NUMERICAL METHOD FOR PREDICTING THE GROWTH OF A CRACK IN ELASTIC-PLASTIC MATERIAL UNDER FATIGUE LOADING

N. RECHO , X.B. ZHANG and T. MORO

In this paper, a numerical method is proposed to predict the growth of a crack in elastic-plastic materials under cyclic loading. The numerical applications have shown that the crack bifurcation angle obtained by the proposed method are in good agreement with the experimental results. However, these bifurcation angles depend on some parameters, such as the ratio of the loading level and the yield stress of the material, the ratio of the mechanical properties of the plastic zone and those of the initial material, the ratio of the stress intensity factors K_I and K_{II} . Some examples have shown the procedure for predicting the crack growth until the failure of the studied structure under a mixed mode fatigue loading.

INTRODUCTION

Under cyclic loading, most of structures are subjected to the phenomenon of fatigue which initiates cracks. These cracks grow by fatigue. The prediction of the crack growth is based on the criteria of bifurcation and propagation according to the fracture mechanics analysis for quasi-static loading. In the case of cyclic loading, these criteria can not be used directly because of the plastification at the vicinity of the crack. The purpose of this work is to propose a methodology for the study of the bifurcation and propagation of a crack in an elastic-plastic material under cyclic loading and to analyse the influence of the plastic zone on the bifurcation angle. This methodology is based on the fact that the mechanical properties of the plastic zone near the crack tip are changed after a high number of cycles and the material becomes harder. The elastic part is considered like a new material with a fictitious Young's modulus. Therefore, the effect of fatigue and the presence of the plastic zone near the crack tip are taken into account easily by using linear elastic finite element calculations.

* Laboratoire d'Etudes et Recherches en Mécanique des Structures
University B.P. of Clermont-Fd - Av. Aristide Briand 03100 Montluçon France

PRESENTATION OF THE PROBLEM

In this work, we analyse the crack bifurcation according to the criterium of the maximum circumferential stress $\sigma_{\theta\theta}$ (1). This leads to :

$$K_I \sin\theta_0 + K_{II}(3\cos\theta_0 - 1) = 0$$

where K_I and K_{II} are the stress intensity factors corresponding to the two fracture modes, θ_0 is the bifurcation angle (Fig. 1).

Case of monotonic loading

It is known that, for an elastic medium under a monotonic loading, Young's modulus E has no influence on the determination of the bifurcation angle. But it is essential to determine the critical loading. When there is plastification near the crack tip, the elastic criterium of bifurcation consists in the use of a fictitious Young's modulus Ee, indicated in figure 2, so that the strains of the structure are considered as elastic ones. The bifurcation angle obtained according to the criterium of the maximum circumferential stress $\sigma_{\theta\theta}$ does not be modified by the value of Ee.

Case of cyclic loading

During cyclic loading, the elastic zone keeps its initial mechanical characteristics. However, the characteristics of the material are modified in the plastic zone. If the fictitious Young's modulus Ee is considered, its value increases during cyclic loading and the material becomes harder in the plastic zone. Therefore, by using a bigger value of Young's modulus, noted Ep, than that in the elastic zone, Ee, the effect of the cyclic loading and the presence of the plastic zone near the crack tip could be taken into account implicitly.

METHODOLOGY

It consists in carrying out an elastic calculation with the adopted value of Young's modulus Ee which allows to obtain the strains equivalent to those obtain after a high number of loading cycles on the unloaded material. The elastic part of the structure keeps tis Young's modulus Ee. The plastic zone near the crack tip is considered like a new material having linear elastic behaviour with Young's modulus Ep which is bigger than Ee (2).

In order to study the bifurcation and the propagation of a crack in mixed mode, the following numerical procedures by finite element method can be used:

a) A model of linear elastic behaviour for whole structure is defined at first.

b) The displacements, strains and stresses are computed and then the plastic zone near the crack tip is determined according to the Von-Mises criterium.

c) A mixed material is defined : linear elastic material with Young's modulus Ep for the found plastic zone and linear elastic material with Young's modulus Ee for the elastic part.

d) Second finite element computation on the mixed material is needed to determine the displacements, strains and stresses in order to calculate the cracking parameters such as the stress intensity factors, the energy release rate G or J-integral (3).
(In this work, the stress intensity factors are calculated by using kinetic method).
e) The bifurcation angle θ_0 can then be determined according to bifurcation criterium.
f) A small extension of the crack Δa is considered in the direction of the calculated bifurcation angle.

The simulation of the crack growth is carried out step by step until the failure of the studied structure.

NUMERICAL APPLICATIONS

The following examples shows the application of the proposed method (4).

