiments finished at  $N=5, 7 \cdot 10^6$

The evolution and spread of damage at the notch root has been examined through thermal imaging the microstructural alternations of the material during fatigue by mean of laser induced thermomicroscopy ALADIN /6/. Different influences (plastic deformations, residual stresses and reached damage) render more difficult an evaluation.

### DISCUSSION

The life time of notched specimens under strainings near endurance limit depends on the local straining amplitudes and the local endurance limit (see eqn. (6),(4),(7)). The local straining amplitudes are only little influenced by overloads (see Fig. 4 and 5). But the residual stresses after overloads have a large influence on damage development (crack initiation and propagation) see Fig. 5. The Figures 4 and 5 show the results of FEM-calculations under plane strain condition. The results under plane stress condition are only little different.

Tension overloads lead to relatively small pressure stresses near at the notch root. Together it gives damage at quasi static loading (see  $\varepsilon_{vm}$  in eq.(6)). This is the reason that there are practically the same number of cycles of incipient crack  $N_A(a_A = 10\mu m)$  at experiments with and without overloads (see table 2, experiments 6, 7 and 8). But the important pressure stresses in front of notch root influence considerably the crack propagation in consequence of different crack closure and mean stresses. General, on sharp notches tension overloads have led to a longer life time (see experiments 1, 2, 3 and 6, 7, 8) and pressure overloads have led to a shorter life time (see experiments 1, 2 and 4, 5).

### CONCLUSION

In the contribution there is examined the damage development (crack initiation and propagation) on specimens made of steel METASAFE 900 with sharp notches under cyclic loading. The influence of overloads before crack initiation was studied with regard to crack initiation and propagation.

Overloads before crack initiation influence especially the crack propagation. Continuum mechanical parameters make it easy to understand the damage development in construction.

### ACKNOWLEDGEMENT

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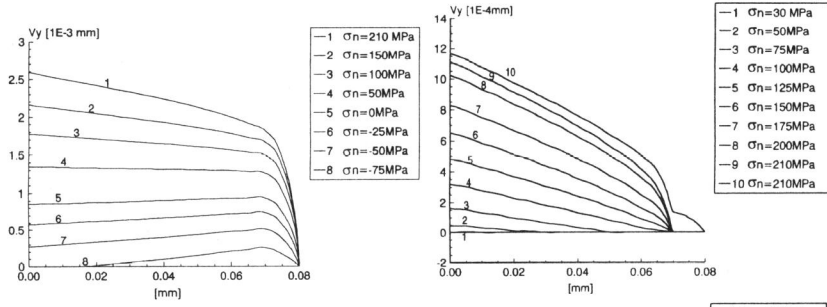


Fig. 1: Crack opening with loading and unloading ( $\rho=0.25\text{mm}$ )

Fig. 2: Crack opening with loading and unloading after crack propagation at maximum loading ( $\rho=0.25\text{mm}$ )

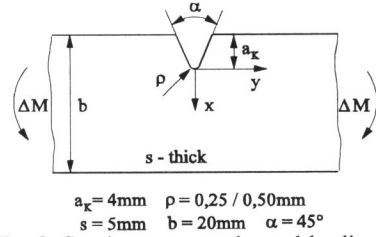
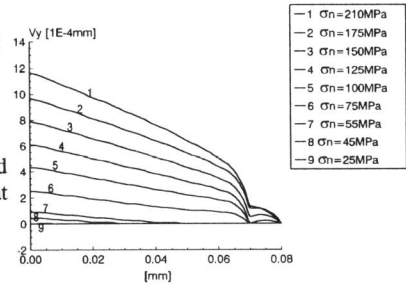


Fig. 3: Specimen geometries and loading

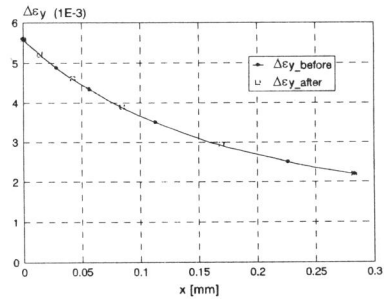
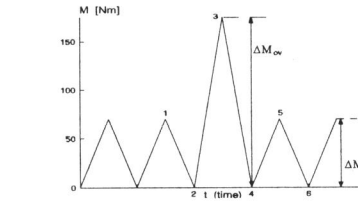


Fig. 4: Strain oscillation range before and after overload (plane strain,  $\rho=0.5\text{mm}$ )

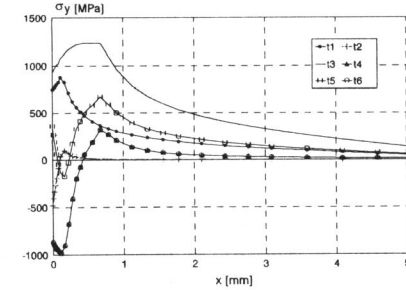


Fig. 5: Stress distribution in the ligament ahead the crack tip (plane strain,  $\rho=0.5\text{mm}$ )