ON FAILURE PROPAGATION IN COMPOSITE MATERIALS IN COMPRESSION. (THREE–DIMENSIONAL CONTINUAL THEORY)

A.N.Guz' *

Mechanism of beginning and propagation of failure in composite materials under three–axial, two–axial and uniaxial compression is considered. The beginning of failure is determined as a result of stability loss in material structure (internal instability). The analysis of internal instability is made within the framework of three–dimensional linearised theory of deformable bodies stability. The value of failure loads and regularities of failure propagation are determined from the condition of transition of main system of equations in the system of hyperbolic type. In continual approximation composite materials are modelled by orthotropic elastic or elastic–plastic bodies with averaged constants.

BASIC STATEMENTS AND EQUATIONS

The first time continual three-dimensional theory of failure of composites in case of uniaxial compression was proposed by Guz' (1) and (2). Basic statement of the continual theory are reduced to the following.

1. In continual approximation laminated and fibrous composites with metal and polymer matrix are modelled by a compressible orthotropic body with the averaged constants. Since compressibility may develop in filler or matrix, an account must be taken of compressibility. In case of elasto–plastic models the averaged constants are calculated at moment of stability loss.

2. Fracture or failure in compression occurs as a result of stability loss in a structure of a composite material (internal instability), quantities of breaking loads (theoretical strength limits) are determined from the condition of transition of the main linearized system of equations into a system of a hyperbolic type. It must be noted, that corresponding linear system of equations

* Institute of Mechanics, Kiev.

for compressible solid is elliptical system. In view of it the surface of theoretical strength limits separates the region of ellipticity from the region of hyperbolicity in space of loading parameters.

3. Investigation of internal instability is carried out within the framework of three–dimensional linearized theory of deformable bodies stability including theory of finite precritical strains and two variants of theory of small precritical strains – according to Guz' (3).

4. In investigation of elasto–plastic materials (for composites with a metal matrix) within the framework of the three–dimensional linearized theory of deformable bodies stability the generalized conception of extending (continuing) loading is used in accordance with monographs (2,3). In this connection the change of off–loading zones in the process of stability loss is not considered. The mentioned statement furnishes an oportunity to consider fracture or failure theory with utilization of elastic and plastic models uniformely.

5. It is assumed, that loading is carried out by "dead" loads. In this connection for investigation of stability the static method of three–dimensional linearized theory of deformable bodies stability is used, since it was strictly proved (3), that in this case for nonlinearly elastic and plastic models sufficient conditions of applicability of the static method are satisfied.

6. Since furthermore investigation is carried out for a point of a material (for a small vicinity of a point – for a small volume element of continuum medium), the precritical state is considered to be homogeneous. Undoubtedly, the precritical state may vary in transition to the neighbouring point, thus, the precritical state should be considered as a local–homogeneous.

7. As applied to the point under consideration, of a material, we will investigate the process of loading by compression of uniformly distributed load (taking into account the statement 6) along axes of symmetry of the material only. In this case, before the process of fracture the homogeneous stress state occurs in the form

$$\sigma^0_{ij} = \delta_{ij}\sigma^0_{jj}; \qquad \sigma^0_{jj} = const; \qquad \sigma^0_{jj} < 0 \ldots\ldots\ldots\ldots \quad (1)$$

All values, related to the precritical state, and corresponding loads before fracture or failure are denoted by "0". It must be noted, that the precritical state (the term accepted in the stability theory) may be callled as the initial state, corresponding to beginning of fracture or failure processes in fracture mechanics under consideration. At the beginning of fracture process in addition to the initial stress state in the form (1) perturbances of the stress state appear, which, apparently, in value, may be considered less than stresssses of the initial (precritical) state. The mentioned consideration, in a certain sense, substantiates the oportunity of application of the linearized theory for description of perturbances behaviour at the beginning of fracture or failure process. The utility of application of three–dimensional linearized theory is seemed to be evident, for it is difficult to propose sufficiently by one of applied two–dimensional theories for description of a relatively delicate phenomenon – stability loss in structure of composite material.

