STOCHASTIC MODELING OF FATIGUE DAMAGE PROCESSES IN METALS

Krzysztof Doliński*

Random material properties and stochastic nature of many load processes make the methods of probability theory and theory of stochastic processes a very useful and rational tool in modeling of fatigue damage process, in particular in modeling of the fatigue crack growth. In the paper some results of stochastic modeling of fatigue crack propagation are presented to show how such a nondeterministic approach enables us to estimate a current value of reliability of a structural element subjected to fatigue while the material parameters are assumed to be some random variables and the loading is considered as a stochastic process with all consequences of its irregularity on the fatigue process.

1. INTRODUCTION

Random nature of material non-homogeneity was always recognized to produce a scatter of results in fatigue crack growth experiments. Even in very well-controlled experiments the fatigue damage, in particular the fatigue crack growth, under deterministic constant amplitude loading is observed to be of stochastic character, e.g. Virkler et al. (1), Ghonem and Dore (2). It inspires many researchers to apply the methods of probability theory and theory of stochastic processes in modeling of the phenomenon.

It is also quite obvious to expect the fatigue process to be of random character due to loading with stochastically varying amplitudes. Moreover, it is experimentally well documented, e.g. a review Kumar (3), that load cycles with single or multiple peak tensile overloads result in retardation of fatigue crack growth or even in crack arrest. The effects of previous cycles makes the fatigue

*Institute of Fundamental Technological Research, Centre of Mechanics
00-049 Warsaw, ul Świętokrzyska 21, POLAND

crack growth models to be load history dependent. A current increment of the crack length or of any other damage parameter depends not only on the actual load, material parameters and on the actual damage intensity (defined by the crack length or damage parameter value) but also on some current effects originated in the past. For randomly varying load amplitudes all load history effects are random both in time instants of their occurrences and in their intensity.

It is also evident that due to both randomness of material properties and stochastic nature of many kinds of real excitations the stochastic approach is the most appropriate one to model the development of fatigue damage effects. Moreover, only such an approach allows us to consider the lifetime of a structural element as a random variable and to estimate the probability of the structural failure by application of methods of structural reliability analysis.

RANDOM MATERIAL PROPERTIES

Among many experimental results which one can find in the literature on the fatigue crack growth under constant amplitude loading there are only a few that allow us to employ evolutionary methods providing an insight into stochastic nature of the fatigue crack propagation process. The most frequently investigated results were published in (1) and (2). They were obtained in tests on 2024-T3 and 7075-T6 aluminum alloy specimens, respectively. It was explicitly shown that even in well-controlled experiments under constant amplitude loading the scatter of the results cannot be negligible. The scatter originates from the randomness of the material properties around the crack tip. The Virkler and Ghonem-Dore data sets were extensively used to identify all statistical characteristics of random variables and of the random fields describing the stochastic material properties and assumed in a stochastic fatigue crack growth model proposed in Doliński (4).

The fatigue crack length increment, Δa_i, due to the i-th load cycle can be written in the following general form

$$\Delta a_i = F\left(a_i, S_i^+, S_i^- \mid \mathbf{x}\right) \tag{1}$$

where a_i denotes the current crack length at the moment of application of the i-th load cycle. The quantities S_i^+ and S_i^- denote, respectively, the stress maximum and minimum in the i-th cycle of the far-field stress applied to a cracked element. The vector $\mathbf{x} = [x_1, x_2, \ldots, x_K]$ represents the material parameters and, in general, means a sample of the random vector of random material parameters, $\mathbf{X} = [X_1, X_2, \ldots, X_K]$.

