

ON THE TEMPERATURE IN DYNAMIC CRACK SURFACE  
LIGAMENTS

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Small ligaments connecting the crack surfaces just behind the moving crack front are assumed to exist during fast cleavage crack growth. The production and conduction of heat during the process of tearing these ligaments are studied. The straining is computed by assuming an elastic visco-plastic material model. The produced heat during plastic work is calculated. Heat conduction is considered. The results reveal that the temperature increases several hundred centigrades for a low yield stress. For a high yield stress the temperature increases to over one thousand centigrades. The appropriateness of the selected constitutive relationship is discussed in view of the result.

INTRODUCTION

At fast cleavage crack growth in structural steels, the rate-dependent material behavior during plastic straining often has to be considered (Freund and Hutchinson(1)). Consider a crack propagating by coalescence of trans-granular micro cracks. The mismatch of crystallography and mechanical properties at grain boundaries will leave unbroken parts connecting upper and lower crack surfaces. During increasing separation of the crack surfaces the unbroken parts will become ligaments, bridging the gap between the separating crack surfaces.

In a previous study by Nilsson *et al.* (2) a ligament model is studied as it deforms plastically to a state where very little remains of its initial cross-sectional area. The dissipated energy is calculated as a function of the ligament extension rate. At crack tip speeds of practical interest for structural steels the energy dissipation rate in the ligaments is found to be comparable to total energy release rates. However, the effects of temperature rise due to plastic work was never examined. It was assumed that the process was adiabatic which resulted in unrealistically high temperatures and temperature gradients.

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In this paper dissipated energy is assumed to be converted into heat. The heat conduction, affecting the temperature distribution, is computed. The deformed geometry is considered as the mechanical and the thermal state is calculated simultaneously.

### THE MODEL

A large body described in the coordinate system  $x_1$  and  $x_2$  is considered. The body is separated along the plane  $x_2 = 0$  except in the region  $|x_1| \leq a$  (see Fig. 1). The surfaces at  $|x_1| > a, x_2 = 0$  are assumed to be traction free. Plane strain is assumed. Large strains are given by Green's strain tensor,  $\epsilon_{ij}$ . The strain rate,  $\dot{\epsilon}_{ij}$ , is decomposed into an elastic part,  $\dot{\epsilon}_{ij}^e$ , and a visco-plastic part,  $\dot{\epsilon}_{ij}^p$ . The elastic strain rates are given by Hooke's law using Young's modulus  $E$  and Poisson's ratio  $\nu = 0.3$ . The visco-plastic strain rates are given by the following constitutive relationship

$$\dot{\epsilon}_{ij}^p = \dot{\gamma}_o \left( \frac{\sigma_e}{\sigma_o} - 1 \right)^n \frac{s_{ij}}{\sigma_e} \quad (1)$$

where  $\dot{\gamma}_o$  is the strain rate sensitivity,  $\sigma_e$  is von Mises effective stress,  $\sigma_o$  is the yield stress,  $n$  is the strain rate exponent and  $s_{ij}$  is the stress deviator (Perzyna (3)). The ratio between Young's modulus and the yield stress  $E/\sigma_o = 251$  and  $502$  is used which is believed to represent common structural steels. The strain rate exponent,  $n = 5$  is chosen as proposed by Malvern (4).

A polar coordinate system,  $r$  and  $\theta$ , is attached to the crack tip as Fig. 1 shows. An elastic solution is employed for a boundary layer analysis of a part bounded by  $r \leq R$ . A prescribed displacement rate  $\dot{v}_o$  control the load. The following boundary conditions are applied

$$u_1 = 0 \text{ and } u_2 = \dot{v}_o t \{ \ln(2R/a) - 1 / [2(1-\nu)] \}, \text{ at } r=R, 0 \leq \theta \leq \pi/2 \quad (2)$$

where  $t$  is the time (cf. Nilsson *et al.* (2)). The ratio  $R/a$  is chosen to be 40 which is assumed to be sufficiently large to make the boundary conditions reasonably well modeled by eq. (2).

The problem of transient heat conduction is computed in the deformed geometry. The temperature rate due to plastic deformation is given as

$$\dot{T} = \frac{\sigma_{ij} \dot{\epsilon}_{ij}^p}{c_v \rho_o} \quad (3)$$

where  $\rho_o$  is the density and  $c_v$  is the heat capacitvity. The crack surfaces at  $|x_1| > a, x_2 = 0$  and the boundary at  $r \leq R$  are assumed to be thermally isolated.

All material parameters are assumed to be temperature independent.

### NUMERICAL MODEL

The commercial finite element code ABAQUS (5) is used for the numerical calculations. An up-dated Lagrangian method for large deformation elastic visco-plastic materials is used. Full integration of element stiffnesses and conductivity's is used. The body is covered by a mesh containing 481 nodes and 432 isoparametric elements.

