PROBABILISTIC MODEL OF FATIGUE CRACK GROWTH

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A generalized B-model which can describe the statistical features of crack growth including mean, variance and cumulative distribution of time to reach a specified crack length is analyzed in this paper. It is based on a semi-Markov chain and called SMC B-model. In contradistinction to SMC B-model presented till now, this model is applied to variable severity fatigue crack growth process, caused by loading interaction effects and leading to retardation or acceleration crack growth. Accuracy of this model is compared with experimental results.

INTRODUCTION

Prediction of fatigue crack growth under spectrum loading is very complicated by load interaction effects leading to retardation or acceleration phenomena. Additionally, agreement of crack-growth predictions with results of experimental tests varies widely, because of the uncertainties of the physical nature of the load interaction effects. In these cases, when duty cycle (DC) severity changes with increasing time, nonstationary computational models can be used, for example nonstationary probabilistic B-model proposed by Bogdanoff-Kozin (1). It is phenomenological model and the crack propagation process is described by a discrete space Markov theory, i.e. cumulative damage (CD) is regarded as a discrete state-discrete time finite Markov process. Thus it can be treated as a Markov chain. The discrete time is measured in numbers of DCs and the fatigue crack growth is described by a transient discrete damage states. Fatigue crack growth is a physical

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observable CD process, thus the model states are a function of crack length. For the known mean and variance of time W_{k_0+j} for a crack to grow from $a_0 = k_0\,\Delta a$ to $a = (k_0+j)\,\Delta a$ Kozin and Bogdanoff (2) constructed the stationary state dependent BK-model of fatigue crack growth whose parameters are determined from the randomized version of Paris-Erdogan equation. In this original model the independent random variables T_j representing the waiting times in each state have the geometric distribution and model states are not proportional to crack length. Therefore these times can not be summing and moreover it is not possible to obtain the model for a new stress level on the ground of another stress level without additional testing. It seems that the best method, particularly useful in modelling of fatigue crack growth is another B-model based on semi-Markov chain (SMC) (Bogdanoff-Kozin (1), (3), (4)). In this work this model is employed not only to analyze the crack growth process under constant cyclic loading, but also under two-stage loading what allowed quantitatively to measure load interaction.

IDEA OF SMC MODEL FOR FATIGUE CRACK GROWTH

In SMC model the independent random variables T_j representing the waiting times in each state have the negative binomial distribution with parameters b_j and q_j and model states $j = 0, 1, 2, \ldots, J$ are proportional to crack length $k_0\,\Delta a = a_0$, $a_0 + \Delta a$, $a_0 + 2\Delta a, \ldots,$ $a_0 + J\Delta a$, respectively, where $a_0 + J\Delta a$ is the largest crack length that must be taken into account. Then we know the expected value and the variance of T_j (Benjamin and Cornell (5)):

$$E[T_j] = E[\Delta W_{k_0+j}] = \frac{b_j}{q_j} \tag{1}$$

$$Var[T_j] = Var[\Delta W_{k_0+j}] = \frac{b_j(1-q_j)}{q_j^2} \tag{2}$$

where: W_{k_0+j} is the total time to reach crack length $(k_0+j)\,\Delta a$

$$W_{k_0+j} = T_0 + T_1 + \ldots + T_{j-1} \quad \text{or} \quad W_{k_0+j+1} - W_{k_0+j} = T_j$$

b_j is the number of sub-states of state j and q_j is the probability of transition to next sub-state in state j.

The cumulative distribution function (CDF) $F_{k_0+J}(x)$ of W_{k_0+J} is generated using a unit-jump probability transition matrix (PTM) P_{J-1} and initial probability distribution p_0 (1):

$$p_x = p_0 \, P_{J-1}^x \qquad (3)$$

where $x \le W_{k_0+J}$

$$P_{J-1} = \begin{bmatrix} p_0 & q_0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & p_0 & q_0 & \dots & 0 & 0 & 0 & \dots & 0 & 0 \\ & & & \vdots & & & & \vdots & & \\ 0 & 0 & 0 & \dots & p_0 & q_0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & p_1 & q_1 & \dots & 0 & 0 \\ & & & \vdots & & & & \vdots & & \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 & \dots & p_{J-1} & q_{J-1} \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & 1 \end{bmatrix}$$

$p_x = \{p_x(1) \; p_x(2) \; \dots \; p_x(b_0+\dots+b_{J+1}\}$ $\qquad F_{k_0+1} = p_x(b_0 + \dots + b_{J+1})$

The estimation of the parameters of the negative distribution b_j and q_j is possible on the basis of a discrete version of the Paris-Erdogan equation, because this parameters are explicit function of the stress intensity function ΔK_j (3)

$$\frac{\Delta a}{E[\Delta W_{k_0+j}]} = \delta \, C \, (\Delta K_j)^n \qquad (4)$$

where δ is the cycle number in one DC, C and n are material parameters. The expected value and the variance of the number of DC to reach a given crack length $a_0 + J\Delta a$ is

$$E[W_{k_0+J}] = \sum_{j=0}^{J-1} \frac{b_j \, \Delta a}{\delta \, C \, (\Delta K_j)^n} \qquad (5)$$

$$Var[W_{k_0+J}] = \sum_{j=0}^{J-1} \frac{b_j\, \Delta a^2}{\delta^2\, C^2\, (\Delta K_j)^{2n}} - E[W_{k_0+J}] \qquad (6)$$

