STRUCTURAL INTEGRITY ASSESSMENT VIA THE UNIFIED THEORY OF FRACTURE IN TENSION AND/OR COMPRESSION

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This paper continues the development of the so-called unified methodology (UM) for comprehensive failure assessment of safety-critical components. The UM permits to correlate in a common manner all kinds of crack response between the two extreme possibilities, crack tip failure under biaxial tension and the same under biaxial compression. The methodology proposed emerges from harmonizing a crack model with an actual crack and they both taken together with a fracture model, laboratory test method and failure assessment procedure. Pre-existing version of the UM is simplified and extended to cover the case of internally cracked finite plate under biaxial loading.

INTRODUCTION

The two-criteria approach to structural integrity assessment was introduced in the framework of the so-called R6 procedure (Harrison et al (1), Milne et al (2)). This approach relates to stress intensity factor, K_I, J_I-integral or CTOD and to standards like ASTM standards E399, E813 or E1290. As a consequence, the effects of load biaxiality and constraint would be included in the failure assessment procedure. However, the available version of such modifications, e.g. Shao and Tan (3), Ainsworth and O'Dowd (4), are not adequate for analyzing mode I fracture under the action of tensile and compressive stress fields with equal facility.

The term UM implies that the effects of loading, crack and component sizes, boundary constraint and preloading history, are evaluated in the context of a single conception. As such, the so-called ρ-theory of brittle fracture (Naumenko (5) and Naumenko (6)) is used in what follows.

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PARAMETERS OF CRACK AND FRACTURE CRITERION

The problem in question is treated as structural integrity assessment of a finite rectangular plate, length $= 2H$ and width $= 2W$, subjected to uniform biaxial loading (Fig.1). Both loads, σ and $q = k\sigma$, may be either tensile (positive), or compressive (negative). Through-the-thickness crack, length $= 2c_u$, is defined as an actual one when the case in point is testing of a laboratory specimen. In fracture analysis the term crack is referred to as an ideal crack, length $= 2l_u$. Index "u" denotes the undeformed state characterized by condition: $\sigma = q = 0$, $\sigma_r(x, y) \neq 0$ and $q_r(x, y) \neq 0$, where σ_r and q_r are residual stresses induced by transformations of a material at the preceding stages of crack extension. In the following, we anticipate that the beginning of nonlinear behaviour of the plate (state "e") came after the start of crack extension (state "i").

For simplicity assume that actual cracks in the so-called basic specimens (6) and postulated crack in the plate are equal in length and all other things, including precracking conditions, are the same. Actual crack parameters for a stress-free specimen are evaluated through measuring an elastic response of the crack border at uniaxial tension $(k = 0)$ and then at uniaxial compression $(k = -\infty)$. The relevant procedure is outlined in (6). As a result of this procedure being performed we have the maximum values of initial displacements (see Fig.1): $\upsilon_u(c, k = 0, x = 0)$; $\upsilon_u(c, k = -\infty, x = 0)$; $u_u(c, k = 0, x = c)$ and $u_u(c, k = -\infty, x = c)$, where υ denotes transverse and u - longitudinal displacements of the crack border.

The displacements of an ideal crack represented by an elliptical hole are determined by the following expressions:

$$\upsilon(l, k, x) = F_\upsilon(\alpha, \beta)\left[1 + 2\left(\frac{l}{\rho}\right)^{1/2} - k\right]\left[\frac{\rho(l^2 - x^2)}{l}\right]^{1/2} \cdot \frac{\sigma}{E^*}, \qquad (1)$$

$$u(l, k, x) = F_u(\alpha, \beta)\left[k + 2k\left(\frac{\rho}{l}\right)^{1/2} - 1\right] \cdot x \cdot \frac{\sigma}{E^*}, \qquad (2)$$

where F_υ and F_u are the correction factors depending only on the dimensionless geometric parameters $\alpha = l / W$ and $\beta = H / W$, $E^* = E$ for hole-tip plane stress conditions and $E^* = E / (1 - \nu^2)$ for plane strain conditions, ν is Poisson's ratio. The quantities l, ρ, F_υ and F_u are defined from the following equalities: $\upsilon_u(l,0,0) = \upsilon_u(c,0,0)$; $u_u(l,0,l) = u_u(c,0,c)$; $\upsilon_u(l,-\infty,0) = \upsilon_u(c,-\infty,0)$ and $u_u(l,-\infty,l) = = u_u(c,-\infty,c)$. They imply that ideal crack displacements coincide with the relevant values for an actual crack at the points: $(x/c_u) = 1$ and $(x/l_u) = 1$; $(x/c_u) = (c_u - r_{pu})/c_u$ and $(x/l_u) = (l_u - r_{lu})/l_u$; $x = 0$. The other parameters of an ideal crack can be computed as follows: $r_l = l - x_l; \delta_l = \delta_{pu} - 2F_\upsilon(\alpha,\beta)\left[2 + (\rho/l)^{1/2}\right](l^2 - x_l^2)^{1/2} \cdot (\sigma_{ru}/E^*)$; $x_l = l\left[1 - \delta_l^2/4\rho l\right]^{1/2}$, where σ_{ru} is the uniform tensile stress that is equivalent to the local stresses $\sigma_r(x,y)$ and $q_r(x,y)$. The values of σ_{ru} and δ_{pu} are estimated by an experimentation as outlined in (6).

