NUMERICAL PATH INDEPENDENT INTEGRAL IN DYNAMICS FRACTURE MECHANICS

M. Attigui* and C. Petit*

A theoretical and numerical analysis of a new path independent integral in fracture dynamics is presented in this paper. For the analysis of fracture problems Destuynder (1983) proposed the G_θ-integral method, based on the Rice J-integral. We have generalized this method for dynamic fracture applications. Numerical tests give us accurate results of identification of the dynamic stress intensity factors.

INTRODUCTION

Path independent integrals are well known in fracture mechanics, and specific methods have been developed for the identification of the stress intensity factors but they are only valid for static fields. Elastodynamic path-independent integrals have been investigated by several authors. J-integral (1) was extended by Bui (2) for Elastodynamic crack problems. It is worthwhile to point out that Bui and Proix (3) have used the same approach to establish the T-integral and A-integral for thermoelasticity fracture problems. In this paper, we generalized the G_θ-integral method (4), for dynamic fracture problems. The numerical modelling is applied to stationary problems (the crack-tip velocity is zero). Our theoretical and numerical approach is validated by comparison with the kinematic method. Several numerical applications are shown to demonstrate the accuracy and the reliability of this method for the identification of the dynamic stress intensity factors. The present method is computed with various paths to demonstrate the path-independence of this integral.

* Civil Engineering Laboratory, University of Limoges, 19300 Egletons, France.

FORMULATION

The conventional J-integral has been extended by Bui (2) for Elastodynamic crack problems. When the crack lips are loaded, this integral becomes :

$$J^d(t) = \int_\Gamma \{\frac{1}{2}\ \sigma_{ij}u_{i,j}\delta_{j1} - \sigma_{ij}u_{i,1} - \frac{1}{2}\rho\dot{u}_i\dot{u}_i\delta_{j1}\ - \dot{a}\rho\dot{u}_i u_{i,1}\delta_{j1}\}n_j \mathrm{d}s + \frac{\mathrm{d}}{\mathrm{d}t}\int_{A(\Gamma)} \rho\ \dot{u}_i u_{i,1}\ \mathrm{d}S\ - \int_L \sigma_{ij}\ n_j\ u_{i,1}\ \mathrm{d}s \tag{1}$$

where Γ is an arbitrary contour around the crack-tip joining two points on the opposite of the crack lips. $A(\Gamma)$ is the domain enclosed within the contour Γ. σ_{ij} and u_i are the components of the stress tensor and displacement, respectively. $\dot{u} = \partial u_i/\partial t$ and ρ is mass density. $\dot{a}$ is the crack-tip velocity and n_j is the unit outward normal to the contour Γ. L is the total of the two crack surfaces. ds is the line infinitesimal and dS is the area infinitesimal. We defined the notations :

$$p_{j,1}n_j = \left[\frac{1}{2}\ \sigma_{ij}\ u_{i,j}\ \delta_{j1} - \sigma_{ij}u_{i,1} - \frac{1}{2}\rho\ \dot{u}_i\ \dot{u}_i\ \delta_{j1}\ - \dot{a}\ \rho\ \dot{u}_i\ u_{i,1}\ \delta_{j1}\right] n_j \tag{2}$$

Consider a crown around the crack-tip delimited by two contours Γ, Γ' and a field $\vec{\theta}$, such as $(\theta_1 = 1, \theta_2 = 0)$ inside the crown and $(\theta_1 = 0, \theta_2 = 0)$ outside (cf. figure 1). We can write $p_{j,1}\ =\ p_{j,k}\ \theta_k$ on the contour Γ. The line integral on Γ is converted to a line integral on the closed contour ∂V due to $\theta_k = 0$ on Γ'. ∂V is the closed contour around the crack-tip, constituted by the two contours Γ, Γ' surrounding the crack-tip and the two parts (AB, $A'B'$) on the crack surfaces (figure 1). The equation (1) can be written as :

$$G^d_\theta(t) = \int_{\partial V} -p_{j,k}n_j\theta_k \mathrm{d}s + \frac{\mathrm{d}}{\mathrm{d}t}\int_{A(\Gamma)} \rho\dot{u}_i u_{i,k}\theta_k \mathrm{d}S - \int_L \sigma_{ij}n_j u_{i,k}\theta_k \mathrm{d}s \tag{3}$$

