NUMERICAL METHODS IN ELASTO-PLASTIC FRACTURE MECHANICS

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A lot of cracked materials have an elasto-plastic behavior; so, the linear elastic fracture mechanics is generally not applicable, due to the fact that the plastic zone near the crack tip is large, mainly when the loading is increasing. In this paper, a global method based on nonlinear analysis with two numerical approaches is developed. The results are compared with those obtained by Irwin method; this last one gives accurate results when the intensity of loading is not very high, corresponding to a confined plastic zone. For higher loading, global nonlinear methods are more accurate.

INTRODUCTION

The linear elastic fracture mechanics (LEFM) is based on the elastic behavior hypothesis of cracked materials. In this case, several characteristic parameters can be determined : the stress intensity factors K_i and the strain energy release rate G (Irwin (1), Griffith (2)). But, when the mechanical behavior is elasto-plastic with a significative plastic zone size, the LEFM is not applicable, due to excessive yielding. In this case, models were established by several authors taking into account a nonlinear elastic behavior law instead of an elasto-plastic one. These methods are used only if the load is applied with increasing steps. The present report compares the results of an approached method (Irwin (1)) with two calculation results based on nonlinear analysis.

ENERGY RELEASE RATE G

When the plastic zone size is small, compared to planar dimensions, several approached methods based on LEFM could be found in literature. Among these different solving ways, three of them are more classically used :

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• In the first one, Dugdale (3) has corrected the plastic zone by an approached loading, corresponding to the yielding stress on an effective distance at the crack tip and then, he determined the crack tip opening displacement δ (CTOD) of real crack.

• In the second one, Eftis and Liebowitz (4) use the displacement-load curve and formulate this one by Ramberg-Osgood model. Then, they calculate the equivalent energy release rate by the nonlinearity factors.

• In the third one, Irwin (1) uses a correction of the plastic zone size by application of the LEFM and considers the redistributed stresses near the plastic zone. The stress field outside the plastic zone is similar to the elastic stress field found with equations for artificial $a_e = a + r_\sigma$ crack length. The term a_e corresponds to an artificial crack with its tip near the center of the plastic zone (see figure 1); r_σ is the radius of the plastic zone defined as follow :

$$r_\sigma = \frac{1}{2\pi}\left(\frac{K}{R_p}\right)^2 \tag{1}$$

R_p is the yielding strength and K is the stress intensity factor. If $K = F\sigma\sqrt{\pi a}$, the modified value K_e becomes :

$$K_e = F\sigma\sqrt{\pi(a + r_\sigma)} \tag{2}$$

F is a dimensionless function F(a_e/b). So, the equivalent G is expressed by :

$$G_{Irw} = \frac{{K_e}^2}{E} \tag{3}$$

Among those methods, the Irwin approached method was chosen and numericaly compared with nonlinear methods allowing the calculation of energy release rate. Dugdale and Irwin methods could be applied only when they verify the following hypotheses :

- elastic, perfectly plastic behavior,
- stress plane hypothesis,
- confined plastic zone,
- fracture in mode I.

When the plastic zone size is large and the material has an elasto-plastic behavior with strain hardening, those methods are not used. Several authors (Rice (5), Hutchinson(6), Rosenberg (5)) have calculated fracture parameters, G and J, for a nonlinear behavior. Those parameters correspond to the loss of potential energy of the cracked field during a unit crack propagation. G may be written as :

$$G = -\frac{\partial \pi}{\partial a} \tag{4}$$

where $\pi = W_\epsilon + W_{ext}$ is the total potential energy (W_ϵ is the strain energy of the material and W_{ext} is the energy of the external forces) and a is the length of the crack. The numerical G_{diff} and $G\theta$ methods permit us to resolve the global methods presented previously.

G_{diff} Method

In this approach, based on the finite difference method, G is obtained from the variation of the total potential energy during the extension Δa. It can be written as :

$$G_{diff} = -\frac{\pi(a + \Delta a) - \pi(a)}{\Delta a} \tag{5}$$

$G\theta$ Method

This global method, using an arbitrary vector field, is founded on a path independent integral. This field varies between (0,1) on the external path and (1,0) on the internal path. The formulation of this method developed by Suo (7) is :

$$G\theta = \int_A [-W\theta_{k,k} + \sigma_{ij}u_{i,k}\theta_{k,j}]dA \tag{6}$$

A is the area delimited by the two paths, and $W = \int_0^\epsilon \sigma_{ij} d\epsilon_{ij}$ is the strain energy density for linear or nonlinear elastic materials.

