ON THE EXPERIMENTAL REALISATION OF MODE-II CRACKS

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The theoretically predicted overlapping of the crack flanks for Mode-II cracks, which makes the linear elastic solution invalid, is presented. The choice of a certain non-zero rigid body rotation at infinity helps to overcome this contradictive situation. However, the experimental realization of pure Mode-II cracks seems to be a difficult task, because of the extremely small magnitude of this rotation. Experimental evidence through the method of caustics supports the theoretical analysis.

INTRODUCTION

Historically, in Fracture Mechanics after the introduction of the three different independent Modes of cracks by Irwin, the way of thinking about these modes has changed. In the early years, scientists were speaking about Mode-I, -II and -III displacement or deformation, defining the crack Modes by the relative displacements of the respective crack flanks in a cracked plate. So, the three Modes of cracks were named as the opening, the sliding or shear and the tearing one. To each one of them a certain singular stress field and the corresponding S.I.F. were assigned.

Recently, scientists speak more often about Modes of loading instead of displacements. They mark, in this way, the cause of the relative displacements of crack flanks, which is not the stress field in the cracked plate but the external loading. Behind this definition of Modes seems to be hidden the following thought: "The introduction of a crack with the appropriate direction in a tensile or shear stress field creates crack-flanks displacements of definite Mode". Surely a crack normal to a tensile stress field is a Mode-I crack. Is it equally true for a Mode-II

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crack and in which direction has the crack to be placed in the shear stress field? Here it must be pointed out that a shear stress field acts always in two perpendicular directions because of the symmetry of the stress tensor unlike the tensile stress field acting in only one definite direction. The answer to this question although unexpected is definitely no.

A detailed theoretical analysis by Theocaris et al (1) assuming zero rigid-body rotation at infinity, of the displacement field along the flanks of an internal slant crack in an infinite elastic plate subjected to uniform biaxial loading at infinity, indicates that, in case of Mode-II loading conditions, these flanks overlap, Pazis et al (2). Thus, according to the linear elastic solution the crack flanks sink into each other, and this overlapping happens as soon as any even small load is applied. Such an overlapping is of course unacceptable. It indicates that the initial boundary condition of stress-free crack borders is violated, making the elastic solution invalid.

There are two main reasons for the general acceptance in fracture mechanics of this Mode-II invalid solution. The first is that overlapping is not detected (2) if one is concerned only with the so called "singular solution", based on one term approximations of the stress field. The second reason is that pure Mode-II tests and measurements of the corresponding K_{IIC} critical stress intensity factors are very few compared to the experimental work done on Mode-I cracks.

Any objection that overlapping is a mathematical scrutinity without relation to the reality, or even that such phenomena do not appear in experiments, where the plates are not infinite, must be rejected. The uniqueness of linear elastic solutions implies that this situation is also valid in cracked plates of finite dimensions. Moreover, experimental evidence on mixed-mode fracture and especially on Mode-II cracks (Banks-Sills et al (3), Poldeshny (4)) indicates difficulties due to compressive-stress concentrations and friction forces between the crack flanks, accompanied with scattering of the experimental results. Finally, calibration calculations (3,4) with finite elements, where the stress intensity factors are computed from the displacements, result to slightly negative K_I-factors, in situations, where (for symmetry reasons) K_I should be zero, exactly because of the overlapping of the crack flanks.

Aim of this paper is to indicate a possible way out from this self-contradicting situation. The overlapping mechanism is presented, as well as, preliminary experimental work.

THEORETICAL CONSIDERATIONS

The plane problem of a Griffith crack under mixed-mode conditions may be investigated on the typical geometry embeded in Fig.1 and from which any K_I, K_{II} combination can be obtained. By choosing a negative value for the biaxiality factor k and an angle $\beta=0.5\cos^{-1}[(1+k)/(1-k)]$ Mode-II loading conditions are established in the plate. For $k=-1$ and $\beta=45^{o}$, it is obtained that $K_I=0$ and $K_{II}=\sigma_0(\pi a)^{1/2}$. Assuming, furthermore, zero rigid-body rotation at infinity, the complex potentials $\Phi(z)$ and $\Omega(z)$ of Muskhelishvili (5) are:

$$\Phi(z)=i\sigma_0[1 - z/(z^2-a^2)^{1/2}]/2, \quad \Omega(z)=-i\sigma_0[1 + z/(z^2-a^2)^{1/2}]/2 \tag{1}$$

where $z=x+iy$.

According to Eqs (1) and assuming no rigid body translation of the whole plate, x and y crack flanks displacements u and v can be derived as follows:

$$u_{\pm}=\pm 2c(a^2-x^2)^{1/2}, \quad v_{\pm}=2cx \tag{2}$$

In Eqs (2) index + indicates the upper crack flank and the index — the lower one. Moreover the dimensionless loading factor c takes the values $c=\sigma_0/E$ for plane-stress and $c=\sigma_0(1-v^2)/E$ for plane-strain conditions, where E and v are the elastic modulus and Poisson's ratio of the material.

