PLASTIC STRAIN DISTRIBUTION NEAR A TIP OF A SHARP V-NOTCH IN A POWER HARDENING MATERIAL

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Based on the numerical solution of the constructed non-linear 4th -order differential eigenvalue equation, regions associated with the local plastic deformation in the vicinity of the tip of a sharp V-notch or a crack in a power-law hardening material are shown which pertain to symmetrical and mixed mode low level loading.

INTRODUCTION

The stress distribution near the tip of a crack or a sharp V-notch in a power-law hardening material was studied earlier by Hutchinson (1) and Kuang and Xu (2) for symmetrical loading, by Anheuser and Gross (3) for longitudinal shear and by Kouzniak et al (4) for antisymmetrical loading.

In this work the authors apply their approach developed in (4) to the construction of the shape of the plastic zone engulfing the tip of a crack or a notch for the case of symmetrical and mixed mode loading.

STATEMENT OF THE PROBLEM

Consider a Cartesian and a polar coordinate system which have their origins attached to the tip of a notch (or a crack) in an unbounded medium where the x-axis coincides with the line of symmetry of the notch opening as shown in Figure 1.The

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Ramberg-Osgood stress-strain relationship is of the following form:

$$\varepsilon_{ij} = \frac{3}{2}\alpha\sigma_e^{\,n-1}S_{ij}, \tag{1}$$

where n is the strain hardening index. The stress deviatoric components are given by

$$S_{ij} = \sigma_{ij} - \frac{1}{3}\sigma_{kk}\delta_{ij},\ \sigma_e^{\,2} = \frac{3}{2}S_{ij}S_{ij},\ \varepsilon_e^{\,2} = \frac{3}{2}\varepsilon_{ij}\varepsilon_{ij}. \tag{2}$$

Consider the case of an incompressible material which is subjected to conditions of plane strain. With reference to cylindrical coordinates r,θ, the following relations hold:

$$\varepsilon_{rr} = -\varepsilon_{\theta\theta} = \frac{3}{4}\alpha\sigma_e^{\,n-1}(\sigma_{rr} - \sigma_{\theta\theta}),\ \ \varepsilon_{r\theta} = \frac{3}{2}\alpha\sigma_e^{\,n-1}\sigma_{r\theta},$$
$$\sigma_e^{\,2} = \frac{3}{4}(\sigma_{rr} - \sigma_{\theta\theta})^2 + 3\sigma_{r\theta}^{\,2}. \tag{3}$$

By means of the stress function $\Phi(r,\theta)$

$$\sigma_{rr} = \frac{1}{r}\frac{\partial\Phi(r,\theta)}{\partial r} + \frac{1}{r^2}\frac{\partial^2\Phi(r,\theta)}{\partial\theta^2},\ \ \sigma_{\theta\theta} = \frac{\partial^2\Phi(r,\theta)}{\partial r^2},$$
$$\sigma_{r\theta} = -\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\Phi}{\partial\theta}\right), \tag{4}$$

the equilibrium conditions are automatically satisfied and the condition of strain compatibility takes the form:

$$\left(\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} - \frac{3}{r}\frac{\partial}{\partial r} - \frac{\partial^2}{\partial r^2}\right)\left[\sigma_e^{\,n-1}\left(\frac{1}{r}\frac{\partial\Phi(r,\theta)}{\partial r} + \frac{1}{r^2}\frac{\partial^2\Phi(r,\theta)}{\partial\theta^2} - \right.\right.$$
$$\left.\left. - \frac{\partial^2\Phi(r,\theta)}{\partial\theta^2}\right)\right] + \frac{4}{r^2}\frac{\partial^2}{\partial r\partial\theta}\left[r\sigma_e^{\,n-1}\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial^2\Phi(r,\theta)}{\partial r^2}\right)\right] = 0. \tag{5}$$

In the limit $r \to 0$, the following asymptotic behaviour is assumed

$$\Phi(r,\theta) = K_i r^{s_i}\tilde{\Phi}(\theta), \tag{6}$$

where K_i is a generalised stress intensity factor with i=1 for mode-I and i=2 for mode-II loading. Quantities with tildes refer to normalised values and s_i denotes the order of the singularity.