Among a great number of fatigue crack growth laws proposed in the literature, see e.g. Kocańda (5), there is a very wide class of equations for which the function $F(a_i, S_i^+, S_i^- | \mathbf{x})$ can be written as a product

$$\Delta a_i = g(a_i | \mathbf{x}) \cdot \Xi(S_i^+, S_i^-) \tag{2}$$

Such a form has the well-known fatigue crack growth equation proposed by Paris and Erdogan (6), $\Delta a_i = C \cdot \Delta K_i^m$. Since the stress intensity factor range corresponding to the i-th stress cycle is given by $\Delta K_i = Y(a_i) \cdot \Delta S_i \cdot \sqrt{\pi \cdot a_i}$ the functions $g(\cdot)$ and $\Xi(\cdot,\cdot)$ in the Paris-Erdogan equation take the following forms $g(a) = C \cdot Y^m(a) \cdot \left(\sqrt{\pi \cdot a}\right)^m$ and $\Xi(S^+, S^-) = (S^+ - S^-)^m = \Delta S^m$ with the coefficient C and the exponent m being some parameters that generally depend on material and load conditions. The dimensionless function Y(a) depends on the crack and specimen geometry. Another fatigue crack growth equation, very often used in application, was proposed in Forman et al. (7), $\Delta a_i = C \cdot \Delta K_i^n / \left[(1 - R_i) \cdot K_{fc} - \Delta K_i\right]$ where $R_i = S_i^- / S_i^+$ is the stress cycle asymmetry parameter and K_{fc} is originally considered as fracture toughness. It is known, e.g. Troščenko & Pokrovskij (8), that the fracture toughness values for monotonic loading, K_c, and in fatigue process, K_{fc}, differ from each other and the latter also depends on the load condition. Introducing consequently a critical fatigue crack length, a_{fc}, instead, and defining $K_{fc} = S^+ \cdot Y(a_{fc}) \cdot \sqrt{\pi \cdot a_{fc}}$ the Forman equation can be written in the product form (2) with the functions, $g(\cdot)$ and $\Xi(\cdot,\cdot)$, as follows (subscript "i" omitted)

$$g(a) = \frac{C \cdot Y^n(a) \cdot \left(\sqrt{\pi \cdot a}\right)^n}{Y(a_{fc}) \cdot \sqrt{\pi \cdot a_{fc}} - Y(a) \cdot \sqrt{\pi \cdot a}} \quad \text{and}$$

$$\Xi(S^+, S^-) = \left(S^+ - S^-\right)^{n-1} = \Delta S^{n-1}$$

The Paris-Erdogan and Forman proposals as well as many other fatigue crack growth equations practically take their origin in fitting of empirical data. There is however a group of proposals based on theoretical reasoning originated by mechanical phenomena occurring in the fatigue crack tip neighborhood. In Doliński (9) the global/local energy balance approach involving plasticity and microstructural effects around the fatigue crack tip due to a load cycle was used in derivation and led to the following equation

$$\Delta a_i = \frac{C_4 \cdot \Delta K_{eff,i}^4 + K_{fc}^2 \cdot C_2 \cdot \left(\Delta K_{eff,i}^2 - \Delta K_{th}^2\right)}{\sigma_Y^2 \cdot \left(K_{fc}^2 - K_{max,i}^2\right)} \cdot X(a_i) \tag{3}$$

where σ_Y denotes the yield stress of material. Comprehensive statistical analysis of the Virkler and Ghonem-Dore from (1) and (2) was performed in (4) and allowed us to specify: $C_4 = 0$, $K_{fc} = S^+ \cdot \sqrt{\pi \cdot A_{fc}}$ and $\Delta K_{th} = \Delta S_{eff} \cdot \sqrt{\pi \cdot A_{th}}$

with A_{fc} and A_{th} as random variables and to identify all statistical characteristics of the random material parameters, $\mathbf{X} = [C_2, A_{fc}, A_{th}]$. Hence, the functions $g(\cdot)$ and $\Xi(\cdot,\cdot)$ in (2) can be written as follows (subscript "i" omitted)

$$g(a|C_2, A_{th}, A_{fc}) = \frac{C_2 \cdot \left(Y^2(a) \cdot a - A_{th}\right)}{\sigma_Y^2 \cdot \left(1 - \dfrac{Y(a) \cdot a}{A_{fc}}\right)} \cdot X(a)$$

$$\Xi(S^+, S^-) = \Delta S_{eff}^2 = U^2(R) \cdot \Delta S^2 = U^2(R) \cdot \left(S^+ - S^-\right)^2 \tag{4}$$