Fig. 1 shows, in an unrealistic magnification for clarity reasons, the way in which the crack deforms and how overlapping occurs. In this figure factor c is taken to be 0.25, while for isotropic elastic brittle materials c is at crack initiation of the order of 10^{-3} or less. According to the second of Eqs (2) a double generic point C on the unloaded crack AB in Fig.1 moves after loading to the double point C′ due to the linear $v_{\pm}$ displacements. Thus, the initial crack deforms to a line segment A′B′ subtenting an angle $\lambda=2c$ with its initial position. Then, according to the first of Eqs (2) the double point C′ splits into two points C^+ and C^- due to the elliptic $u_{\pm}$ displacements. So, the line segment A′B′ deforms to an ellipse with overlapping sections. It is now obvious that overlapping occurs by any whatever small load, provided the crack flanks are initially in touch, as it is the case for natural cracks.

This overlapping occurs not only in the case where $k=-1$ and $\beta=45^{o}$. From Eq. (16) in ref. (2) it can be readily concluded that if Mode-II loading conditions prevail in the plate overlapping occurs anyway. Obviously, the strain set up in the plate under Mode-II loading conditions is what Love (6) calls "pure shear" and not that what here is assumed, i.e. a strain of "simple shear" The difference between these two strain-states is based on the rigid-body rotation. That is the reason why a calculation of the complex potentials potentials $\Phi(z)$ and $\Omega(z)$ for the crack problem shown in Fig.1, without the assumption of zero rigid-body rotation at infinity, has been undertaken. Such a non-zero rigid-body rotation ε_0 does not affect the stress distribution, so that its choice is irrrelevant for the first fundamental problem, although it affects the displacement field.

Following the solution procedure given in ref. (5) the complex potentials have been calculated from the beginning. For the present case ($k=-1$ and $\beta=45^{o}$), they are given by:

$$\Phi(z)=i\sigma_0[1 - z/(z^2-a^2)^{1/2}]/2 + iR, \quad \Omega(z)=-i\sigma_0[1 + z/(z^2-a^2)^{1/2}]/2 + iR \tag{3}$$

where the rotation constant R takes the values $R=\varepsilon_0 E/4$ for plane stress, $R=\varepsilon_0 E/4(1-v^2)$ for plane strain and ε_0 represents rigid-body rotation at infinity. It is well known that an additional imaginary constant iR in the complex potentials leaves the stress distribution unaffected but it creates an additional linear displacement field of the form: $u=-\omega y$ and $v=\omega x$, corresponding to an infinitesimal rotation ω of the whole plate. Accordingly the crack flanks displacements obtained from the complex potentials, given by Eqs (3), are:

$$u_{\pm}= \pm 2c(a^2-x^2)^{1/2}, \quad v_{\pm}= \varepsilon_0 x+2cx \tag{4}$$

It becomes obvious, that by putting $\varepsilon_0= -\lambda= -2c$, the y crack-flanks displacements disappear leaving the crack in its initial position. Thus, the $u_{\pm}$ displacements can be realized without any hindrance, resulting to a sliding Mode, i.e. to a real Mode-II crack.

EXPERIMENTAL CONSIDERATIONS

A question rises now: "How could be realized experimentally a rigid body rotation and relatively to which frame must it be considered?" The only one fixed frame in space is given by the loading machine, which determines the loading direction. Thus, rigid body rotation of the plate must take place relatively to the loading direction. Such a rotation, should take place clockwise, leaving (x_0, y_0) directions fixed in space (see Fig.1). Moreover, this rotation should go progressively with load application, realizing at each loading step a Mode-II crack.

Preliminary experiments have been performed with precracked perspex plates and a holding fixture shown in Figure 2. Details on this fixture are given in (6). Cracks were either machined to their full length of 0.05m by a fine saw or by pushing a sharp blade into a shorter pre-existing crack. Tensile load applied by pins introduced in the 90° holes produce in the plate Mode-II loading conditions, while mixed-mode loading conditions with Mode-II dominance can be produced by load application through the 82.5° holes. Finally, the well known method of caustics has been used as observation method.

