THE CHARACTERIZATION OF MIXED–MODE CRACK TIP LOADING BY NEAR-FIELD DISPLACEMENTS ON DAMAGE MECHANICS MODELS

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An attempt is made to characterize the critical states of a cracked plate under mixed mode I/II loading by defined opening and sliding near field displacements $\delta_{o,c}$ and $\delta_{s,c}$ which are equivalent to the critical states of the void growth model of Rice/Tracey or the continuum damage mechanics model of Wang in the relevant element close to the crack tip. Since both the models describe the critical states by critical combinations of the effective plastic strain and the stress triaxiality the derived displacements depend on the current constraint. The analysis presented bases on Shih's HRR-field solution for mixed-mode I/II. To demonstrate the influence of constraint the stress triaxiality is varied realizing the effect of a variable T-stress according to the J-Q-field of o'Dowd/Shih.

INTRODUCTION

Systematic investigations connecting experimental results with Finite-Element analysis have shown that ductile fracture on a macroscopic scale can be put down to reaching critical combinations of the effective plastic strain p and the triaxiality of the stress state usually defined by the ratio $h = \sigma_m/\sigma_{eq}$, where σ_m is the hydrostatic stress and σ_{eq} is the von Mises equivalent stress. The microstructural phenomena on this stage are characterized by the coalescence of micro-voids or micro-cracks. The empirically found correlation corresponds to various micromechanical and continuum damage mechanics models (CDMM), such as the void growth model (VGM) of Rice and Tracey (1) or the CDMM of Wang (2).

Considering cracked structures fracture initiation is determined by critical couples of the strain p and the constraint parameter h acting over a significant interval in the near field and represented by the states in an element at a certain distance r_c from the crack tip.

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Reflecting the experimentally and theoretically supported assumption of an essentially proportional relation between crack tip displacements δ and the effective plastic strain p at a fixed distance r_c from crack tip it seems to be a well-founded alternative to describe the critical local states indirectly on an extended CTOD-concept including the dependence on stress triaxiality.

GENERALIZATION OF THE CTOD-CONCEPT FOR MIXED-MODE I/II

To get a consistent definition of crack tip displacements for the opening and sliding effects of Mixed-Mode loading we refer to the displacements on the line $\theta = \pm\pi/2$ to the crack axis. Analogous to the definition of Rice (3) the reference points for the displacements may be determined by equating the length of the vector $\boldsymbol{u}$ with its distance r to the crack tip

$$\theta = +\pi/2: \qquad |\boldsymbol{u}^*| = \sqrt{u_{r+}^{*2} + u_{\theta+}^{*2}} = r^* \tag{1}$$

$$\theta = -\pi/2: \qquad |\boldsymbol{u}^*| = \sqrt{u_{r-}^{*2} + u_{\theta-}^{*2}} = r^* \tag{2}$$

The mode assigned crack tip displacements are obtained summarizing the components for $\theta = +\pi/2$ and $\theta = -\pi/2$: $\delta_o = u_{r+}^* + u_{r-}^*$ and $\delta_s = u_{\theta+}^* + u_{\theta-}^*$.

In the context of Shih's HRR-field solution for Mixed-Mode (4) the opening and sliding displacements δ_o and δ_s are given by

$$\frac{\delta_o}{J/\sigma_0} = \frac{u_{r+}^* + u_{r-}^*}{J/\sigma_0} = \hat{d}_{no} = (\alpha\varepsilon_0)^{1/n}\hat{D}_{no} \tag{3}$$

$$\frac{\delta_s}{J/\sigma_0} = \frac{u_{\theta+}^* + u_{\theta-}^*}{J/\sigma_0} = \hat{d}_{ns} = (\alpha\varepsilon_0)^{1/n}\hat{D}_{ns} \tag{4}$$

where

$$\hat{D}_{no} = \frac{1}{I_n}\left[\tilde{u}_{r+}\left(\tilde{u}_{r+}^2 + \tilde{u}_{\theta+}^2\right)^{1/(2n)} + \tilde{u}_{r-}\left(\tilde{u}_{r-}^2 + \tilde{u}_{\theta-}^2\right)^{1/(2n)}\right] \tag{5}$$

$$\hat{D}_{ns} = \frac{1}{I_n}\left[\tilde{u}_{\theta+}\left(\tilde{u}_{r+}^2 + \tilde{u}_{\theta+}^2\right)^{1/(2n)} + \tilde{u}_{\theta-}\left(\tilde{u}_{r-}^2 + \tilde{u}_{\theta-}^2\right)^{1/(2n)}\right] \tag{6}$$