The subscript "eff" at the stress intensity factor range, ΔK_{eff}, in (3) and, consequently, at the stress amplitude, ΔS, in (4) indicates the effective stress amplitude, $\Delta S_{eff} = S^+ - S_{op}$, where S_{op} denotes the crack opening stress, to be more relevant to the fatigue process description than the whole stress amplitude, ΔS. This substitution follows the Elber observations, Elber (10), who noticed the fatigue crack closure before reaching S^- by the stress process and delayed crack opening after the stress minimum. In the literature there is no universal formula relating S_{op} to load condition and material properties. Most of the proposals are based on experimental data, see e.g. Bulloch (11), and introduce a function, $q(R)$, depending on the stress asymmetry ratio, R, so that $S_{op} = q(R) \cdot S^+$. Among many proposals the bilinear form

$$q(R) = \frac{S_{op}}{S^+} = \max\left\{q_0 \cdot \left(1 + \frac{R}{|R_0|}\right), R\right\} \tag{5}$$

suggested by Veers (12) is admitted. The material-dependent parameters, q_0 and R_0, remain usually within the following intervals $q_0 \in [0.2, 0.5]$ and $R_0 \in [-5,-2]$. The factor U(R) in (4) reduces the stress cycle, ΔS, to the effective one, $\Delta S_{eff} = U(R) \cdot \Delta S$, while $U(R) = (1 - q(R))/(1 - R)$.

Empirical plots of the crack length, a, versus the number of cycles, N, show some stochastic fluctuations for every sample. They apparently result from the stochastic non-homogeneity of material along the crack path. In Doliński (9) some stochastic fields describing the fluctuations of material properties around the crack tip were assumed. They led to the stochastic function, X(a), in (3) that describes the effect of stochastic material non-homogeneity on local fluctuations of the fatigue crack growth rate. The stochastic properties of these fields and eventually of the function X(a) were completely identified for the Virkler and Ghonem-Dore results. The effect of stochastic material non-homogeneity on probabilistic characteristics of lifetime estimates decreases very quickly with increasing size of the admissible increment and may be often neglected in engineering applications, i.e. X(a) = 1.

STOCHASTIC LOADING

Fatigue damage process due to variable amplitude loading is subjected to some load history effects resulting in retardation of fatigue growth after overloads. The physical nature of this phenomenon has not been completely explained, yet. Among various mechanisms involved the plasticity-induced fatigue crack closure is generally considered as a dominant cause of the retardation in Mode I of fatigue crack growth, Shin & Fleck (13). Most of the models proposed in the literature to predict the fatigue crack growth with regard to the load sequence effects refer to the overload-induced plastic zone and a diminution of the effective stress intensity factor range after an overload, see e.g. Wanhill & Schijve (14). Such an approach was also applied in modeling of fatigue crack growth under stochastic loading in Ditlevsen & Sobczyk (15), Doliński (16), Veers (12), Veers et al. (17). Mathematical tools and solution methods differ, however, substantially in the quoted papers. Birth process, averaged load characteristics, diffusion Markov process, numerical simulation were there, respectively, used in derivation of statistical characteristics of the structural lifetime when a critical fatigue macrocrack length defines the structural failure due to stochastic loading. In the present paper a mixed, partially numerical, partially analytical approach proposed in Doliński & Colombi (18) is presented. It allows us to derive the lifetime probability distribution with account for retardation effects due to stochastic loading as well as for random material properties.