Applying the Gauss-Ostrogradski theorem to equation (3), results in :

$$G^d_\theta(t) = \int_V -\left(p_{j,kj}\theta_k + p_{jk}\theta_{k,j}\right)\mathrm{d}S + \frac{d}{dt}\int_{A(\Gamma)} \rho\dot{u}_i u_{i,k}\theta_k \mathrm{d}S - \int_L \sigma_{ij}n_j u_{i,k}\theta_k \mathrm{d}s \tag{4}$$

The surface integral on $A(\Gamma)$ is the difference between the total surface and the surface of the crown ($A(\Gamma) = \Omega - V$). The equation (4) becomes :

$$G^d_\theta(t) = \int_V -\left[p_{j,kj} + \frac{\mathrm{d}}{\mathrm{d}t}\rho\dot{u}_i u_{i,k}\right]\theta_k \mathrm{d}S + \int_\Omega \left[-p_{j,k}\theta_{k,j} + \frac{\mathrm{d}}{\mathrm{d}t}\rho\dot{u}_i u_{i,k}\theta_k\right]\mathrm{d}S - \int_L \sigma_{ij}n_j u_{i,k}\theta_k \mathrm{d}s \tag{5}$$

After the development of the previous integrals, using equation (2), the dynamic G_θ-integral is given :

$$G_\theta^d(t) = \int_\Omega \Big\{ -\frac{1}{2}\sigma_{ij}u_{i,j}\theta_{k,k} + \sigma_{ij}u_{i,k}\theta_{k,j} + \frac{1}{2}\rho\dot{u}_i\dot{u}_i\theta_{k,k} + \frac{\mathrm{d}}{\mathrm{d}t}\rho\dot{u}_i u_{i,k}\theta_k \Big\}\mathrm{d}S - \int_L \sigma_{ij}n_j u_{i,k}\theta_k \mathrm{d}s \qquad (6)$$

The well-know relationship between the stress intensity factors and J-integral is establish by Irwin and extended in dynamic problems by Bui (5) as :

$$G_\theta^d(t) = J^d(t) = \frac{1}{E'}\Big[f_I(\dot{a})\Big(K_I^d(t)\Big)^2 + f_{II}(\dot{a})\Big(K_{II}^d(t)\Big)^2\Big] \qquad (7)$$

where $E' = E/(1-\nu^2)$ and $E' = E$ for plane strain and stress conditions, respectively, E and ν are Young's modulus and Poisson's ratio, respectively. $f_i(\dot{a})$ are the crack-tip velocity-dependent functions (Freund (6)). In this paper, the numerical modelling is applied to stationary problems so that the crack-tip velocity is zero ($\dot{a} = 0$) and ($f_i(\dot{a}) = 1$) in the previous equations.

NUMERICAL RESULTS

Numerical modelling was carried out, using finite elements code CASTEM 2000. The G_θ-integral is developed in the code as a procedure. The static integral (the two first terms in equation (6)) was implemented in the code by Suo (7). To valid the G_θ-integral method, the numerical results of this last are compared to the results determined by crack opening displacement (COD) method. This method is a local approach to evaluate the stress intensity factors. The K_I and K_{II} factors are proportional to the crack opening,

$$[u_2]^2 = K_I^2\,\frac{(k+1)^2}{2\,\pi\,\mu^2}\,r \quad \text{and} \quad [u_1]^2 = K_{II}^2\,\frac{(k+1)^2}{2\,\pi\,\mu^2}\,r \qquad (8)$$

where $k = 3 - 4\nu$ in plane strain condition or $k = (3-\nu)/(1+\nu)$ in plane stress condition.