NUMERICAL RESULTS

In order to validate the previously presented methods, we consider a center-cracked plate, with the following dimensions : $a = 50\ mm$, $b = 100\ mm$, and $h = 300\ mm$. The behavior law is elastic perfectly plastic, with a yielding strength $R_p = 400MPa$ and a loading varying between 20 and $180N/mm^2$. The fracture is in Mode I and it is a plane stress analysis (see figure 2). In this example, the parameter of fracture G is calculated by different methods presented previously, using the finite elements code CASTEM2000 developed by CEA. Energy release rate calculated by G_{diff} method depends on the virtual crack extension Δa. In our case, the value of Δa is equal to 1/100 of the element length near the crack tip.

TABLE1 - Values of G using different methods

Loading N/mm^2	$G\theta$(ela.) MPa.mm	G_{Irw}(pla.) MPa.mm	$G\theta$(pla.) MPa.mm	G_{diff}(pla.) MPa.mm
20	0.440	0.439	0.440	0.440
40	1.760	1.762	1.760	1.761
60	3.961	3.999	3.987	3.986
80	7.041	7.199	7.173	7.178
100	11.002	11.433	11.418	11.440
120	15.843	16.800	16.858	16.921
140	21.564	23.432	23.768	23.887
160	28.165	31.505	32.669	32.897
180	35.646	41.247	44.997	46.380

G, calculated by $G\theta$ method, depends lightly on the path considered when the plastic zone is large (Baouch (8)). The results listed in the previous table present some differences between the various methods when the loading is increased. The values of energy release rate calculated by Irwin approached method are very close to those of $G\theta$ and G_{diff} methods. Irwin method is reliable as far as the plastic zone is confined at crack tip. But, when this area becomes important and tends to the nearest free edge (see figure 3), this method does not give satisfactory results. It was also noted, according to the values of $G\theta$ obtained in elasticity and in elasto-plasticity as shown in column 2 and 4 of table 1, that the plastic zone appears for a loading near from $60N/mm^2$. The values of $G\theta$ and G_{diff} are not so different at each loading step. It can be noted that the value of G_{Irw} is similar to the values obtained by $G\theta$ and G_{diff} methods until loading near than $140N/mm^2$; after that, the large plastic zone induces a lower value by such a method because the confined plastic zone hypothesis is not respected and, moreover, Irwin (1) considers a circular plastic zone instead of a butterfly wing shape.

CONCLUSION

The previously methods allow the computation of energy release rate G. The advantage of the approached method of Irwin resides in elastic calculation without important differences, compared to $G\theta$ and G_{diff} methods, while the plastic zone is not very large. This approach can be used when the hypotheses presented previously, are respected. Meanwhile, G_{diff} method allows to take into account the elastic-plastic behavior of the materials and determines G value by integration on the total structure. Its major advantage is in the fact that the calculation is independent with regard to the plastic zone position. The $G\theta$ method is more economical than G_{diff} method because it calculates G in only one configuration. Its accuracy is correct.

SYMBOLS USED

r_σ = radius of the plastic zone

R_p = yielding strength (MPa)

K = stress intensity factor ($MPa.\sqrt{mm}$)

a = crack length (mm)

F = dimensionless function

G = energy release rate (MPa.mm)

W = strain energy density (MPa)

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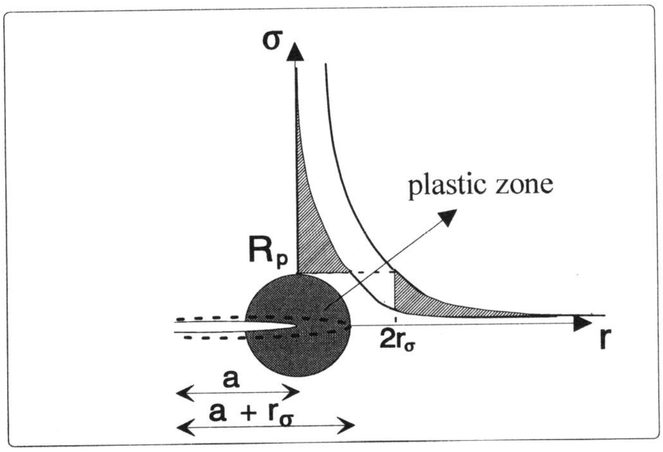

Figure 1 Dimension of the plastic zone : Mode I (Irwin)

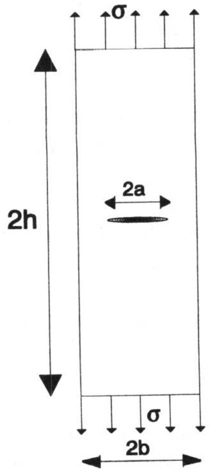

Figure 2 Center-cracked plate

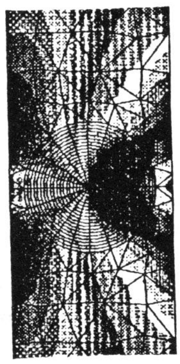

Figure 3 Plastic zone size (200 MPa)