Following the Willenborg model, c.f. Willenborg et al. (19), the reduction of the fatigue crack growth rate after an overload, s_{ol}, occurring at $a = a_{ol}$ is associated with the stress, s_r, called the reset stress, necessary to create a plastic zone of range $r_Y(a,s_r)$ that would reach the boundary of the overload-induced plastic zone, $r_Y(a_{ol},s_{ol})$. The reset stress is calculated from the equality $a_{ol} + r_Y(a_{ol},s_{ol}) = a + r_Y(a,s_r)$, where a is the current crack length, and is given as follows

$$s_r = s_r(a_{ol}, a; s_{ol}) = \frac{\sigma_Y}{\gamma \cdot Y(a)} \cdot \sqrt{\frac{a_{ol}}{a} \cdot \left(1 + \frac{\gamma \cdot Y^2(a_{ol}) \cdot s_{ol}^2}{\sigma_Y^2}\right) - 1} \tag{6}$$

Large compressive stresses around the fatigue crack tip after unloading associated with the large overload-induced plastic zone hinders the crack opening due to the subsequent stress cycles. The opening stresses corresponding with the post-overload maxima transitorily increase while the current plastic zones move within the overload-induced one. In order to specify the retardation intensity Veers in (12) assumed the augmented opening stress, $S_{op,r}$, to be equal to $S_{op,r} = (s_r/S^+)\cdot S_{op}$, where $S_{op} = q(R)\cdot S^+$ denotes the opening stress corresponding with the current stress cycle (S^-,S^+) (without retardation effects). Considering the effective amplitude, ΔS_{eff}, relevant to the fatigue process as a difference between the stress maximum and the effective minimum, $\Delta S_{eff} = S^{+} - S^{-}_{eff}$, the latter is explicitly defined as follows

$$S^{-}_{eff} = S^{-}_{eff}\left(a, S^{-}, S^{+} | a_{ol}, s_{ol};\right) = \tag{7}$$
$$= \begin{cases} q(R)\cdot S^{+} & \text{if } s_r < S^{+} \text{ and } S^{-} < S^{+} \\ q(R)\cdot s_r(a|a_{ol}, s_{ol}) & \text{if } q(R)\cdot s_r < S^{+} < s_r \\ S^{+} & \text{if } q(R)\cdot s_r > S^{+} \text{ or } S^{-} > S^{+} \end{cases}$$

Introducing the reset stress, $s_r(a|a_{ol},s_{ol})$, as a variable governing the retardation effects and depending on the current crack length, a, on the crack length, a_{ol}, at the time of application of the last overload, s_{ol}, and on the overload value itself we loose a very convenient product form property of the fatigue crack growth equation as given in (2). The crack length increment, Δa_i, due to a single stress cycle application has to be written as follows

$$\Delta a_i = g(a_i|\mathbf{x})\cdot \Xi\left(S_i^{+}, S_i^{-}; s_r\left(a_i|a_{ol}, s_{ol}\right)\right) \tag{8}$$

The phenomenon of fatigue crack growth retardation introduces a memory effect. It excludes a separation of variables allowing for definition of any damage parameter, Γ, depending on the load process alone and satisfying the Palgrem-Miner hypothesis on linear accumulation of damage. Thus, an involved cycle-by-cycle incremental analysis has to be always performed to determine the development of damage and, eventually, the fatigue crack length due to variable-amplitude loading. Such a time consuming procedure is apparently impractical for stochastic loading for which any load path is only a sample of a stochastic process. The entirely incremental approach to complete statistical information about stochastic behavior of the fatigue process adequate for reliability analysis, eventually, would require extensive numerical simulation. Some features of the fatigue crack growth retardation phenomenon under stochastic loading and some statistical properties of extremes of stochastic processes can be, however, used to significantly simplify and restrict the numerical simulation that furthermore, provides some initial but sufficient data to continue the probabilistic analysis analytically.

Simulation procedure

Considering the fatigue loading as a stochastic process it is easily seen that the retardation phase of the fatigue crack growth may be initialized by any random maximum provided that it is greater than the current reset stress and followed by lower maxima. This overload maximum starts a retardation phase which continues as long as the current reset stress is not violated. If this condition is not satisfied for a maximum then this maximum can be the next overload or it can start a post-retardation phase which will continue as long as any subsequent maximum is not lower than the previous one. If the condition for continuation of the post-retardation phase is not satisfied for a maximum then this maximum is assumed to start the next retardation phase. This scheme can be extended on the whole fatigue crack propagation process which appears to alternately consist of retardation and post-retardation phases. The couples of these successive phases are considered as blocks starting and terminating with overloads.