We consider a Compact Compression Specimen (CCS) represented in figure 2. The CCS is used previously by Rittel (8) and Maigre (9) for experimental studies. This bar is loaded dynamically in the vertical direction by uniform tension $P(t) = 400$ MPa with heaviside-function time dependence. The boundary conditions given correspond to plane strain loading. The material of the strip is linear elastic with Young's modulus $E = 210$ GPa, Poisson's ratio $\nu = 0.285$ and density $\rho = 7800 kg/m^3$. The finite elements mesh, used in this example, is shown in figure 2.

The normalized mode I dynamic stress intensity factor $K_I' = K_I/P(t)\sqrt{\pi a}$ is plotted against t in figure 3, (time step $\Delta t = 0.5\mu s$). The numerical results obtained by COD method are also plotted in figure 3. However the value of the present method are in excellent agreement with those obtained by COD method. This last are lightly lower.

To verify the path-independence of the present method, five different crowns enclosed the crack-tip are selected. The normalized mode I dynamic stress intensity factors K_I' obtained for the five crowns, (at the times $t_1 = 80\mu s$, $t_2 = 100\mu s$ and $t_3 = 120\mu s$) are shown in figure 4. These results clearly demonstrate the path-independence of the present method.

CONCLUSION

A theoretical formulation of the dynamic G_θ-integral is presented. This integral is very interesting to obtain the stress intensity factors in fracture elastodynamic problems. To test the present method, a procedure is implemented in the finite elements code CASTEM 2000. The G_θ-integral is computed over various crowns. The results clearly demonstrate the path-independence of the present method. Different examples are tested, comparisons are made against results obtained by other numerical methods (COD methods). The numerical results demonstrate the accuracy and the reliability of the present method.

REFERENCES

(1) Rice, J. R., J. Applied Mechanics, Vol. 35, 1968, pp. 379-386.

(2) Bui, H. D., J. of Mech. and Phy. of solids, Vol. 31, 1983, pp. 439-448.

(3) Bui, H. D. and Proix, J.M., Actes du Troisième Colloque Tendances Actuelles en Calcul de Structure, Bastia, 1985, pp. 631-643.

(4) Destuynder P., J. de Méc. Théor. Appl., Vol. 2, 1983, pp. 113-135.

(5) Bui, H. D., "Introduction aux problèmes inverses en mécanique des matériaux", Eyrolles, France, 1993.

(6) Freund, L. B., J. of Elasticity, Vol. 2, 1972, pp. 341-349.

(7) Suo, X. Z., and Combescure, A., Nuclear Eng. Design, Vol. 135, 1992, pp. 207-224.

(8) Maigre, H. and D. Rittel, Int. J. of Solids Structures, Vol. 30, N° 23, 1993, pp. 3233-3244.

(9) Rittel, D., Maigre, H. and Bui, H. D., Scripta Metallurgica et Materialia, Vol. 26, 1992, pp. 1593-1598.

