THE SOLID PARTICLE IMPACT EROSION OF BRITTLE SOLIDS

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An approach based on the system of fixed material constants describing macro-strength properties of the material is considered. The corresponding incubation time criterion allows one to manage without the a priori given rate dependences of dynamic strength and fracture toughness. The problem of erosion on the basis of incubation time concept is analyzed.

BASIC STRUCTURAL CHARACTERISTIC OF STATIC FRACTURE

One of the principal parameters of linear fracture mechanics is the material structure size d describing the elementary cell of failure. The elementary cell of fracture has no unique physical interpretation. It may be interpreted in various ways, depending on the class of problems. The corresponding Griffith-Irwin criterion is a universally recognized critical condition of brittle and quasi-brittle fracture. This criterion is based on the square root singularity of the stress field at the crack tip. Therefore its field of applicability is limited. For instance, in the case of an angular notch in a plate, the general energy balance equation can not be satisfied for all methods of loading (Morozov (1)).

Novozhilov (2) suggested to consider the material structure directly. The corresponding criterion requires that the mean normal stress in the range of material structure size d must be equal to the static strength of the material. In the plane deformation state case we have:

$$\frac{1}{d}\int_0^d \sigma(r)\,dr \le \sigma_c \qquad (1)$$

Assuming that in the simplest cases the criterion (1) gives the same results as the Irwin's critical stress intensity factor criterion, we obtain for the material structure size d the expression:

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$$d = \frac{2}{\pi}\frac{K_{Ic}^{2}}{\sigma_c^{2}} \qquad (2)$$

Criterion (1) may be used in various cases in which the square root singularity and the appropriate energy balance do not work. Results obtained by means of the criterion (1) under the condition (2) are well confirmed by experiments in static cases (Morozov (1)).

DYNAMIC FRACTURE CRITERION

Analysis of the dynamic experiments shows that the main contradictions of the traditional models appear when failure occurs during the short time intervals after the start of the loading process. Morozov and Petrov (3) proposed an approach to the analysis of dynamic brittle failure based on the incubation time criterion:

$$\frac{1}{\tau}\int_{t-\tau}^{t}\frac{1}{d}\int_{0}^{d}\sigma(r,\theta,t')\,dr\,dt' \le \sigma_c \qquad (3)$$

Here d and τ are material structure size and structure time of failure respectively, σ_c is static strength of the material, (r,θ) are polar coordinates, $\sigma(r,\theta,\tau)$ is tensile stress at the crack tip ($r=0$). The material structure size d is to be determined in accordance with the data of quasi-static tests of specimens containing cracks and in the case of plane strain it may be expressed by the simple formula (2). The material structure time τ is responsible for the dynamic peculiarities of the macro-fracture process, and for each material it should be found from experiments. In accordance with this approach σ_c, K_{Ic} and τ constitute the system of fixed material constants describing macro-strength properties of the material (see Petrov and Morozov (4)). Petrov (5) has shown that the criterion (3) reflects the discrete nature of dynamic fracture of brittle solids.

In the case of virgin materials, the criterion (3) reduces to the form:

$$\frac{1}{\tau}\int_{t-\tau}^{t}\sigma(t')\,dt' \le \sigma_c \qquad (4)$$

This form will now be used for the analysis of a particular problem.

The analysis of the particular problems of dynamic fracture mechanics is associated with the appropriate choice of the parameter τ. We shall here consider the case when the incubation time is defined by the material structure size of fracture:

$$\tau = \frac{d}{c} = \frac{d\sqrt{\rho}}{k}, \qquad (5)$$

where c is the maximum wave velocity, ρ is the density of continuum, k is the constant depending on the deformation material properties. According to this definition the incubation time has a physical meaning of the minimum time period required for the interaction between two neighboring material structure cells. The incubation-time criterion with the parameter τ selected according to the formula (5) allows us to describe effectively the time dependence of strength and the fracture zone geometry in conditions of spalling (Morozov et al. (6)). Thus, the definition (5) provides a good analogy between the incubation time criterion and the well-known experiments in the case of "defectless" materials.

APPLICATION TO THE PROBLEM OF EROSION

The solid particle impact velocity at the beginning of target material loss in the steady state erosion process can be considered as a critical or threshold velocity. It is a principal characteristic that bears an information about dynamic strength properties of materials subjected to the impact loading. In this paper the relation between the threshold velocity W and the incubation time τ is investigated. The possibility of using the incubation time criterion in determining the threshold erosion characteristics is established.

One of the principal features of the erosion process is that the target material surface is subjected to extremely short impact actions. The evaluation of failure in these conditions may be done only on the basis of special criteria reflecting the specific nature of fast fracture phenomenon. The incubation time criterion (4) is an effective instrument for this analysis. Here we shall consider the simplest way to obtain some of the basic threshold erosion characteristics.