Extremes of the stress process are the only load parameters involved in fatigue crack growth equations. Therefore, just their probabilistic characteristics are desired to predict the structural lifetime due to the macrocrack propagation. A full stochastic description of sequence of extremes is not available except very specific cases of stochastic processes. Recently Frendahl & Rychlik (20) have shown on a very wide numerical simulation basis that a homogeneous Markov chain is a very good approximation of the random sequence of extremes of stationary Gaussian and non-Gaussian processes with various spectral characteristics and proposed a numerical procedure to derive the transition probability density function of extremes. Any homogeneous Markovian sequence, $[S_k] = [S_1, S_2, \ldots]$, is fully described by a transition probability density function $p(s_k|s_{k-1})$ of any current term of the sequence, e.g. S_k, given the last previous term $S_{k-1} = s_{k-1}$. Since analytical derivation of stochastic process is, in general, hardly possible a numerical procedure proposed in (20) is applied.

The Markov property of stress extremes and the retardation+post-retardation block structure of fatigue growth process allows us to apply a simple numerical simulation scheme to estimate the probabilistic properties of all variables that describe the fatigue features within a block, in particular the joint probability density function $p_{B,N_B}(b_i, n|a_{ol}, \mathbf{x})$ of the retardation+post-retardation block length, $B(a_{ol},\mathbf{x})$, and number of cycles within the block, $N_B(a_{ol},\mathbf{x})$, given $a = a_{ol}$ and $\mathbf{X} = \mathbf{x}$. This joint probability density function allows us to calculate the statistical moments of $B(a_{ol},\mathbf{x})$ and $N_B(a_{ol},\mathbf{x})$ that appears to be sufficient to proceed with lifetime calculation.

Cycles to failure

We introduce a damage parameter, $\Gamma(N_B|a_{ol},\mathbf{x})$, corresponding with the block length $B(a_{ol},\mathbf{x})$

$$\Gamma(N_B|a_{ol},\mathbf{x}) = \int_{a_{ol}}^{a_{ol}+B(a_{ol},\mathbf{x})} \frac{da}{g(a|\mathbf{x})} = \sum_{n=1}^{N_B(a_{ol},\mathbf{x})} \Delta\Gamma(S_n^-, S_n^+; a_n|a_{ol}) \tag{9}$$

Its statistical moments appear practically constant and independent of a_{ol} and $\mathbf{x}$, i.e. $\overline{\Gamma^k}(a_{ol},\mathbf{x}) = \overline{\Gamma^k} = \text{const}$, so that the number of cycles to failure when the crack, initially of the length a_0, reaches its critical size, a_F, can be calculated from the following equation

$$\gamma_F(\mathbf{x}) = \gamma(a_0, a_F|\mathbf{x}) = \int_{a_0}^{a_F} \frac{da}{g(a|\mathbf{x})} \approx \sum_{m=1}^{M_F(\mathbf{x})} \Gamma_m$$

where the subscript "m", m = 1,2,...,M(**x**), numbers the retardation+post-retardation blocks to failure. The number of blocks, $M_F(\mathbf{x})$, is a random variable with probability distribution approaching, for the sufficiently great mean, $\overline{M_F(\mathbf{x})} = \gamma_F(\mathbf{x})/\overline{\Gamma}$, to the inverse Gaussian one and having the variance $\sigma^2_{M_F}(\mathbf{x}) = \nu^2_\Gamma \cdot \overline{M_F(\mathbf{x})}$, where $\nu_\Gamma = \sigma_\Gamma/\overline{\Gamma}$. The total number of cycles to failure, $N_F(\mathbf{x})$, given **X** = **x** is a sum of $M_F(\mathbf{x})$ independent random numbers of cycles, N_B, within blocks. The central limit theorem modified by Renyi (21) for a sum of a random number of random variables may also be applied to its probability distribution resulting in