Example 1 : A plate under a tensile loading σ_y= 100 MPa with a central crack. The crack (length 2a=3.6 mm) is inclined at an initial angle α with respect to X-axis (figure 3). The yielding stress of the material is 360 MPa. The bifurcation angle θ_0 with respect to the axis of the crack for different values of α is calculated and then compared with the results of some experiments (5). In order to analyse the influence of the ratio Ep/Ee, the bifurcation angle θ_0 is calculated for different ratio Ep/Ee. The value of Ep is fixed to 210000 MPa. Figure 5 shows the results of the comparison with experimental results. One can note that the bifurcation angle decreases as the ratio Ep/Ee increases. The obtained results are in good agreement with experimental results particularly when the initial inclined angle α is small, with a big ratio Ep/Ee, and when α is big, with a small ratio Ep/Ee.

Example 2 : A plate under a tensile loading σ_y = 100 MPa with an inclined edge crack (Figure 4). The initial crack length equals to 4 mm and the inclined angle α equals to 45°. The ratio Ep/Ee = 2 is considered. The yielding stress of the material is 300 MPa The crack growth is simulated numerically by the use of a finite element code capable of automatically tracking until the failure of the studied structure. It is assumed that the cracking parameters of the structure get their critical values during the propagation. So, only the track of the crack growth is investigated. The form of the plastic zone around the crack tip changes during the propagation. Figure 6 illustrates the track of the crack growth and some steps of the simulation during the crack growth. The evolution of the plastic zones can be observed from the blake parts in the figure.

Example 3 : Figure 7 represents another studied plate under a tensile loading σ_x = 200 MPa with an inclined crack at one of its corners. The crack with the initial length of 100 mm is inclined at 45° with respect to X-axis. The values of Young's modulus in the elastic part and in the plastic zone are chosen as follows : Ee =105000 MPa and Ep = 210000 MPa. The yielding stress of the material is 300

MPa. The crack growth is followed by the same procedure as that in example 2 until the failure of the plate. The track of the crack growth is shown in Figure 8.

CONCLUSION

The proposed method allows to study the bifurcation and propagation of a crack in a 2D elastic-plastic material under a mixed cyclic loading. The influence of the plastic zone near the crack tip is considered by means of two linear elastic finite element calculations. The bifurcation angles obtained by using the presented method are in good agreement with those of experiments.

It has been found that the value of the fictitious chosen Young's modulus Ee depends on the level of the applied load. So, the obtained bifurcation angle depends not only on K_I/K_{II} (as in the case of monotonic elastic loading) but also on some parameters, such as the ratio of the loading level and the yield stress of the material and the ratio Ep/Ee.

REFERENCES

(1) Sih G.C.; "*Introductory Chapter : A Special Theory of Crack Propagation*" Mechanics of fracture *1*" Noordhoff, Leyden ,1972.

(2). Zhang X.B., Recho N "Modélisation numérique de la bifurcation et propagation des fissures en fatigue" 12ème congrès Français de Mécanique, Strasbourg.1995

(3) Recho N, "Rupture par Fissuration des Structures " Edited by Hermès, Paris, 1995.

(4). Moro T., Zhang X.B. and Recho, "Critère de bifurcation - Modélisation d'un critère de bifurcation en fatigue", Rapport du Laboratoire d'Etudes et Recherches en Mécanique des Structures, Juil, 1995

(5) Recho N., Brozzetti J., "Crack Criteria under Fatigue Loading", Third International Conference on Numerical Methods in Fracture Mechanics, Swansea, UK, 26-30 March 1984