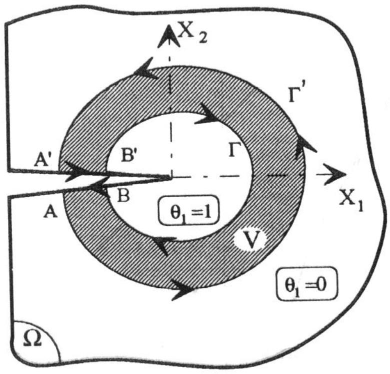

Figure 1. Domain and contour around the crack-tip

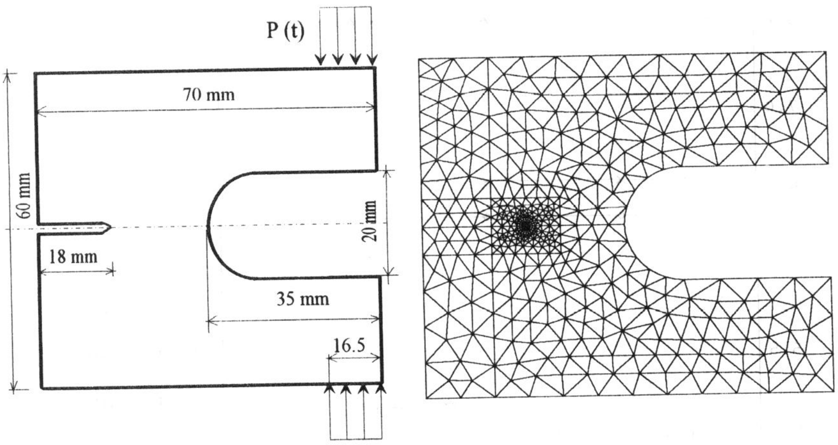

Figure 2. Geometry and finite elements mesh of the CCS

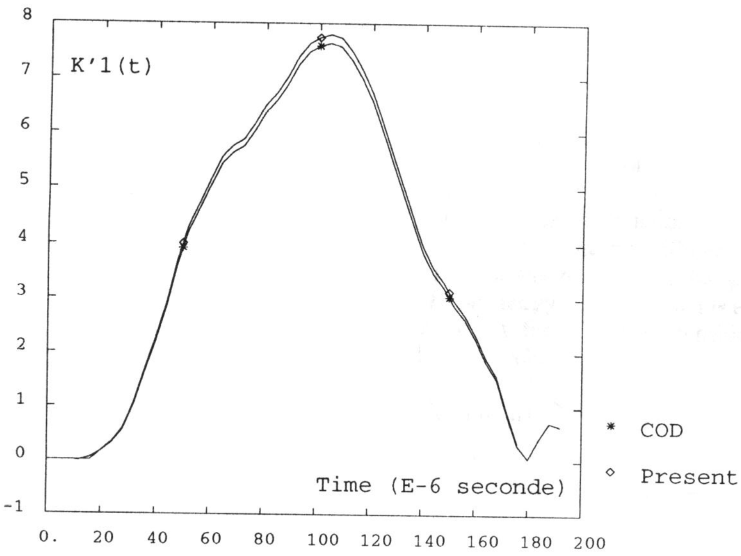

Figure 3. Normalized factor $K_I'(t)$

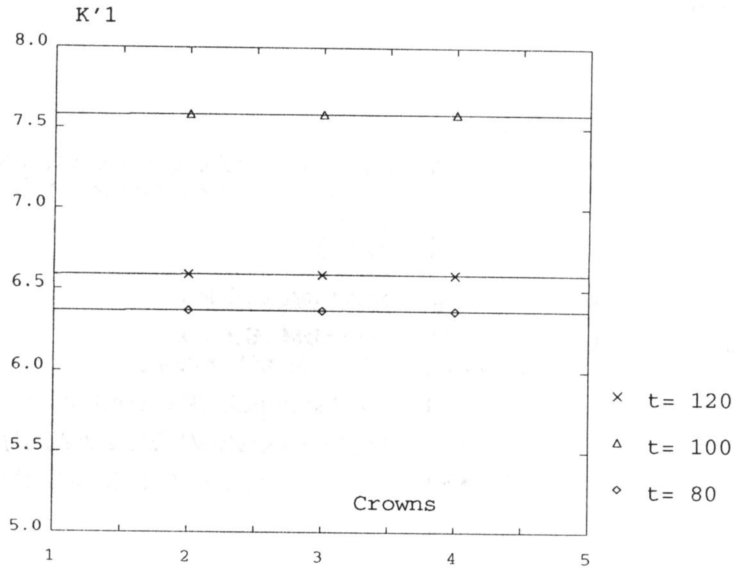

Figure 4. Normalized factor K_I' against the crowns