The angular functions $\tilde{u}_i$ and the constant I_n both depending on the hardening exponent n and the mixity factor M_p are listed in (5).

CRACK TIP DISPLACEMENTS ON DAMAGE MECHANICS MODELS

The critical states of the VGM of Rice and Tracey (1) modified by Wang (6) to cover the influence of void interaction reads

$$p_c \cdot \exp(k\, h_c) = W_{VG,c} = \text{const.} \tag{7}$$

A quite similar relation is given by Wang's CDMM (2)

$$p_c^{\varphi} \cdot f(h_c) \quad = \quad W_{CD,c} \quad = \quad \text{const.} \tag{8}$$

where $f(h_c) = 2(1+\nu)/3 + 3(1-2\nu)h_c^2$. In Equ. (7) and (8) is p_c the critical effective plastic strain, $h_c = (\sigma_m/\sigma_{eq})_c$ the critical constraint parameter, ν the Poisson ratio. The quantities $W_{VG,c}$ and $W_{CD,c}$ denotes the critical void growth and damage constants, and k or φ respectively are material constants usually established by use of circumferencially notched round specimen.

Referring to the HRR-field solution of Shih (4) for mixed mode I/II loading the effective plastic strain is given by

$$p \quad = \quad \frac{\alpha \varepsilon_0 \tilde{p}(\theta^*)}{(\alpha \varepsilon_0 I_n)^{n/(n+1)}} \cdot \left(\frac{J/\sigma_0}{r}\right)^{n/(n+1)} \tag{9}$$

where θ^* gives the direction for which $\tilde{p}_c \cdot exp(k\, h_c)$ or $\tilde{p}_c^{\varphi} \cdot f(h_c)$ becomes maximum. The load quantity J/σ_0 can be substituted by Equ. (3) and (4) leading to

$$p \quad = \quad \tilde{p}(\theta^*) \cdot \left(\frac{\delta_o}{\hat{\Gamma}_{no}\, r}\right)^{n/(n+1)} \tag{10}$$

$$p \quad = \quad \tilde{p}(\theta^*) \cdot \left(\frac{\delta_s}{\hat{\Gamma}_{ns}\, r}\right)^{n/(n+1)} \tag{11}$$

where $\hat{\Gamma}_{no} = \hat{D}_{no} \cdot I_n$ and $\hat{\Gamma}_{ns} = \hat{D}_{ns} \cdot I_n$. It should be noted that Equ. (10,11) with respect to the correlation between near tip displacement and strain have not to be restricted on small scale yielding as this is postulated for Equ. (9). Inserting Equ. (10,11) into (7) and assuming that $\delta = \delta_c$ if $p = p_c$ and $h = h_c$ gives

$$\delta_{o,c} \exp(k_n\, h_c)\, \frac{\tilde{p}^{(n+1)/n}(\theta^*)}{\hat{\Gamma}_{n,o}} \quad = \quad r_c \cdot W_{VG,c}^{(n+1)/n} \tag{12}$$

$$\delta_{s,c} \exp(k_n\, h_c)\, \frac{\tilde{p}^{(n+1)/n}(\theta^*)}{\hat{\Gamma}_{n,s}} \quad = \quad r_c \cdot W_{VG,c}^{(n+1)/n} \tag{13}$$

where $k_n = k(n+1)/n$. On the assumption that the distance r_c is a microstructural length the right side of Equ. (12) and (13) becomes constant for the material considered

$$\delta_{o,c} \exp(k_n\, h_c)\, \frac{\tilde{p}^{(n+1)/n}(\theta^*)}{\hat{\Gamma}_{n,o}} \quad = \quad \phi_{VG} \tag{14}$$

$$\delta_{s,c} \exp(k_n\, h_c)\, \frac{\tilde{p}^{(n+1)/n}(\theta^*)}{\hat{\Gamma}_{n,s}} \quad = \quad \phi_{VG} \tag{15}$$