Experiments have shown a diverging behaviour of the cracked plates under Mode-II loading conditions according to the two kinds of initial crack machining, i.e. saw-crack and extended-crack respectively. The fracture load of the plates with extended cracks was about 2/3 or less than the respective load of identical plates with saw-cracks. Moreover, the development of the stress field at the crack-tip, as it was traced by the respective caustic curve-formation, was qualitatively distinguishable.In the plates containing saw-cracks a mixed-mode stress field was initially observed. Following load increase, the stress field was changing rapidly to a stress-field with evident Mode-II dominance and until fracture no hindrance of the stress-field development was observed. On the contrary, in plates containing extented-cracks, Mode-II stress field dominated from the beginning. During loading the development of this field seemed to face a kind of obstacles mainly near the crack flanks, as it was observed at the limit points of the caustic, and fracture was observed under smaller loads. The impression was that this hindrance supported fracture. These preliminary observations support the theoretical analysis presented in this paper. However, no definite results have been experimentally obtained yet. More experiments are needed and probably a refinement of the method of caustics, which ignores in its theoretical development the in-plane displacements.

CONCLUSIONS

It was shown that the introduction of a crack in a shear stress field does not result to a Mode-II crack but rather to a crack with theoretically overlapping flanks i.e. to a crack with unknown stress distribution. To realize a Mode-II crack a rigid-body rotation of the plate with the shear stress field is necessary. This rigid-body rotation depends on the stress level, the elastic properties and the prevailing plane-stress or plane-strain conditions. Moreover, in real situations it is so small, that its realization in experiments seems to be cumbersome. On the other hand there is some experimental evidence that uncontrolled and unknown stress distributions occur in plates with cracks under Mode-II loading conditions, depending on the friction forces developed between the crack flanks, which have the tendency to overlap.

REFERENCES

(1) Theocaris, P.S., Pazis, D. and Konstantellos, B.D., Int. J. of Fracture, Vol.30, 1986, pp. 135-153.

(2) Pazis, D., Theocaris, P.S. and Konstantellos, B.D., Int. J. of Fracture, Vol. 37, 1988, pp. 303-319.

(3) Banks-Sills, L., Arcan, M. and Gabay, H., Engng. Fracture Mech., Vol. 19, 1984, pp. 739-750.

(4) Podleschny, R., Ph.D. Thesis, Ruhr-University, Bochum, 1993.

(5) Muskhelishvili, N.I., "Some Basic Problems of the Mathematical Theory of Elasticity", Noordhoff Int.Publishing, Leyden, 1975.

(6) Love, A.E.H., "A Treatise on the Mathematical Theory of Elasticity", 4th Edition, Dover Publications, New York, 1974.

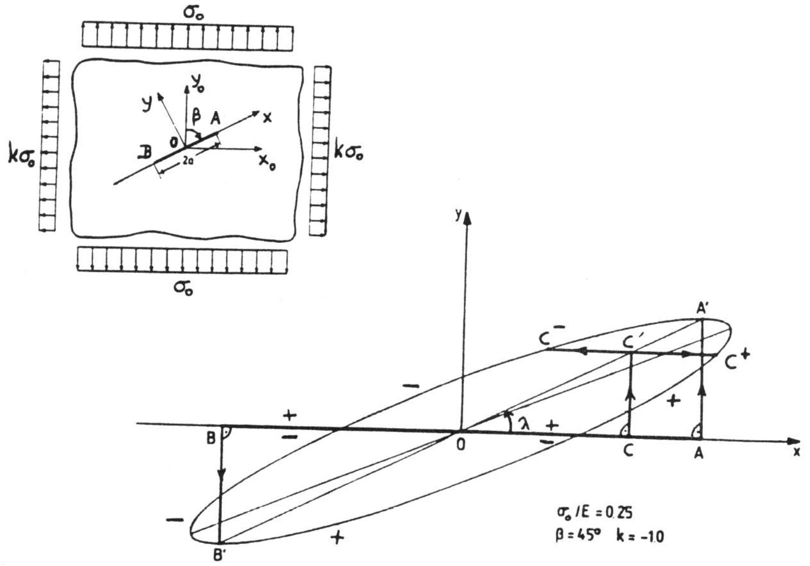

Figure 1 Overlapping geometry and the configuration of the problem

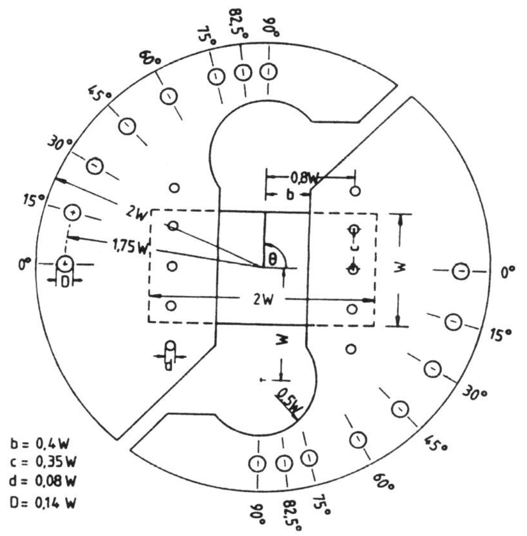

Figure 2 The holding fixture for mixed-mode cracks