STRESS INTENSITY FACTOR, COMPLIANCE AND INVERSE COMPLIANCE FORMULA FOR THREE-POINT BEND SPECIMENS

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An expression for stress intensity factor, load-point compliance and inverse compliance of three-point bending beams with straight through crack are presented here. The stress intensity factor expression was calculated using the well known Srawley expression, and correcting the effect of the concentrated force of the loading point in finite specimens. The compliance formula was obtained by direct integration of this stress intensity factor, using the classic Irwing method. Finally, a method is proposed for calculation of the inverse compliance, of general application for any span-width ratio.

INTRODUCTION

The three-point bend (TPB) specimen with straight through crack is a geometry widely used for the determination of elastic fracture parameters of many materials. It is usually employed with a span-width (S/W) ratio equal to four according to the standards (1), and sometimes with S/W= 8.

This paper gives new high accuracy expressions for the stress intensity factor, load-point compliance, and inverse compliance for the standard three-point-bend specimen, with span-width ratio of four as a function of the crack length (a). The compliance results are compared with others available in the literature, and with the data obtained from a modelization by finite elements. An inverse compliance expression was also fit to S/W=4; the proposed method is absolutely general and valid for any span-width ratio.

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STRESS INTENSITY FACTOR DETERMINATION

The well known expression for the stress intensity factor for three-point-bend specimens with S/W = 4 given by Srawley (2) overestimates the value of the stress intensity factor for short cracks (Pastor *et al.* (3), Bakker (4)) since it do not account for the disturbance of the stress distribution due to the concentrated force. This error is far larger than the accuracy claimed for this expression (0.5 percent).

A more accurate expression for this stress intensity factor can be obtained taking the original points obtained by Srawley (2) for relative crack depth (α=a/W) greater than 0.2, and modifying the value for α=0 according to the suitable stress distribution along the central cross-section due to the concentred load, P, proposed by Thimoshenko and Godier (5). With this correction a new value for α=0 has been derived and a new stress intensity factor, K_I, fitting was carried out:

$$K_I = \frac{6}{W^{3/2}B}\frac{PS}{4}\alpha^{1/2}Y(\alpha) \tag{1}$$

where B is the specimen thickness, and $Y(\alpha)$ a geometric dimensionless function given by.

$$Y(\alpha) = \frac{1.945 - 1.256\alpha^2 + 3.300\alpha^3 - 3.225 + 1.224\alpha^4)}{(1+2\alpha)(1-\alpha)^{3/2}} \tag{2}$$

This equation is valid for the entire range of the relative crack depth, $0 \leq \alpha \leq 1$, with a maximum deviation from the original points of ± 0.07 %. Figure 1 shows a graphical representation of the stress intensity fitting in relation to the original points, and their relative difference.

LOAD-LINE DISPLACEMENT DETERMINATION

The relation between the compliance and the stress intensity factor can be obtained using the energetic approach to linear elastic frecture mechanics. The available energy per unit increase in crack surface area, G, is given by:

$$G = \frac{K^2}{E} = \frac{P^2}{2WB}\frac{dC}{d\alpha} \tag{3}$$

where C is the compliance of the TPB specimen, and E' is the generalizated elastic modulus (E' = E for plane stress, and E' = E/(1 – ν^2) for plane strain).

An alternative expression for the specimen compliance can be written as:

$$C = \frac{u - u_0}{P} = \frac{3S^2}{2BE'W^2} V(\alpha) \tag{4}$$

where $u - u_0$ is the load-line displacement due only to the presence of the crack, and $V(\alpha)$ is a nondimensional function depending only on the specimen geometry. To evaluate $V(\alpha)$ one can make use of equations (3) and (4):

$$\frac{dV(\alpha)}{d\alpha} = 3\alpha Y(\alpha) \tag{5}$$

and by integration obtain:

$$V(\alpha) = 3\int_0^{\alpha} \alpha Y(\alpha) d\alpha = 24\left\{3.5182 - \frac{0.4774}{(1\alpha)} - \frac{0.6594}{(1\alpha)2} + \frac{3.3361}{(1+2\alpha)} - \right.$$
$$8.6930\alpha + 3.2375\alpha_2 - 2.1308\alpha^3 + 0.9190\alpha^4 - \tag{6}$$
$$\left. 0.2242\alpha^5 - 1.2377\ln(1-\alpha) + 6.6432\ln(1+2\alpha)\right\}$$

This result is compared in Figs. 2 and 3 with those proposed by Chen Chich *et al.* (6), Wu (7) and Tada *et al.* (8), and with a modelization by finite element method (FEM) made by the authors (9).

Our results agree quite well with the results of Wu, except for $\alpha<0.2$. This is because Wu did not consider the correction due to the concentrated force. The FEM results show a constant difference of about 2 %. On the other hand, the Tada equation diverges from the others, as already reported Underwood *et al.* (10). The divergence of Chen´s values for $\alpha<0.45$ and $\alpha>0.75$ suggest that Chen's expression does not fit the right asymptotic behaviour.

INVERSE COMPLIANCE CALCULATION METHOD

For three-point bend specimens, the crack length may be computed from crack mouth opening displacement (CMOD) measurements by the elastic compliance method. Nevertheless, this method is not always suitable when working with very small samples, at high temperatures or in aggressive environments. In these circumstances, the crack length from the load-point displacement, u, or from the specimen compliance, C, could be computed.