Substituting relation (6) into eq. (5) leads to a 4th-order non-linear differential equation:

$$\left\{\frac{\partial^2}{\partial\theta^2} - n(n-2)[n(s_i-2)+2]\right\}\left\{\tilde{\sigma}_e^{\,n-1}\left[s_i(2-s_i)\tilde{\Phi}(\theta)+\frac{\partial^2\tilde{\Phi}(\theta)}{\partial\theta^2}\right]\right\}+ \\ +4(s_i-1)[n(s_i-2)+1]\frac{\partial}{\partial\theta}\left(\tilde{\sigma}_e^{\,n-1}\frac{\partial\Phi}{\partial\theta}\right)=0, \tag{7}$$

and the stress components may be represented in the form

$$\begin{aligned} \sigma_e &= K_i r^{s_i-2}\tilde{\sigma}_e(\theta) = K_i r^{s_i-2}\left[\frac{3}{4}(\tilde{\sigma}_r-\tilde{\sigma}_r)^2+3\tilde{\sigma}_{r\theta}^{\,2}\right]^{0.5}, \\ \sigma_{rr} &= K_i r^{s_i-2}\tilde{\sigma}_r(\theta) = K_i r^{s_i-2}\left(s_i\tilde{\Phi}(\theta)+\frac{\partial^2\tilde{\Phi}(\theta)}{\partial\theta^2}\right), \\ \sigma_{\theta\theta} &= K_i r^{s_i-2}\tilde{\sigma}_\theta(\theta) = K_i r^{s_i-2}s_i(s_i-1)\tilde{\Phi}(\theta), \\ \sigma_{r\theta} &= K_i r^{s_i-2}\tilde{\sigma}_{r\theta}(\theta) = K_i r^{s_i-2}(1-s_i)\frac{\partial\tilde{\Phi}(\theta)}{\partial\theta}. \end{aligned} \tag{8}$$

The condition of having zero stresses along the notch boundaries requires that

$$\tilde{\Phi}(\theta)\Big|_{\theta=\pi-\beta}=0,\quad \frac{\partial\tilde{\Phi}(\theta)}{\partial\theta}\Big|_{\theta=\pi-\beta}=0. \tag{9}$$

Conditions of symmetric (mode-I) and antisymmetric (mode-II) stress distributions around the V-notch tip with $\tilde{\Phi}(\theta)$ a symmetric or skew-symmetric function, respectively, lead to the following requirements:

$$\frac{\partial\tilde{\Phi}(\theta)}{\partial\theta}\Big|_{\theta=0}=0,\quad \frac{\partial^3\tilde{\Phi}(\theta)}{\partial\theta^3}\Big|_{\theta=0}=0 \text{ or } \tilde{\Phi}(\theta)\Big|_{\theta=0}=0,\quad \frac{\partial^2\tilde{\Phi}(\theta)}{\partial\theta^2}\Big|_{\theta=0}=0. \tag{10}$$

As this would reduce the problem to the solution of a homogeneous differential equation, eq.(7), with homogeneous boundary conditions, eq.(10), this set of equations must be complemented by the following normal conditions (see Refs (2), (4))

$$\tilde{\Phi}(\theta)\Big|_{\theta=0}=1 \quad\text{or}\quad \frac{\partial\tilde{\Phi}(\theta)}{\partial\theta}\Big|_{\theta=0}=1. \tag{11}$$

NUMERICAL RESULTS

Small-scale yielding in the vicinity of a notch tip will be studied on the basis of the numerical solution of the boundary value problem, eqs.(7), (10), and (11).

Case 1. Symmetrically loaded notch. Assuming an approach based on the von Mises yield criterion

$$\sigma_e = \sigma_Y{}^n, \tag{12}$$

the expressions (8) lead to the following relation

$$K_1 r^{s_1 - 2} \widetilde{\sigma}_e(\theta) = \sigma_Y{}^n, \tag{13}$$

from which the shape of the plastic zone, $r(\theta)$ can be extracted. The cases for $\beta = \pi / 12$ (very sharp notch) and $\beta = \pi / 6$ (sharp notch) are shown in Figs 2a and 2b, respectively. The shapes have been normalised with respect to coordinates $\{x,y\}/r_0$, $r_0 = (\sigma_Y{}^n / K_2)^{1/s_1 - 2}$ for $n = 3$ and $n = 5$.

Case 2. Mixed-mode loaded crack. For a crack ($\beta = 0$) the order of the singularity is the same for mode-I and mode-II loading, i.e. $s_1 = s_2 = s$. This fact facilitates the construction of the plastic zone around the crack tip under biaxial loading. Recasting the yield criterion, eq. (12), in the form

$$r^{s-2}\left\{\frac{3}{4}\left[K_1(\widetilde{\sigma}_r{}^I - \widetilde{\sigma}_\theta{}^I) + K_2(\widetilde{\sigma}_r{}^{II} - \widetilde{\sigma}_\theta{}^{II})\right]^2 + 3(K_1\widetilde{\sigma}_{r\theta}{}^I + K_2\widetilde{\sigma}_{r\theta}{}^{II})^2\right\}^{\frac{1}{2}} = \sigma_Y \tag{14}$$

where I and II refer to values of the functions calculated for the corresponding mode, plastic zones have been evaluated for a sequence of ratii K_1 / K_2: 0 (Fig.3, a), 0.5 (Fig.3b), 1 (Fig.3c) and 2 (Fig.3d).

