STEADY-STATE ANALYSIS OF AN ARRAY OF SEMI-INFINITE EDGE CRACKS IN A TRANSFORMATION TOUGHENING CERAMIC

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This paper presents an analysis of arrays of semi-infinite edge cracks in transformation toughening ceramics under steady-state conditions. It is demonstrated that the transformation zones between the cracks cannot coalesce, but that for transformation densities above a critical value two transformation zone solutions are possible. One solution pertains to quasi-static crack growth and the other to pretransformed materials. The latter can cause excessive transformation to appear during loading before crack growth is initiated. The multiplicity of solutions is a consequence of the semi-infinite crack length.

INTRODUCTION

A model for periodical arrays of semi-infinite edge cracks in transformation toughening ceramics is studied. This model can be considered as a prelude towards analyzing surface damage of ceramic materials. An array of finite surface cracks very effectively shields the crack tips in comparison with single surface cracks. For crack distances less than about 5 times the crack lengths the stress intensity factor for an array of finite surface cracks is within 2% of the stress intensity factor for a similar array of semi-infinite surface cracks. The multiplicity of solutions that emerges from the study of steady-state growth of semi-infinite edge cracks is not found in a similar study of finite surface cracks, Andreasen and Karihaloo (1).

Surface grinding of transformation toughening ceramics can induce a certain strengthening of the component if the grinding gives rise to transformation. The grinding-induced transformations can be the result of at least two mechanisms. First, as the contact stresses between the grinding agent and the ceramic are locally very large, and possibly singular if the grinding agent consists of irregular particles, transformation is likely to take place in the vicinity

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of contact. A second, less direct mechanism is that the grinding just induces small cracks in the surface, but the transformation is brought about by subsequent loading of the ceramic during either the grinding process or service. The latter mechanism can be expected to give rise to transformation in a thicker surface layer in comparison with the former mechanism. Limited crack growth can be sustained by an array of cracks, where R-curve behaviour induced by transformation prevents the instability of this configuration that would otherwise occur. In the following, this mechanism where the transformation is a result of crack growth will be considered.

MODEL DESCRIPTION AND THEORY

The model of an array of semi-infinite edge cracks is depicted in Fig. 1. An infinite array of equally spaced parallel cracks C (spacing d) is loaded at infinity by a constant normal stress σ^{∞}. Each crack is bounded by a zone S of transformation formed during loading and crack growth. The zones are assumed to continue along the crack faces to infinity along the negative x-axis, such that steady-state conditions prevail. The transformation strains are assumed to be constant and purely dilatational in the zones in accordance with the supercritical transformation assumption. The transformation zone boundary ahead of a crack is determined by the critical mean stress criterion. Effects from elastic mismatch between the matrix and transforming particles are neglected, and reverse transformations are assumed not to occur.

The cracks are modelled by a pile-up of appropriate dislocations. The density of the dislocations in the pile-up is adjusted to meet the traction free crack condition. This condition together with the critical mean stress criterion is expressed through the following two coupled integral equations which are solved numerically

$$
\begin{aligned}
0 &= \frac{\sigma^{\infty}}{\sigma_c^{\infty}} + \frac{\omega}{18d}\int_{-H}^{H} g\left(\frac{x - x_0(y_0)}{d}, \frac{y_0}{d}\right)dy_0 + \frac{1}{d}\int_{-\infty}^{0} D_0(t) f\left(\frac{x-t}{d}\right)dt \\
1 &= \frac{\sigma^{\infty}}{\sigma_c^{\infty}} + \frac{2}{d}\int_{-\infty}^{0} D_0(t) g\left(\frac{x_0 - t}{d}, \frac{y_0}{d}\right)dt \qquad (1)
\end{aligned}
$$

The normalized dislocation density function $D_0(t)$ is defined such that the Burgers vector between t and $t+dt$ is $b = (12(1+\nu)\sigma_m^c D_0(t)/E)dt$. The weight functions $g(\zeta, \xi)$ and $f(\eta)$ are defined by

$$
\begin{aligned}
g(\zeta,\xi) &= \frac{\sinh(2\pi\zeta)}{\cosh(2\pi\zeta) - \cos(2\pi\xi)} - 1 \\
f(\eta) &= 2\coth(\pi\eta) - (\pi\eta)\mathrm{cosech}^2(\pi\eta) - 2 \qquad (2)
\end{aligned}
$$

From (1) the dislocation density function $D_0(t)$ and the transformation zone boundary S can be obtained for a given load σ^{∞} and value of the transformation strength parameter ω given by

$$\omega = \frac{Ec_v\theta_p^T}{\sigma_m^c}\left(\frac{1+\nu}{1-\nu}\right) \tag{3}$$

θ_p^T and c_v are the dilatation and volume fraction of transformable particles respectively, and ν is Poisson's ratio.

