ON STABILITY LOSS OF FIBRES IN COMPOSITE MATERIALS NEAR THE BOUNDARIES

Yu.N. Lapusta*

A rigorous approach to investigation of fibre instability near a boundary based on application of three-dimensional linearized stability equations to each component of a unidirectional fibrous composite is developed .It includes statement of the problems for an arbitrary disposition of parallel fibres, construction of solutions, exact satisfaction of all of the boundary conditions and derivation of characteristic equations. Examples of numerical results are presented and analysed for some particular cases.

INTRODUCTION

Investigations of fracture mechanisms of composites aimed at prevention of the loss of their load-carrying capacity are of a great importance nowadays. In many cases failure may be a result of fibre instability phenomena occuring near the border of a composite or product. This fracture mechanism is most commonly encountered in unidirectional fibrous composites subjected to axial compressive loads along the reinforcement. Related phenomena may also take place at compression of composites with insignificant reinforcement in the directions perpendicular to the direction of the main reinforcement, at multiaxial and biaxial loading when the compressive loads along the reinforcement are more intensive than that in the transverse directions, and in compressed zones of composites or products subjected to bending or or other action of forces.

* Institute of Mechanics, National Academy of Sciences of Ukraine

The most precise theoretical results on fibre instability phenomena may be obtained by means of employment of three-dimensional linearized stability equations and of the model of a piecewise-homogeneous medium. At that, the three-dimensional equations mentioned above should be applied to each element of a composite material and all the boundary conditions should be satisfied exactly. Special issue edited by Guz (1) gives an insight into the results which were obtained in the direction of investigation of the internal instability (i.e. instability in a structure of a composite without taking into account the influence of boundaries). However, allowing for the influence of a boundary surface is often necessary because all the real bodies have boundaries. So, the purpose of this work is to present some results on stability loss of fibres in composite materials obtained with allowance for the influence of a free plane boundary on the basis of the model of a piecewise-homogeneous medium with the employment of the three-dimensional linearised stability equations.

PROBLEM STATEMENT

We consider a unidirectional fibrous composite material with free plane surface. We introduce rectangular (x,y,z) and cylindrical coordinate systems $(r_p, 0_p, z_p)$ in such a way that the fibre with number "q" occupies the region $r_q \leqslant R$ and the matrix occupies the region $y \geqslant 0$. All the quantities related to the fibres will be denoted by superscript "a" and, if necessary, by subscript "q" indicating the number of a fibre.

It is assumed that compression along the reinforcement results in equal shortening e of the fibres and the matrix in this direction.

We also suppose that there exists total

$$P^a_{r,q} = P^m_r, \quad P^a_{\theta,q} = P^m_\theta, \quad P^a_{z,q} = P^m_z, \quad u^a_{r,q} = u^m_r, \tag{1}$$
$$u^a_{\theta,q} = u^m_\theta, \quad u^a_{z,q} = u^m_z \quad (r_q = R, \; q = 1, 2, 3, \dots)$$

or sliding contact between the fibres and the matrix.

At the plane surface of the matrix we require the satisfaction of the zero force conditions

$$P^m_x = 0, \quad P^m_y = 0, \quad P^m_z = 0 \tag{2}$$

We consider the case of homogeneous subcritical states of the fibres and the matrix and we will use general solutions of the three-dimensional linearised stability theory of deformable bodies proposed by Guz according to which the pertubations of the displacement components can be determined through functions Ψ and X by means of the formulas

$$u_x = \frac{\partial}{\partial y}\Psi - \frac{\partial^2}{\partial x \partial z} \; ; \quad u_y = -\frac{\partial}{\partial x}\Psi - \frac{\partial^2}{\partial y \partial z}X \; ; \quad u_z = A\left(\Delta_1 + B\frac{\partial^2}{\partial z^2}\right)X$$

$$\Delta_1 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \tag{3}$$

Where Ψ and X are the solutions of the equations

$$\left(\Delta_1 + \zeta_1^2 \frac{\partial^2}{\partial z^2}\right)\Psi = 0 \; ; \left[\Delta_1^2 + (\zeta_2^2 + \zeta_3^2)\Delta_1 \frac{\partial^2}{\partial z^2} + \zeta_2^2 \zeta_3^2 \frac{\partial^4}{\partial z^4}\right]X = 0 \tag{4}$$

In (3),(4) A, B, ζ_1^2, ζ_2^2, ζ_3^2 are coefficients depending on the subcritical state and on the properties of the material.