On Wang's CDMM the same procedure leads to

$$\delta_{o,c} f^{1/\varphi_n}(h_c) \frac{\tilde{p}^{(n+1)/n}(\theta^*)}{\Gamma_{n,o}} = r_c \cdot W_{CD,c}^{(n+1)/(\varphi n)} = \phi_{CD} \tag{16}$$

$$\delta_{s,c} f^{1/\varphi_n}(h_c) \frac{\tilde{p}^{(n+1)/n}(\theta^*)}{\Gamma_{n,s}} = r_c \cdot W_{CD,c}^{(n+1)/(\varphi n)} = \phi_{CD} \tag{17}$$

where $\varphi_n = \varphi n/(n+1)$.

The quantities ϕ_{VG} and ϕ_{CD} can be determined on the results of an experimentally investigated cracked specimen even under Mode I loading if the material parameter k or φ and the constraint measure h at fracture initiation is known. Using the established initiation value J_i the term $\delta_{o,c}$ is given by

$$\frac{\delta_{o,c}}{\Gamma_{n,o}} = \frac{(\alpha\varepsilon_0)^{1/n}}{I_n} \cdot \frac{J_c}{\sigma_0} \tag{18}$$

in Mode I valid up to fully plastic states.

THE FRACTURE TOUGHNESS OF STEEL HY 80

To demonstrate the influence of stress triaxiality on fracture toughness formulated by Equ. (14,15) and (16,17) we refer to a steel HY 80 investigated by Sun et al. (7). The material parameters obtained are $k = 0.812$ or $\varphi = 1.092$ respectively. The hardening exponent n is about 10, the averaged yield stress σ_0 is 800 N/mm^2 and the strain-offset chosen is 0.006. The experiments with C(T)- and SE(B)-specimen characterized by a high stress triaxiality of $h_c = 2.8$ yield J_i-Values around 150 N/mm. With $I_n = 4.53$ for plane strain state Equ. (18) gives $\delta_{o,c}/\Gamma_{n,o} = 0.0248$. From the tables in (5) $\tilde{p}$ can be taken as 0.018 and solving Equ. (14) and Equ. (16) we get $\phi_{VG} = 3.635 \cdot 10^{-3}$ mm and $\phi_{CD} = 3.116 \cdot 10^{-3}$ mm.

The stress triaxiality of the HRR-field is described by the angular function

$$h = \frac{\sigma_m}{\sigma_{eq}} = \frac{\dfrac{\tilde{\sigma}_m}{(\alpha\varepsilon_0 I_n)^{1/(n+1)}} \cdot \left(\dfrac{J/\sigma_0}{r}\right)^{1/(n+1)}}{\dfrac{\tilde{\sigma}_{eq}}{(\alpha\varepsilon_0 I_n)^{1/(n+1)}} \cdot \left(\dfrac{J/\sigma_0}{r}\right)^{1/(n+1)}} = \frac{\tilde{\sigma}_m}{\tilde{\sigma}_{eq}} \tag{19}$$

but a wide spectrum of constraint can be realized by the variation of the hydrostatic stress according to the J-Q-field of Shih (7). The parameter Q describes a difference of the hydrostatic stress which can be put down to the T-stress in Mode I but seems to hold also in mixed mode loading. The change of the hydrostatic stress has no effect on the plastic strain and the near field displacements.