The specific solution for quasi-static crack growth at steady-state conditions are obtained by imposing the following side conditions

$$\begin{aligned} 1 &= \lim_{x\to 0^-} 2D_0(x)\sqrt{\frac{-x}{L}} \qquad && L = \frac{2}{9\pi}\left(\frac{K^c(1+\nu)}{\sigma_m^c}\right)^2 \\ 0 &= \left.\frac{dy_0}{dx_0}\right|_{y_0=H} && (x_0, y_0) \in S \end{aligned} \tag{4}$$

The first condition ensures quasi-static crack growth by fixing the value of the stress intensity factor at the crack tip to the inherent toughness of the material, $K^{tip} = K^c$. The second condition ensures steady-state conditions on the transformation zone shape by defining the height of the wake.

RESULTS AND DISCUSSION

On the basis of the model described above for an array of parallel semi-infinite edge cracks some results relating to the strengthening of ceramics with damaged surfaces and transformation induced by crack growth are presented in the following.

An upper limit on the transformation zone height H can be obtained by considering the second of the two equations (1) for the critical mean stress criterion. As the applied stress σ^∞ is less than the critical applied stress σ_0^∞ the mean stress σ_m^D due to the dislocations given by the integral in the second equation (1) must give a positive contribution to the mean stress, so that the following inequality must hold

$$0 \le \int_{-\infty}^{0} D_0(t)\left(\frac{\sinh(2\pi\frac{x_0-t}{d})}{\cosh(2\pi\frac{x_0-t}{d}) - \cos(2\pi\frac{y_0}{d})} - 1\right) dt \tag{5}$$

Numerical studies show that the dislocation density function $D_0(t)$ is always positive. For $y_0 > d/4$ the integrand increases monotonically for fixed t, and tends to zero from below as x_0 is allowed to increase. Therefore the integral is negative for $y_0 > d/4$ and the inequality (5) is violated. The limiting value of the transformation zone height is therefore $H = d/4$, and coalescence of neighbouring transformation zones cannot take place. At this limit for H, the zone front diverges, i.e. $x_0 \to \infty$. The critical value of the transformation parameter ω_c for which a diverged transformation zone is a solution is

$$\omega_c = 18\left(1 - \sqrt{\frac{\pi L}{d}}\right) \tag{6}$$

For nondiverging transformation zones, solutions to equations (1-4) are obtained numerically. The strengthening effect for various crack spacings d/L is depicted in Fig. 2. It is seen that solutions to equations (1-4) can be obtained for the transformation parameter ω equal to the critical transformation parameter ω_c of equation (6) but with lower strengthening than that corresponding to a diverged zone. For these solutions, the transformation zones remain finite, and for the transformation parameter ω greater than the critical value ω_c but less than a certain maximum ω_{max}, two finite transformation zones are solutions to (1-4). The limits on ω are shown in Fig. 4.

The result for the crack spacing $d/L = 50$ shown in Fig. 2. is redrawn in a slightly more explicit form in Fig. 3a. The stable region is now above the curve, the unstable region below it, and the curve itself pertains to quasi-static crack growth. The transformation strength ω is fixed by the parameters entering (4) and is thus a material constant for a specific microstructure. For $\omega = 22$, the line A-D is indicated in the diagram. This line is followed from A to D as the applied load σ^∞ is increased. The part from A to B is in the stable region, and as the load is increased from point A no crack growth appears. When point B is reached quasi-static crack growth is possible. A further increase in the load will lead to unstable crack growth as indicated by the broken line between B and C. The derivative of the crack tip stress intensity factor K^{tip} with respect to the applied load σ^∞ is positive at the point B as indicated in Fig. 3b. Therefore it is not possible to go from B to C just by increasing the load on the specimen. If however the situation pertaining to point C is brought about by some other means, quasi-static crack growth is possible at a higher load at C compared to the load at B. Increasing the load from point C towards point D leads to a decrease in the crack tip stress intensity factor as indicated by the negative derivative in Fig. 3b at point C. Therefore a new stable region is reached and the point C is a "superstable" point at which an increase in the load stops crack growth by enhancing the transformation, i. e. the toughening effect of the transformation grows more rapidly than the increase in applied stress intensity factor. Under these circumstances failure will initiate first by divergence of the transformation zones and thereafter by crack growth as the surrounding matrix material loses its ability to enclose the transformation zone. Due to the assumption of no reverse transformation the configuration of larger transformation zones pertaining to the left branch cannot revert to the right branch simply by lowering the applied load, as the derivative of the crack tip stress intensity factor K^{tip} with respect to the applied load σ^∞ is positive for fixed transformation zone shapes, as indicated by the dotted line in Fig. 3b.

The transformation zone sizes in terms of the transformation zone boundary intercept with the crack line extension x_c and the height of the transformation zone $y_c = H$ associated with the quasi-static solutions of Fig. 3a are shown in Fig. 3c with the points B and C indicating the load cases just described. Transformation zone shapes for crack spacing $d/L = 50$ and various loadings σ^∞ are depicted in Fig. 3d.

The toughening ratio K^∞/K^c corresponding to the strengthening of Fig. 2 is depicted in Fig. 5. The broken curve is the limiting result for a single semi-infinite crack obtained by Amazigo and Budiansky (2) and the dotted line pertains to the critical value of the transformation parameter ω_c given by (6).

REFERENCES

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(2) Amazigo J. C., and Budiansky, B., Int. J. Solids Structures, Vol. 24, No. 7, 1988, pp. 751-755.