The suggested initially sharp crack tips at  $|x_1| = a, x_2 = 0$  are modeled as notches with a small finite root-radius,  $\rho = 0.2a$ . The center of the half circle forming the notch bottom, is at  $x_1 = 1.2a$  and  $x_2 = 0$ , se Fig. 2. The finite radius improves the convergence rate. The modification has little influence on the result since the radius increases several times during the loading (cf. McMeeking (6)).

### RESULTS

Temperature distribution in the ligaments as a function of the ligament extension rate is studied. The heat conductivity  $\lambda = 84 \text{ N/s}^\circ\text{C}$  is used. Calculations are performed for constant extension rates from  $\dot{v}_o = 0.08\dot{\gamma}_o a$  to  $80\dot{\gamma}_o a$ . The force in the ligament versus extension for five ligament extension rates can be studied in Fig. 3. The results are presented for  $E/\sigma_o = 502$  and 251, the latter are included for comparison reasons and for one extension rate only. The calculations are interrupted when the load has decreased to 30% of its maximum value.

The temperature distributions for the case  $\dot{v}_o = 7.79\dot{\gamma}_o a$  is shown at  $v_o = 0.03a, 0.06a$  and  $0.12a$  in Fig. 4. Rupture is assumed to occur at the latter displacement. Early during the straining (Fig. 4a) the temperature gradient is observed to be very large at the notch bottom. The gradient has completely vanished later during the process (Fig. 4c).

Figure 5 shows temperatures at the center of the ligament,  $x_1 = x_2 = 0$  as function of ligament extension. An observation is that the temperature is low during the initial essentially elastic phase. At onset of plastic deformation, i.e. when a substantial deviation from the linear elastic response occurs (see Fig. 3), temperature is increasing rapidly and is strongly dependent of the ligament extension rate. The heat flow from the ligament region is probably responsible for the decreasing temperature slopes in Fig.5. For low extension rates this even leads to a decreasing temperature at the end of the calculation. Fig. 5 also compares temperatures at the notch bottom, i.e., at  $x_1 = a, x_2 = 0$ , with those at the center of the ligament. It is interesting that the temperature difference, at the end of the calculation, is very small even for the largest extension rates.

The normalized temperature at the time of rupture at the center of the ligament is shown in Fig. 6 as function of extension rate for two

values of the parameter  $E/\sigma_0$ . Only a small difference in normalized temperature is observed for the different ratios between  $\sigma_0$  and  $E$ . As temperatures are normalized by  $\sigma_0/\rho_0 C_v$  and assuming that different steels have approximately the same  $E$ ,  $\rho_0$ ,  $C_v$  and  $\lambda$  this means that temperature rise essentially scales with  $\sigma_0$ .

The difference between the temperatures for different ratios between  $E/\sigma_0$  is small and is believed to be due to the chosen criterion to interrupt the calculations.

### DISCUSSION AND CONCLUSIONS

It is shown by Campbell and Ferguson (7) that the strain rate dependence decreases with increasing temperatures. Therefore the dissipated energy in a ligament is lower than what is predicted by Nilsson *et al.* (2) especially at high extension rates and yield stresses. It should be noted that dependent on how the measurements of the constitutive behavior are done heat develops during the measurements. A constitutive relation taking strain, strain rate and temperature into consideration explicitly would be of great interest for modeling the material behavior.

The constitutive model used by Nilsson *et al.* (2) coincide well with measurements done by Campbell and Ferguson (7) and Huang and Clifton (8). The tests reported in (7) are dynamic shear tests done with a drop weight testing technique. The temperature increase during those tests is estimated to be approximately 1 °C in the medium strain rate region (1 to 10<sup>4</sup> 1/s ). The tests presented in (8) are pressure-shear impact tests on high purity iron. There numerical simulations of the experiments indicate a temperature increase of several hundred centigrades due to large strains.

It is believed that the constitutive relation chosen by Nilsson *et al.* (2) have to be refined in future research except maybe for the highest ratios between  $E/\sigma_0$  and the lowest extension rates.

### REFERENCES

- (1) Freund, L. B. and Hutchinson, J., High-strain rate crack growth in rate-dependent plastic solids, *Journal of the Mechanics and Physics of Solids* , Vol. 33 (1985) 169-191.
- (2) Nilsson, P., Ståhle, P. and Sundin, K-G. "On the Behavior of Crack Surface Ligaments." To be published in *Nuclear Engineering and Design*, 1996.
- (3) Prezyna, P., The constitutive equations for rate sensitive plastic materials, *Quarterly of Applied Mathematics* Vol. 20 (1963) 321-332.