In accordance with monograph (3) the system of differential equations in partial derivatives for compressible body has following form

$$\omega_{im\alpha\beta}\frac{\partial^2}{\partial x_i \partial x_\beta}u_\alpha = 0; \qquad i, m, \alpha, \beta = 1, 2, 3, \ldots \ldots \ldots \quad (2)$$

where: u_α - perturbances of a displacements; ω - tensor components of averaged constants, which are shown for various models in above mentioned monograph. In case of three–axial compression there are following conditions

$$\omega_{im\alpha\beta} = \omega_{im\alpha\beta}(\sigma^0_{11}, \sigma^0_{22}, \sigma^0_{33}); \qquad \sigma^0_{11}, \sigma^0_{22}, \sigma^0_{33} = const \ldots \ldots \quad (3)$$

FAILURE PROPAGATION

It must be noted, that we consider internal fracture in a composite material, which is determined by stability loss in structure of a composite material, i.e. by stability loss in microvolume. In the mechanism of fracture under consideration, changes must one way or another develop in macrovolume, and in the last case, therewith, they should not be of a local character, since only in this case fracture of the whole specimen will be considered (macrofracture). The mentioned changes in microvolume must be reflected in properties, which do not depend on boundary conditions, since process of material fracture is being investigated (internal fracture corresponds to an "infinite" material) instead of influence of grips of testing machines, the form of transverse section etc. It is also apparent that changes in microvolume are determined by perturbances of displacements, which are described by the system of equations (2). Thus, we may consider, that beginning of fracture corresponds to an appearing of solutions of the system equations (2) with the notation (3), which do not depend on boundary conditions and do not have a local character. It is obvious, that we should not give up solution of homogeneous stress states type (1). Given conditions for solution of the system of equations (2) may be satisfied only in case, if the mentioned solutions contain solutions of hyperbolic type, i.e. when the system of equations (2) becomes the system of hyperbolic type. As it was mentioned, in case the absence of initial stresses (1) ($\sigma^0_{ij} = 0$) corresponding linear system of equations (linear classical mechanics for compressible bodies) is a system of elliptical type, which

provides the uniqueness of solving of linear problems. *Therefore the beginning of fracture or failure process may be identified with that moment in history of loading, when the system of differential equations (2) from elliptical system transforms to hyperbolic one, i.e. when the system (2) losses the property of ellipticity; theoretical strength limits, therewith, are also determined from this condition.*

Within the framework of the continual theory of fracture or failure of composite materials in compression, under consideration it is possible not only to determine quantities of theoretical strength limits in compression, which are calculated from above formulated conditions, but to describe the nature of fracture or failure of composite materials in compression as well. We note, that in elucidated here continual theory the beginning of fracture or failure process is associated with appearence of perturbance, which do not have a local character and are not determined by boundary conditions. In this connection it is quite natural to associate (identify) the nature (regularity) of fracture or failure propagation with the nature of perturbances propagation (in a material), which has the above formulated properties. Since, perturbances in hyperbolic system of differential equations propagate over characteristic planes and surfaces, relative to the continual theory under consideration, we come to conclusion, that *fracture or failure of composite materials in compression occurs along characteristic planes and surfaces.* **For determination just directions at a point with coordinates $x_j^0 (j = 1,2,3)$ in which fracture or failure may propagate, it will suffice to find characteristic directions, which are specified by planes, the equations of which are represented in the following form**

$$\alpha_j(x_j - x_j^0) = 0; \qquad j = 1,2,3, \ldots \tag{4}$$

where $\alpha_j (j = 1,2,3)$ are roots of the characteristic equation

$$\Delta(\alpha) \equiv \det \|\omega_{irs\beta}\alpha_i\alpha_\beta\| = 0; \qquad i,r,s,\beta = 1,2,3 \ldots \tag{5}$$

with additional condition

$$|\alpha_1|^2 + |\alpha_2|^2 + |\alpha_3|^2 = 1 \ldots \tag{6}$$

Thus main equations for determination of failure propagation are presented, additional information is presented in monographs (2,4).