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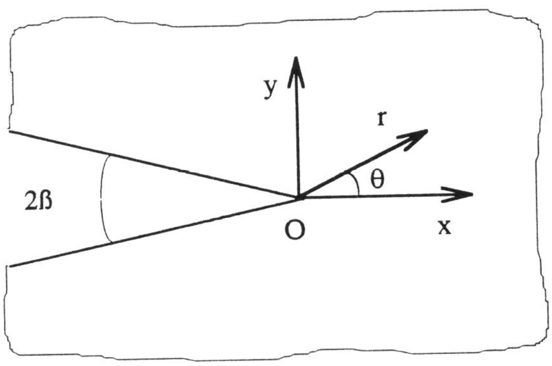

Figure 1 Coordinate systems at the tip of a sharp V-notch

a) b)

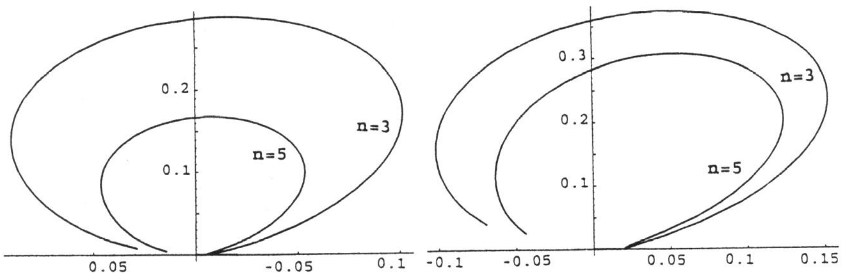

Figure 2 Shape of the plastic zone around a notch tip for the case of mode-I loading

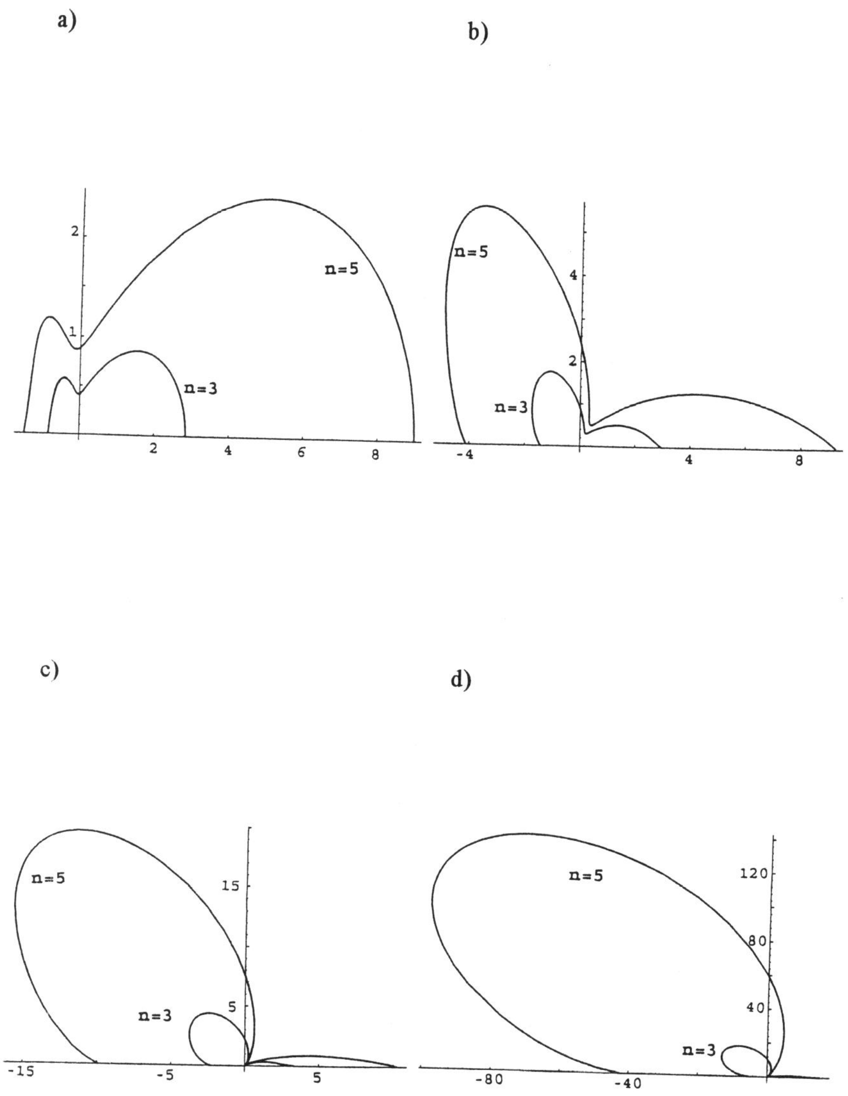

Figure 3 Shape of the plastic zone around a crack tip for the case of mixed mode loading