- (4) Malvern, L. E., Plastic wave propagation in a bar of material exhibiting a strain rate effect, *Quarterly Journal of Applied Mathematics*, Vol. 8 (1951) 405-411.
- (5) ABAQUS, User's Manual V 5.3-1 Hibbit, Karlsson and Sorensen, Providence, Rhode Island, USA, 1993.
- (6) McMeeking, R. M., Finite deformation analysis of crack-tip opening in elastic-plastic materials and implications for fracture, *Journal of the Mechanics and Physics of Solids*, Vol. 25 (1977) 357-381.
- (7) J. D. Campbell and W. G. Ferguson, The temperature and strain rate dependence of shear strength of mild steel, *Phil. Mag.* Vol. 21 (1970) 63-82.
- (8) Huang, S. and Clifton, R. J. for IUTAM Symposium Tokyo/Japan 1985 on Macro- and Micro-Mechanics of High Velocity Deformation and Fracture, Dynamic Response of OFHC Copper at High Shear Strain Rates.

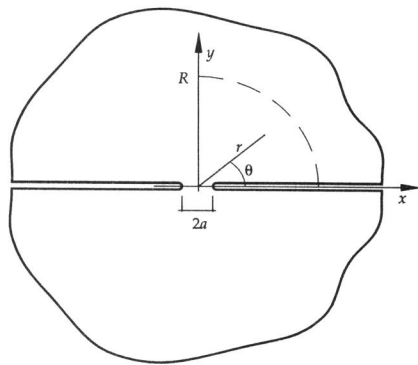


Figure 1 Geometry

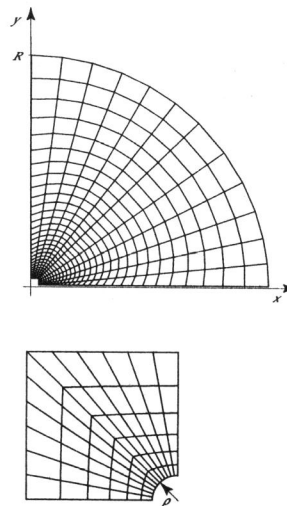


Figure 2 Finite element mesh

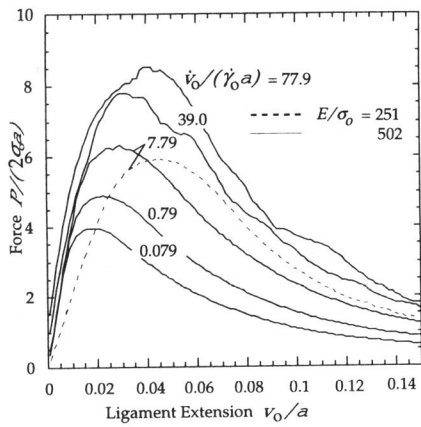


Figure 3 Ligament extension forces vs extension

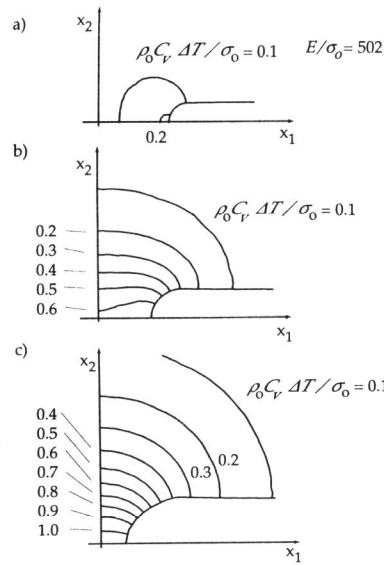


Figure 4 Temperature distribution at a)  $v_0 = 0.03a$ , b)  $0.06a$  and c)  $0.12a$

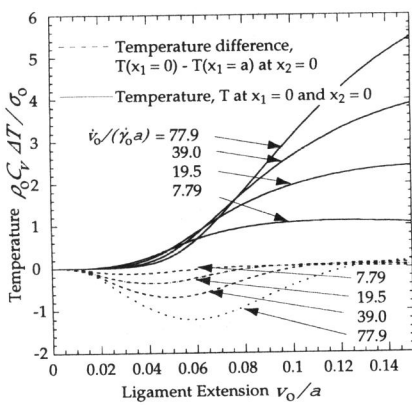


Figure 5 Temperature in the ligament for  $E/\sigma_0 = 502$

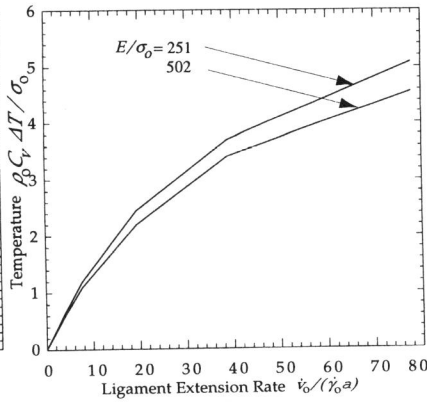


Figure 6 Temperatures at the center of the ligament at the time of rupture