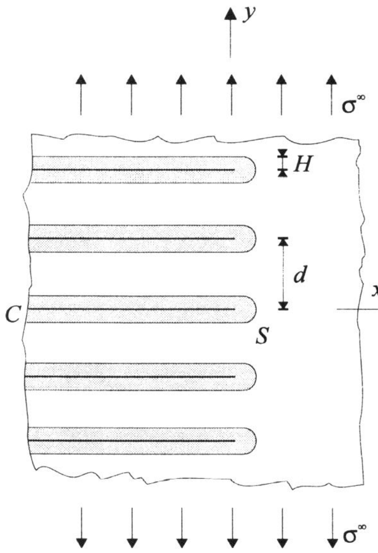

Figure 1: Model configuration

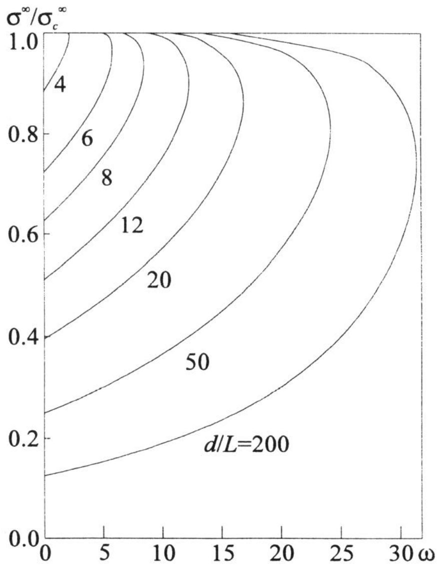

Figure 2: Strengthening for arrays of semi-infinite cracks

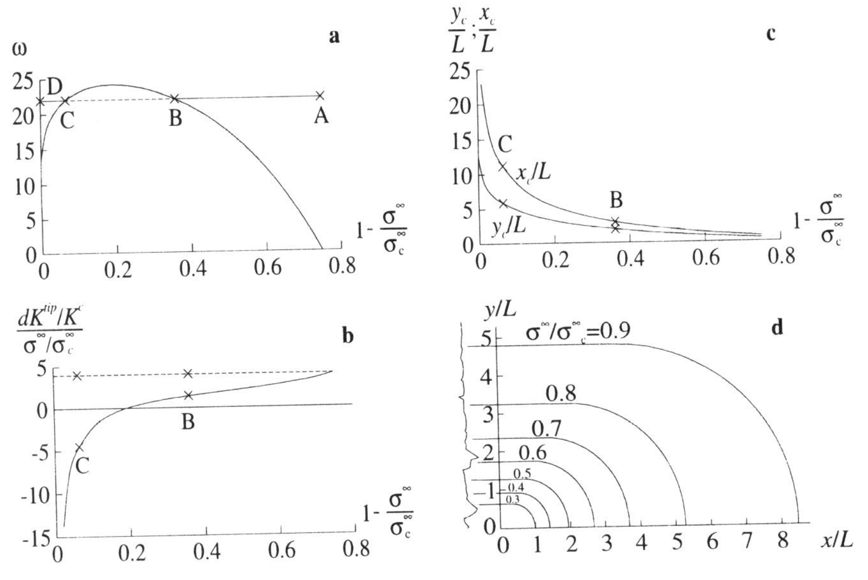

Figure 3: Characteristic results for d/L=50

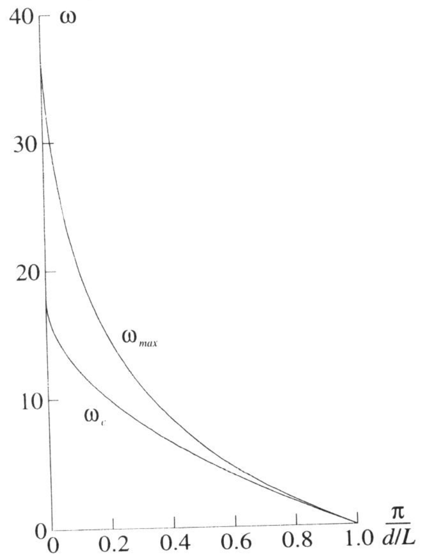

Figure 4: Critical and maximum value of the transformation parameter ω

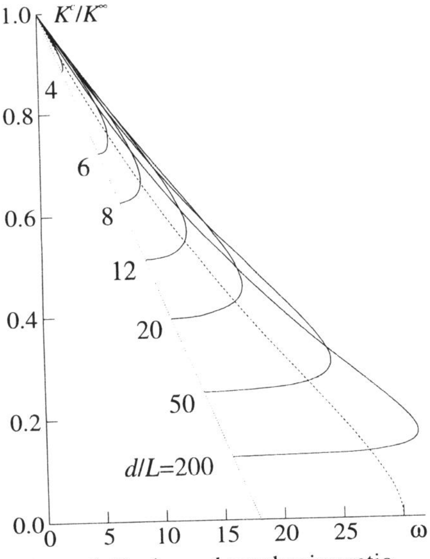

Figure 5: Reciprocal toughening ratio for arrays of semi-infinite cracks