An inverse compliance expression may be derived from equation (6) by setting:

$$\alpha = \frac{\alpha}{W} = A_0(x)A_1(x) \qquad \text{with} \tag{7}$$

$$A_0(x) = \frac{\sqrt{\dfrac{x}{p_0}}}{1+\sqrt{\dfrac{x}{p_0}}} \qquad \text{and} \tag{8}$$

$$A_1(x) = 1 + \frac{x_c}{d(1+f\,x_g)^h} \tag{9}$$

where $x = V(\alpha)$, and the coefficients (p_0, c, d, f, g, h) are fitted to obtain the same values of pairs (α, $C(\alpha)$) form predicted by equation (4).

The following expression is then derived for three-point bend specimens with a span width ratio of four,

$$\alpha = \frac{\sqrt{\dfrac{x}{5.415}}}{1+\sqrt{\dfrac{x}{5.415}}}\left(1+\frac{x^{0.5246}}{1.184(1+0.7770x^{0.8269})^{1.1186}}\right) \tag{10}$$

This fitting was found to be accurate to within less than 0.3 percent of the original values for any relative crack length. The relative error between the original and the computed value for α is displayed in Fig. 3.

The results described here are suitable for general use in fracture testing using TPB specimens, including fatigue crack growth rate tests, and unloading compliance J_{IC}. This method can also be used in the determination of the crack length from the load-line displacement for any span-width relation with similar accuracy (9).

CONCLUSIONS

This paper gives a corrected fitting of the stress intensity factor for the of three-point bend specimens with span-width ratio of 4 proposed by Srawley (2).

This fitting has proved more accurate than ±0.07 % in the entire relative crack depth interval ($0 \leq \alpha \leq 1$). With this new stress intensity expression, an equation for the load point compliance has been derived by direct integration, which is accurate within one percent for any relative crack length. This expression generalised the Wu (7) and Underwood (8) equations for compliance, and should be useful for fracture testing, and for elastic crack length determination from the inverse compliance expression. Finally, presented a general method is proposed for the calculation of the inverse compliance formula with an accuracy better than 0.5 percent.

ACKNOWLEDGEMENTS

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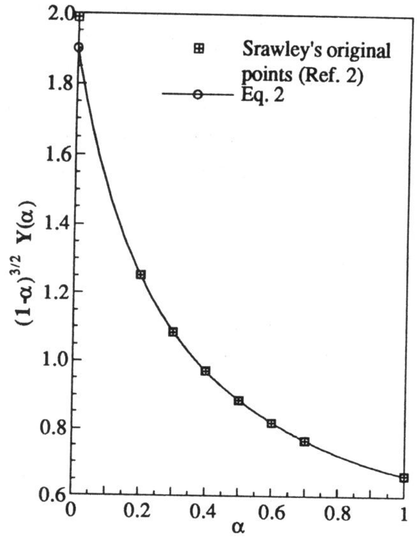

Figure 1. Dimensionles stress intensity factor versus relative crack length.

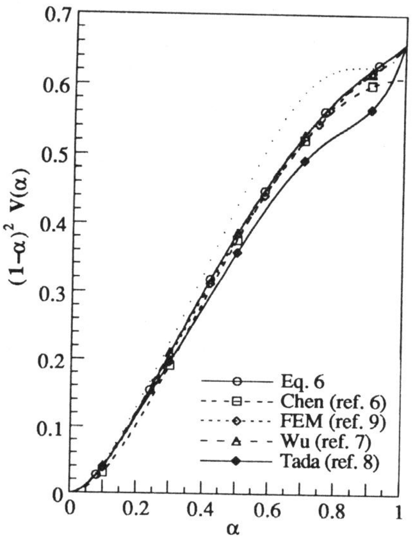

Figure 2. Dimensionless compliance versus relative crack length.

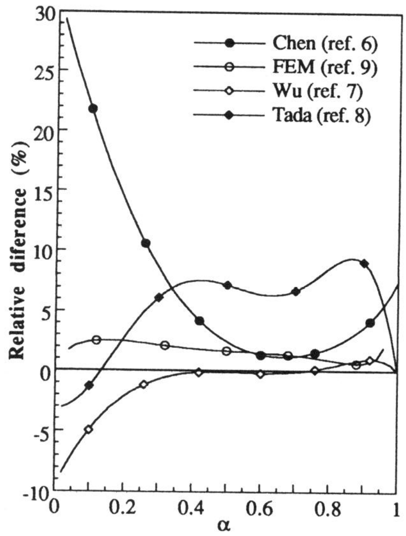

Figure 3. Relative difference between various compliance results and eq. (4).

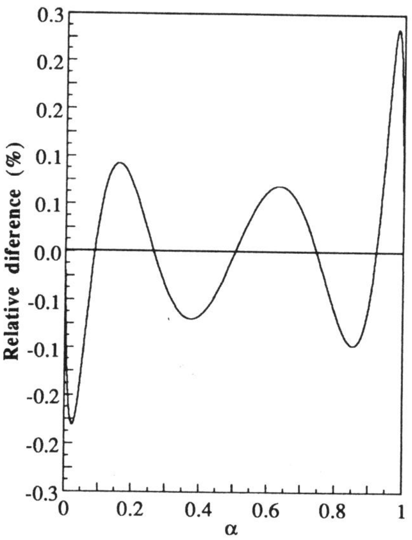

Figure 4. Relative error in the inverse determination of the crack length.