Thus above-mentioned statistical parameters of fatigue crack growth data can be compared with empirical data. To this end the fatigue tests were performed on the hydraulic testing machine MTS. Case-hardened gear wheels (z = 39, m = 2) made of 20NCT2 steel were selected as test pieces. The initial crack length $k_0\,\Delta a$ = 0.1 *mm*. Tests under constant loading were conducted at the stress of σ_1 = 778 and σ_2 = 936 *MPa*. Fatigue crack growth tests under two-stage loading were conducted for the high-low case. First was applied σ_2 = 936 *MPa* and then σ_1 = 778 *MPa*. The fixed number of cycle at the first stage loading was chosen x_2 = 140 000 (140 in DC). Three sets of sample functions from fatigue crack growth experiments under these loadings are shown in Fig. 1 to 3. Figures 4 - 6 and 7 show fit of empirical (EDF) to cumulative distribution functions generated by BK-model and SMC B-model, respectively.

CONCLUSION

Parameters of SMC B-model are explicitly related to the load as in the PE equation what allows to accommodate this model in case of the load change. Then Wheeler's retardation factor should be applied in the P-E equation additionally in order to increase the usefulness of the SMC B-model to the analysis of variable severity of fatigue crack growth.

REFERENCES

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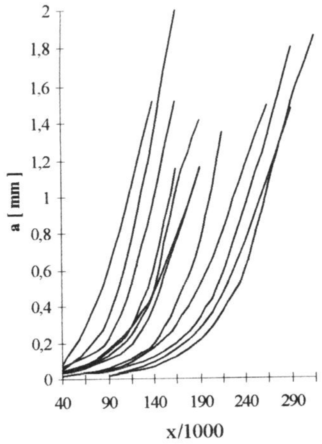

Figure 1 Sample function at σ_2=936 MPa

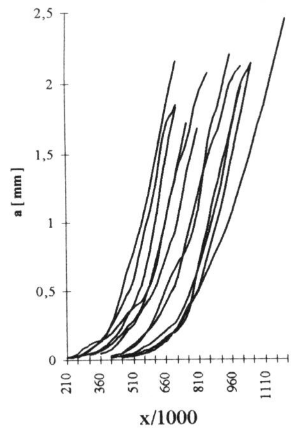

Figure 2 Sample function at σ_1=778 MPa

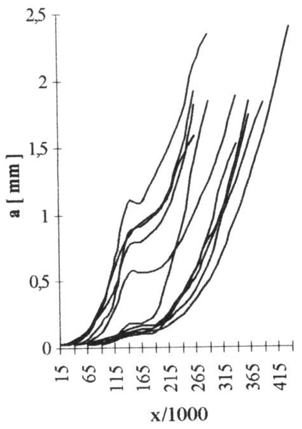

Figure 3 Sample function at two - stage loading σ_2 , σ_1

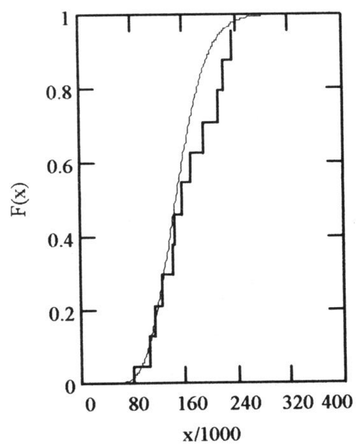

Figure 4 CDF and EDF at σ_2=936 MPa, a=0.5 mm

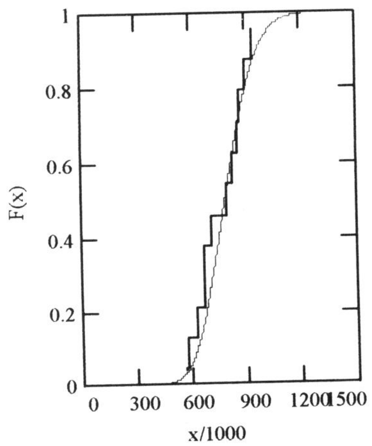

Figure 5 CDF and EDF at σ_1=778 MPa, a=1 mm

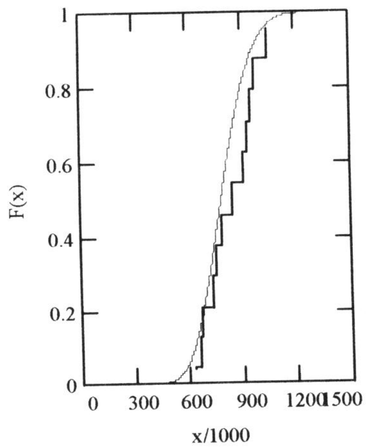

Figure 6 CDF and EDF at σ_1=778 MPa, a=1.5 mm

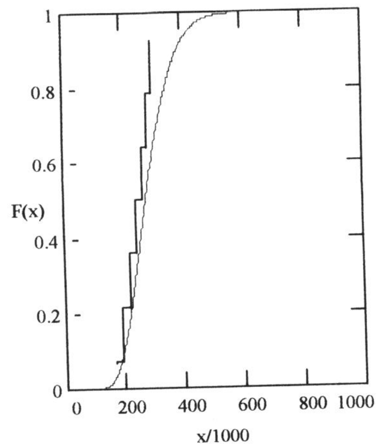

Figure 7 EDF and CDF(SMC) under two - stage loading σ_2 , σ_1