The crack field displacements $\delta_{o,c}$ and $\delta_{s,c}$ equivalent to the critical void growth constant W_{VG} or the critical damage constant W_{CD} in the element at r_c on the angular θ^* were calculated varying the mixed mode ratio and the triaxiality of the stress state. Fig. 1 and 2 show the critical combinations in form of interaction curves $\delta_{s,c}$ versus $\delta_{o,c}$ for h_c = const. The dashed lines refer to the unchanged HRR-field solution and depicts the variable triaxiality in dependence on the mixity factor M_e

$$M_e = \frac{2}{\pi} \tan^{-1} \left| \frac{K_I}{K_{II}} \right| \tag{20}$$

The curves reveal the increase of the critical crack field displacements with decreasing constraint. The corner around $M_e = 0.8$ is obviously put down to the discrete values ot the angular functions. Besides this unsteadiness the curves may be approximated by an ellipse little deviating from a circle.

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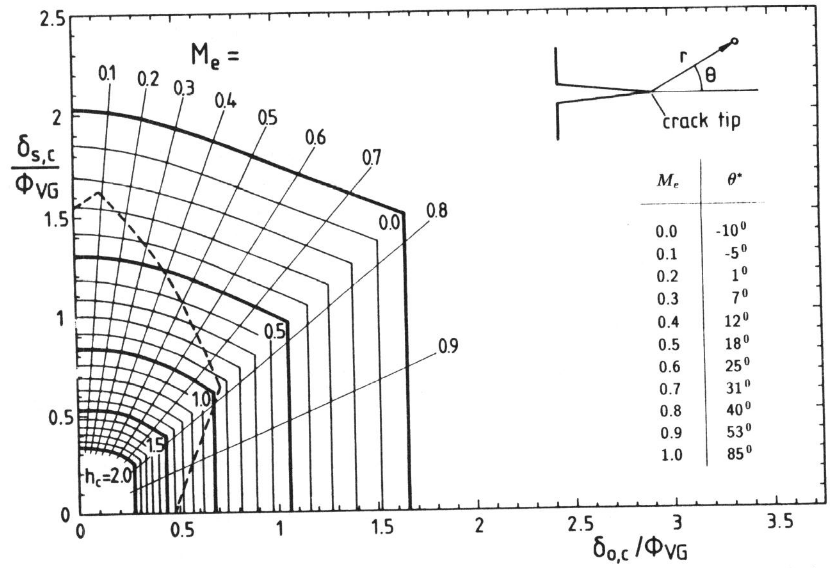

Figure 1 Critical near field displacements $\delta_{s,c}$ versus $\delta_{o,c}$ on the basis of the void growth model of Rice and Tracey

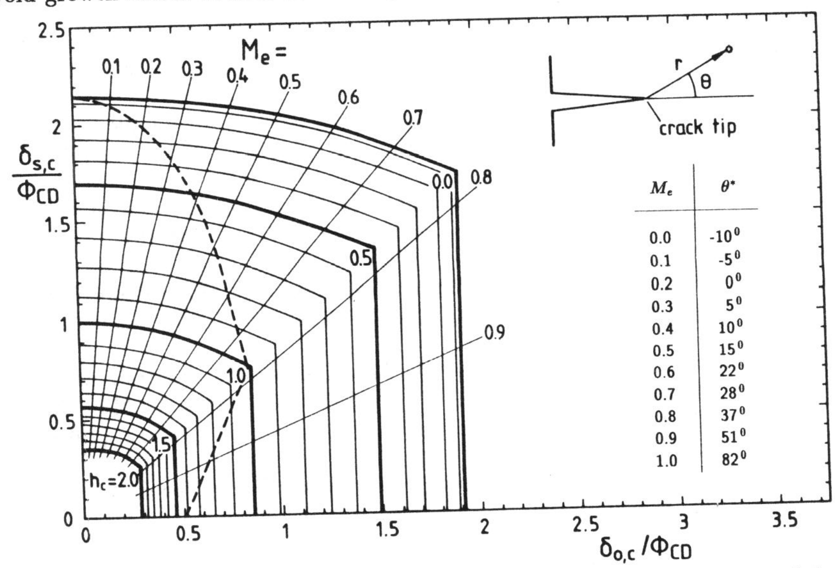

Figure 2 Critical near field displacements $\delta_{s,c}$ versus $\delta_{o,c}$ on the basis of the continuum damage mechanics model of Wang