As applied to the plate in Fig.1, the criterion of crack growth initiation takes the form,

$$\rho_i = \rho \frac{\left\{1 + F_\upsilon(\alpha,\beta)\left[1 + 2(l/\rho)^{1/2} - k\right]\sigma_i / E^*\right\}^2}{1 + F_u(\alpha,\beta)\left[k + 2k(\rho/l)^{1/2} - 1\right]\sigma_i / E^*} \qquad (3)$$

The value $\rho_i = \rho_{mat}$ is treated as an "inherent material property". In other words, it is implied that the start of crack-extension occurs at the same value of ρ_i irrespective of the crack length, signs of loads and k-ratio. Fracture parameters, such as K_I and J_I-integral, have many uses, among which is failure assessment of components. In the present context we shall restrict our consideration to the increment of the radius of a hole at its ends, ρ_I. This fracture parameter is determined as $\rho_I = \rho C_l(\sigma/E^*)^2$, where $C_l = (\sigma_l/\sigma) = F_\upsilon(\alpha,\beta)\left[1 + 2(l/\rho)^{1/2} - k\right]$ is the stress concentration factor at hole ends and σ_l is the local stress.

STRUCTURAL INTEGRITY ASSESSMENT

In the ρ-theory failure assessment diagram (FAD) approach, fracture is assessed in terms of $R_\rho = (\rho_I/\rho_{mat})^{1/2}$ and $S_\rho = \sigma/\sigma_{mat}$. Material's property, $\sigma_{mat} = 2F_\upsilon(\alpha,\beta)(l/\rho)^{1/2} \cdot \sigma_i(l, k=1)$, relates to the start of crack-extension. It is equal to a local tensile stress, σ_l, under equibiaxial tension. R_ρ vs S_ρ relationship takes the form:

$$R_\rho = S_\rho C_l \left[1 + \frac{F_u(\alpha,\beta)}{F_\upsilon(\alpha,\beta)} \cdot \frac{\rho}{l} \cdot \varepsilon_{mat}\right]^{1/2} \left(\frac{\varepsilon_{mat}}{1+\varepsilon_{mat}}\right) \quad (4)$$

where $\varepsilon_{mat} = \sigma_{mat} / E^*$. Straight-line FADs (Fig.2) corresponding to acceptable cracks are determined by the following expressions:

$$S_{\rho\min} = -q_i(l,-\infty) / 2\left(\frac{l}{\rho}\right)^{1/2} \left[\sigma_{mat} + F_\upsilon(\alpha,\beta) \cdot q_i(l,-\infty)\right], \quad (5)$$

$$S_{\rho\max} = \left[4F_\upsilon^2(\alpha,\beta)\frac{l}{\rho}\right]^{-1/2}, \quad (6)$$

$$R_{\rho\sigma\max} = \left[1 + \frac{F_u(\alpha,\beta)}{F_\upsilon(\alpha,\beta)} \cdot \frac{\rho}{l} \cdot \varepsilon_{mat}\right]^{1/2} \left(\frac{\varepsilon_{mat}}{1+\varepsilon_{mat}}\right) \quad (7)$$

where σ_{mat} and $q_i(l,-\infty)$ are calculated from Eq. (3). Some preliminary results for a plate made of PMMA (Naumenko (7)) set a lower limit on the ideal crack length as $l_{\min} \approx 80\rho$. A possible buckling places an upper limit, $l_{\max}$, on the crack length.

By way of illustration we consider fracture initiation of a PMMA plate under plane strain conditions. Its characteristics are: $l = 40.42$ mm, $\rho \approx 12.5$ μm, $\alpha \le 0.1$, $\beta = 1$, $E^* = 3.47$ GPa, $\sigma_{mat} = 200.7$ MPa and $\rho_i \approx 14\mu$m. The relevant FAD is specified by three quantities: $S_{\rho\min} = -1.59 \cdot 10^{-2}$, $S_{\rho\max} = 0.88 \cdot 10^{-2}$ and $R_{\rho\sigma\min} = 5.47 \cdot 10^{-2}$. Local tensile stresses, σ_{li}, appropriate to the points $S_\rho = S_{\rho\min}$, 0 and $S_{\rho\max}$, are equal: 0, $0.652\sigma_{mat}$ and σ_{mat}, respectively.