Let a spherical particle of radius R fall with velocity v on the surface of an elastic half-space (see Figure 1). Using the classical Hertz impact theory approximation (see Kolesnikov and Morozov (7)), we describe the motion of the particle by the following equation:

$$m\frac{d^2h}{dt^2} = -P, \qquad (6)$$

where:

$$P(t) = k(R)h^{3/2}(t), \qquad k(R) = \frac{4}{3}\sqrt{R}\frac{E}{(1-\nu^2)} \quad \text{...(7)}$$

At the beginning of the impact event we have $dh/dt=v$. The maximum penetration h_0 occurs when $dh/dt=0$. Solving the equation (6), we obtain:

$$h_0(v,R) = \left(\frac{5mv^2}{4k}\right)^{2/5}, \qquad t_0(v,R) = \frac{2h_0}{v}\int_0^1 \frac{d\gamma}{\sqrt{1-\gamma^{5/2}}} = 2{,}94\frac{h_0}{v} \quad \text{....................(8)}$$

where t_0 is the duration of the impact event. The penetration function $h(t)$ can be approximated by the simple formula (Kolesnikov and Morozov (7)):

$$h(t) = h_0 \sin(\pi\, t / t_0) \quad \text{...(9)}$$

The maximum tensile stress occurring at the edge of the contact area is given by the expression (Lawn and Wilshaw (8)):

$$\sigma(t,v,R) = \frac{1-2\nu}{2}\frac{P(t,v,R)}{\pi\, a^2(t,v,R)}, \qquad a(t,v,R) = \left[3P(t,v,R)(1-\nu^2)\frac{R}{4E}\right]^{1/3} \quad \text{...(10)}$$

where the contact force $P(t,v,R)$ can be found by means of equations (7)-(9).

Let $v=W$ denote the threshold velocity corresponding to the beginning of failure. We consider the function:

$$f(\tau,v,R) = \max_t \int_{t-\tau}^{t} \sigma(s,v,R)ds - \sigma_c\tau \quad \text{...(11)}$$

According to (4), we determine the threshold velocity $v=W$ as the minimum positive root of the equation:

$$f(t,v,R)=0 \quad \text{...(12)}$$

where τ is the incubation time for the target material.

The corresponding calculations were performed for the aluminum alloy B95 and the incubation time was determined according to formulae (2) and (5): $\sigma_c = 460MPa$, $K_{Ic} = 37MPa\sqrt{m}$, $c = 6500m/s$, $\tau = 2K_{Ic}^2/(\pi\sigma_c^2 c) \approx 0.6\,\mu s$ The calculated dependence of the threshold velocity W on the value of radius R is presented in the Figure 2 by the solid curve. Its static branch shows a weak dependence of the threshold velocity on the value of radius. On the contrary, the dynamic branch, corresponding to the small particles and very short loading pulses, represents a strong dependence of the critical velocity on the radius of particles. This behavior of the threshold velocity is observed in numerous experiments (see, for instance, Polezhaev (9)), but it can not be explained on the basis of the traditional fracture mechanics. The dependence following from the conventional critical stress theory is also presented on the Figure 2 by dashed line.

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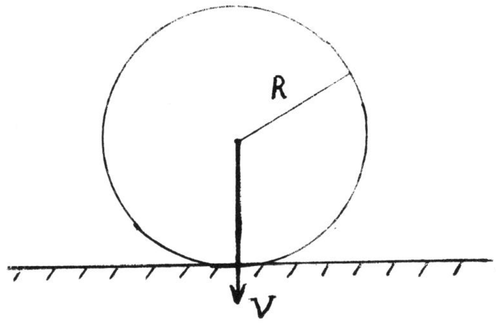

Figure 1 Dynamic interraction between the solid particle and the half-space.

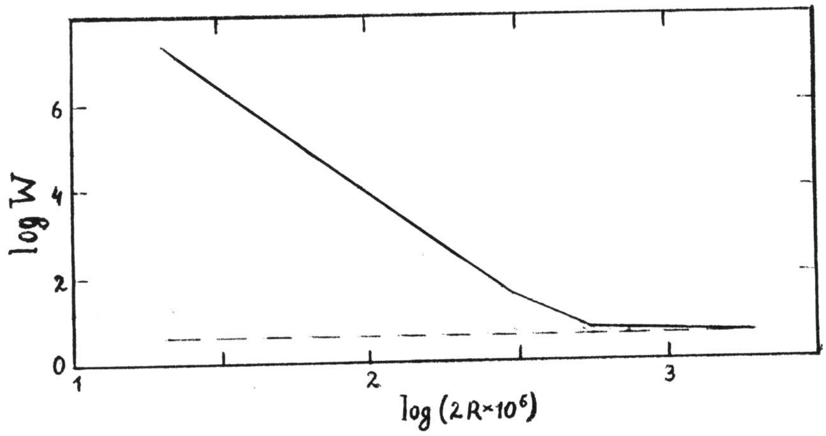

Figure 2. Dependence of the threshold velocity W(m/s) on the value of radius R(m) of erodent particles calculated for aluminium alloy B95. The dependence corresponding to the classic fracture criterion: $\sigma \leq \sigma_c$ is plotted by dashed line.