So, we arrive at the following mathematical formulation of the problems: it is necessary to determine the minimal shortening for which equations (4) for the matrix and equations

$$\left(\Delta_1 + \zeta_1^{a\,2} \frac{\partial^2}{\partial z^2}\right)\Psi_q^a = 0 ; \left[\Delta_1^2 + (\zeta_2^{a\,2} + \zeta_3^{a\,2})\Delta_1 \frac{\partial^2}{\partial z^2} + \zeta_2^{a\,2} \zeta_3^{a\,2} \frac{\partial^4}{\partial z^4}\right]X_q^a = 0 \; (r_q \le R) \tag{5}$$

for each of the fibres have nontrivial solution satisfying boundary conditions (1),(2) and attentuation conditions for the displacement perturbation components at $y \to \infty$.

DESCRIPTION OF THE PROCEDURE OF THE PROBLEM SOLUTION

We construct solutions for the matrix in the form of superposition of expressions represented in rectangular and each of the cylindrical coordinate systems as follows

$$\begin{aligned} \Psi = \gamma \sin \gamma z \sum_{t=1}^{2} \sum_{p=1} \Big\{ & \sum_{n=\delta_{t1}}^{\infty} A_{n,1}^{tp} K_n(\zeta_1 \gamma r_p)(\delta_{t1} \sin n\theta_p + \delta_{t2} \cos n\theta_p) + \\ & + \sum_{j=1}^{3} \sum_{n=n_{jt}}^{\infty} \int_{-\infty}^{\infty} V_{n,ij}^{tp}(\tilde{t}) \exp\left(-\gamma y \sqrt{\zeta_1^2 + \zeta_2^2 \operatorname{sh}^2 \tilde{t}}\right)\left[\delta_{t1} \sin(\zeta_j \gamma x_p \operatorname{sh} \tilde{t}) + \right. \\ & \left. + \delta_{t2} \cos(\zeta_j \gamma x_p \operatorname{sh} \tilde{t})\right] d\tilde{t} \Big\}, \quad \gamma = \pi/\ell, \; n_{jt} = \delta_{t1}\delta_{j1} + \delta_{t2}(1-\delta_{j1}); \; x_p = x - a_p \end{aligned} \tag{6}$$

Solutions for the fibres are constructed in the form

$$\Psi_q^a = \gamma \sin \gamma z \sum_{t=1}^{2} \sum_{n=\delta_{t1}}^{\infty} A_{n,1}^{atq} I_n(\zeta_1^a \gamma r_q)(\delta_{t1} \sin n\theta_q + \delta_{t2} \cos n\theta_q) \quad (7)$$

In (6) and (7) (a_p, h_p) denote the coordinates of the centre of the cross-section of the fibre No. p in the coordinate system (x,y), l is the length of a half-wave of the mode of instability, $A_{n,j}^{tp}$, $A_{n,j}^{atp}$ are unknown coefficients and $V_{n,ij}^{tp}(\tilde{t})$ are unknown functions. Functions X_q^a for the fibres and X for the matrix are constructed in a similar manner. Representing the solutions for the matrix in a rectangular coordinate system (x,y,z) and introducing them into (2) we obtain systems of equations for determination of the unknown functions under the integral sign.

To satisfy the conditions on the interfaces between the fibres and the matrix we represent the solutions for the matrix by turns in the coordinate systems (r_q, θ_q, z_q) and insert them jointly with the solutions for the corresponding fibre into (1). After the change of variables we arrive at infinite homogeneous system of algebraic equations in a matrix form

$$A_{rm}^{tu} X_m^{tu} + B_{rm}^{tu} X_m^{atu} + \sum_{w=1}^{2} \sum_{v=1}^{N} \sum_{n=0}^{\infty} C_{rmn}^{tuwv} X_n^{wv} = 0 \quad (8)$$

$$(k, r = 1,2; \quad u = 1,2,3,\dots,N; \quad m = 0,1,2,\dots)$$

for the case of total contact on the interfaces and at a similar system for the case of sliding contact. From condition of existence of nontrivial solutions we derive characteristic equations which can be written in the form

$$D(e,k) = 0 \quad (9)$$

where D(e,k) is the determinant of the corresponding infinite system of algebraic equations.

So, we can fix initial parameters of the problems and obtain correspondences between e and k for different forms of stability loss by means of numerical solution of equations (9). After solving (9) it is necessary to find the minimal shortening value for the fixed set of initial parameters of the problem considered.