The ρ-theory FAD differs essentially from the available FADs not only in shape but in significance too. Contrary to them, Eq. (4) enables failure assessment at overall compression. Once the start of crack-extension under biaxial compression is the critical event, the margin between the design stress, σ_D, and the critical stress, σ_i, can be obtained by linear projections (see Fig.2) $\sigma_i = \left(\overline{OC} / \overline{OD}\right)\sigma_D$ and $h_\sigma = (\sigma_i / \sigma_D)$, where h_σ is reserve stress factor. Safe triangle ABE is seen to grow smaller as crack elongates. It must be emphasized that the main influence factors originating from the material behaviour, loading, crack sizes, as well as boundaries constraints fit naturally into the ρ-theory FAD.

GENERAL OBSERVATIONS

In the context of a single concept of fracture the unified approach to structural integrity assessment of a cracked plate under tensile and compressive stress fields has been demonstrated. It appears as if the application of the ρ-theory presented here is viable and does capture both qualitatively and quantitatively the main features of the problem.

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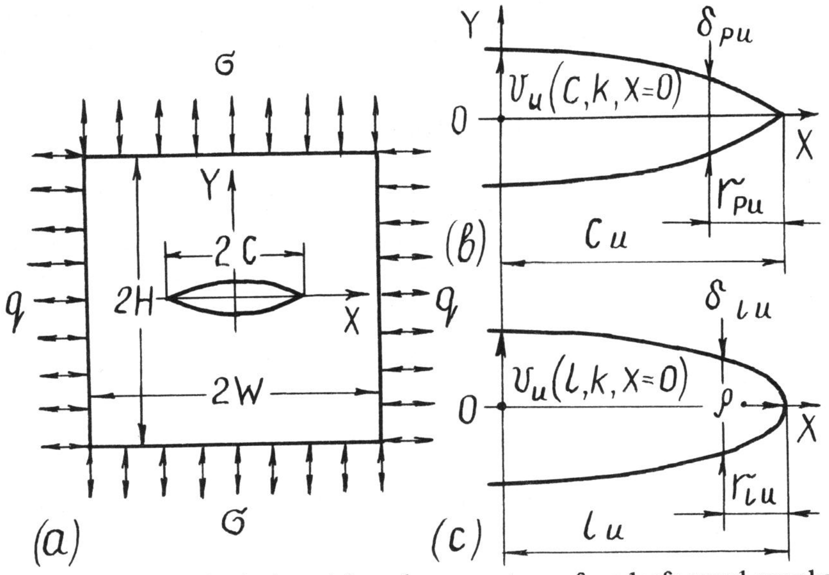

Figure 1. A cracked plate (a) and parameters of underformed cracks: (b) - actual, (c) - ideal.

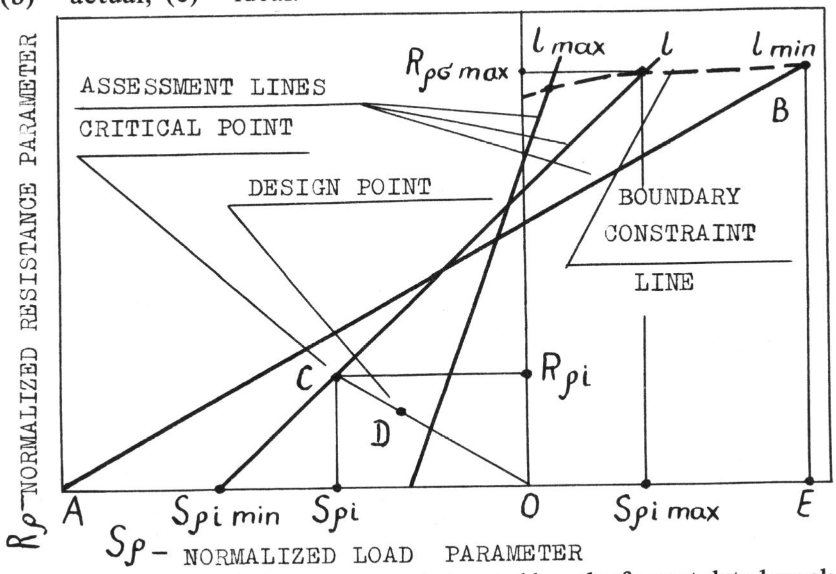

Figure 2. Scheme of failure assessment at varied length of a postulated crack.