EXPERIMENTAL RESULTS FOR UNIAXIAL COMPRESSION

In case of uniaxial compression *the theoretical result (on base of above elucidated theory) is strictly proved that the failure propagates almost perpendicular to direction of compression.* **This result (monographs (2,4)) was**

obtained for general models of brittle and plastic fracture or failure. Consider the results of experimental studies for laminated and unidirectional fibrous composites.

Fig.1 shows the view after fracture of laminated composite. Fig.2 shows the view after fracture of unidirectional fibrous composite (square cross–section). Fig.3 shows the view after fracture of unidirectional fibrous composite (circular cross–section).

Experimental results (Fig.1–Fig.3) correspond to main theoretical result for case of uniaxial compression presented (formulated) in the beginning of this paragraph.

It should be noted, that in experimental investigations of fracture or failure mechanism, considered in this paragraph (Fig.1-Fig.3), some design and technological techniques, excluding occurence of fracture or failure mechanism in the form "of bearing strain in end faces" were utilized. So, in case in Fig.1 in specimens near ends transverse cross–section increased and in cases in Fig.2 and Fig.3 ends of specimens were fixed into rigid holders. If above mentioned and similar techniques do not use fracture or failure mechanism in the form "of bearing strain in end faces" may arise. For an example in Fig.4 the character of fracture or failure in the form "of bearing strain in end faces" of unidirectional boron– aluminium composite is shown (plastic failure 50% contents of boron fibers).

It should be noted, that in case of failure in the form "of bearing strain in end faces" failure propagates almost perpendicular to direction of compression, i.e. result of experimental investigation corresponds main theoretical conclusion in the framework of theory under consideration.

REFERENCES

(1) Guz', A.N., Dokl. AN UkSSR, Ser.A, No 3, 1969, pp.236–238 (in Ukrainian).

(2) Guz', A.N., "Stability of three–dimensional deformable bodies", Naukova Dumka, Kiev, Ukraine, 1971. (272 pp., in Russian).

(3) Guz', A.N., "Foundations of three-dimensional theory of deformable body stability", Vyshcha Shkola, Kiev, Ukraine, 1986. (512 pp., in Russian).

(4) Guz', A.N. Mechanics of composite materials failure in compression, Naukova Dumka, Kiev, 1990. (630 pp., in Russian).

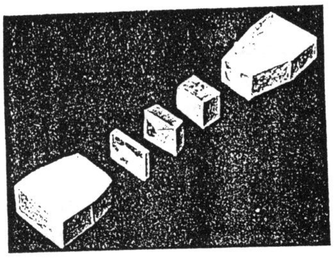

Figure 1. Specimen of a glass–cloth–base laminate.

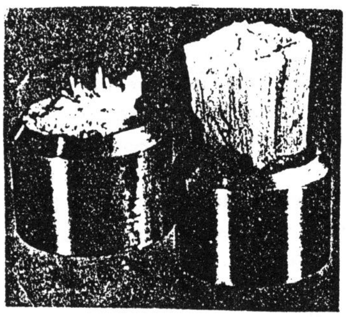

Figure 2. Specimen from a glass–plastic (square cross–section).

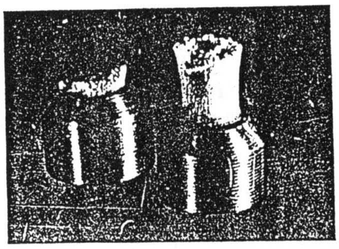

Figure 3. Specimen from a glass–plastic (circular cross–section).

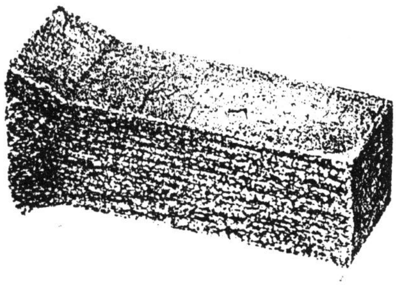

Figure 4. Specimen from a boron–aluminium.