NUMERICAL RESULTS AND CONCLUSIONS

As a result of computerized solution of the characteristic equation for the problem for a separate fibre in a semiinfinite matrix for different values of Ea/Em and h/R the functions e=e(k) were obtained. Some of these functions for h/R=2,4,8 and for the case of infinite matrix are depicted in Figure 1. These results make it possible to conclude that a free surface of a binder may influence significantly the stability of a single fibre even if the influence of other fibres is not taken into account.

As an example of results obtained for a pair of fibres near a free boundary, relations e=e(k) for four different forms of stability loss are presented in Figure 2. These results were obtained for Ea/Em=1000, $h_1/R=1.5$; $h_2/R=4$. Curve No.1 corresponds to the stability loss form (s.l.f.) in the plane of the fibres in the phase, No.2 - to s.l.f. out of this plane in the phase. No.3 and No.4 correspond to s.l.f. in the antiphase in and out of this plane, respectively. While in the case of the infinite matrix the s.l.f. out of the plane of fibres in the phase (dashed curve in Figure 2) is realized, here, due to the influence of the free surface, the s.l.f. in the plane of the fibres is realizable.

Figures 3 and 4 present the values of critical shortenings for a reriodic row of fibres for Ea/Em=50 (Figure 3) and Ea/Em=1000 (Figure 4). Curves No. 1-5 correspond to the values h/R= 1.5; 2;3;5;9. Results for the case of infinite matrix are depicted by the curves No.6. Solid curves correspond to the s.l.f. in phase for which the axes of the fibres remain in the planes perpendicular to the free border, and dashed curves -to the s.l.f. in phase for which the axes of the fibres go out of these planes From the results presented one can see that in the cases of small distances between the fibres and the free surface the critical shortenings in the problems considered differ from the critical shortenings calculated for large distances between the fibres and the free surface by 30-40%. The joint effects of the free surface and of mutual interaction of fibres may result in a reduction of critical shortenings by three times and more. This points out the necessity of taking into account of the mentioned phenomena in the calculations of the near-the-surface fibre instability in composites..

REFERENCES

(1) Guz, A.N. (guest editor) "Micromechanics of Composite Materials: Focus on Ukrainian Research", Special Issue, Applied Mechanics Reviews, Vol.45, No.2, pp. 13-101.

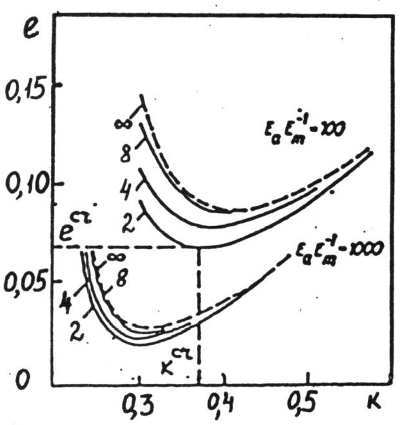

Figure 1 Relations e=e(k) for a separate fibre

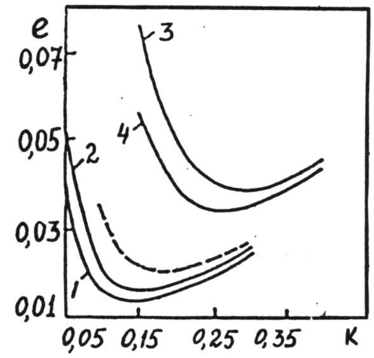

Figure 2 Relations e=e(k) for four s.l.f. of a pair of fibres

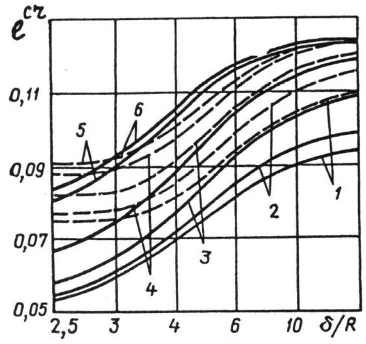

Figure 3 Critical shortenings for a row of fibres (Ea/Em=50)

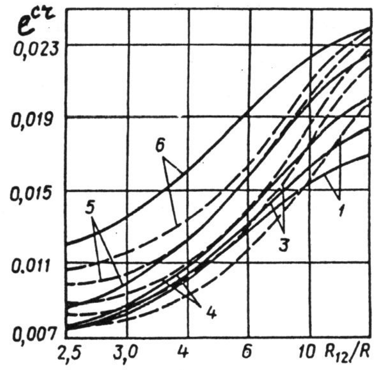

Figure 4 Critical shortenings for a row of fibres (Ea/Em=1000)