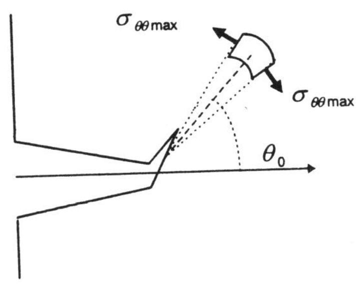

Figure 1 : Bifurcatuion angle of a crack

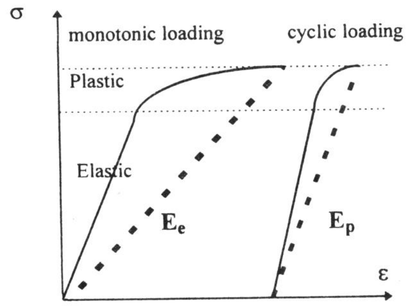

Figure 2:E_e and E_p in monotonic and cycling loadings

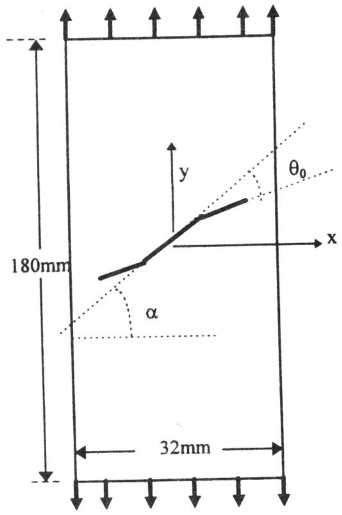

Fig. 3. Plate with a central crack

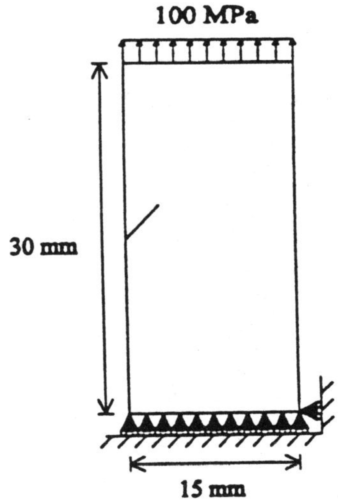

Fig. 4. Plate with an inclined edge crack

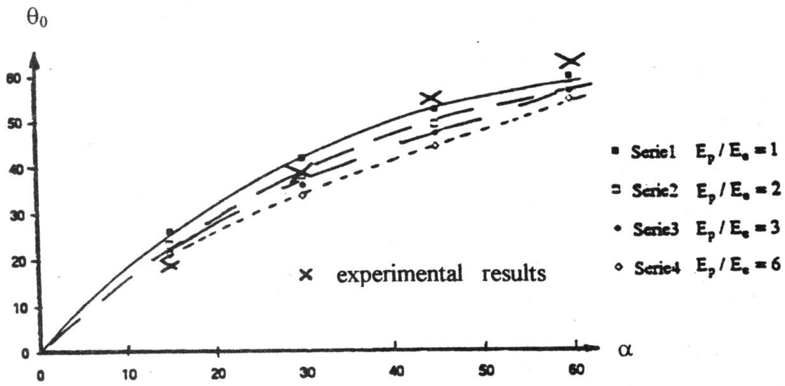

Fig. 5. Bifurcation angle against initial inclined angle for different ratio Ep/Ee

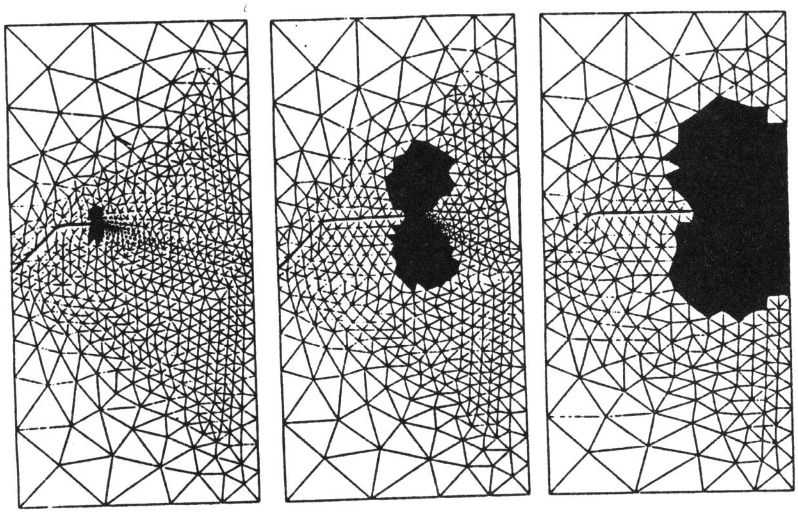

Fig. 6. Track of the crack growth and some simulation steps

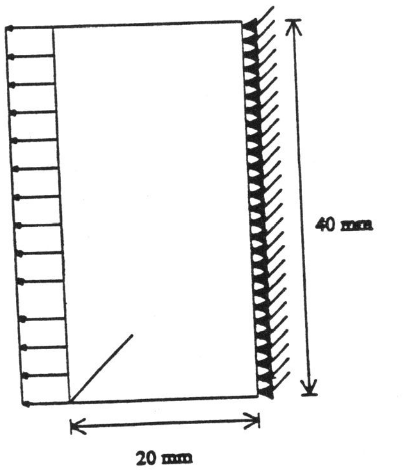

Fig. 7. Plate with an inclined crack at its corners

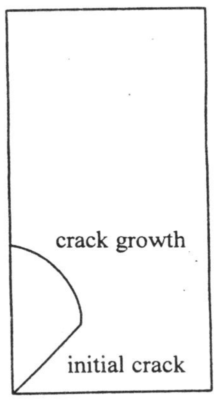

Fig.8. Crack growth track