$$F_{N_F}(n;\mathbf{x}) = P[N_F(\mathbf{x}) \le n] \approx \Phi\left[\frac{n - \overline{M_F(\mathbf{x})}\cdot\overline{N_B}}{\sigma_{N_F}(\mathbf{x})}\right] \tag{10}$$

where the variance of $N_F(\mathbf{x})$ is given as follows

$$\sigma^2_{N_F}(\mathbf{x}) = \overline{M_F(\mathbf{x})}\cdot\sigma^2_{N_B} + \sigma^2_{M_F}(\mathbf{x})\cdot\overline{N_B}^2 = \frac{\gamma(\mathbf{x})}{\overline{\Gamma}}\cdot\left(\sigma^2_{N_B} + \nu^2_\Gamma(\mathbf{x})\cdot\overline{N_B}^2\right) \tag{11}$$

<u>Time to failure</u>

For a stochastic process the number of cycles, $N_t(n|t)$, within a given time interval, [0,t], can be approximated by a Gaussian random with the mean, $\overline{N^+_t(t)} = \nu^+ \cdot t$, and variance, $\sigma^2_{N^+_t}(t) = S^+_N \cdot t + \overline{S^+_N}$, both linearly dependent of time for great t. The mean rate of maxima, ν^+, and coefficients S^+_N and $\overline{S^+_N}$ can be derived from spectral density of stochastic load process or from its empirical samples. The probability distribution of time to failure given **X** = **x** is calculated as follows

$$\begin{aligned}
F_{T_F}(t|\mathbf{x}) &= P[T_F \le t|\mathbf{x}] = \int_{-\infty}^{\infty} F_{T_n}(t|n)\cdot f_{N_F}(n|\mathbf{x})\,dn = \\
&= 1 - \int_{-\infty}^{\infty} F_{N_t}(n|t)\cdot f_{N_F}(n|\mathbf{x})\,dn \approx \\
&\approx 1 - \int_{-\infty}^{\infty} \Phi\left[\frac{n - \overline{N_t(t)}}{\sigma_{N_t}(t)}\right]\cdot\frac{1}{\sigma_{N_F}(\mathbf{x})}\cdot\varphi\left[\frac{n - \overline{M_F(\mathbf{x})}\cdot\overline{N_B}}{\sigma_{N_F}(\mathbf{x})}\right] dn = \\
&= \Phi\left[\frac{\overline{N_t(t)} - \overline{M_F(\mathbf{x})}\cdot\overline{N_B}}{\sqrt{\sigma^2_{N_t}(t) + \sigma^2_{N_F}(\mathbf{x})}}\right]
\end{aligned} \tag{12}$$

The probability (12) has got the form similar to the so-called Birnbaum-Saunders probability distribution. Depending on the dimension, K, of the material parameter vector $\mathbf{X} = [X_1, X_2, ..., X_K]$ the unconditional probability distribution of the lifetime can be calculated by direct integration

$$F_{T_F}(t) = \int_{-\infty}^{\infty} F_{T_F}(t|\mathbf{x}) \cdot f_{\mathbf{X}}(\mathbf{x})\, d\mathbf{x} \qquad (13)$$

where $f_{\mathbf{X}}(\mathbf{x})$ denotes the probability density function of the parameter vector $\mathbf{X}$ or by employing some approximate methods of reliability analysis involving a search for design points, first and second order reliability method, importance sampling, say.

CONCLUSIONS

The division of the fatigue growth process into blocks and Markov assumption about the sequence of extremes of the loading process shorten the simulation procedure providing all necessary parameters involved in the lifetime probability distribution. The load sequence effects as well as the random material properties are taken into account. The approach allows us also to account for load sequence effects in fatigue damage process due some trains of random cycles resulting from some stochastic extreme excitations of random duration and randomly occurring during the structural seervice time.

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