NUMERICAL EVALUATION OF THE ANVIL FORCE FOR PRECISE PROCESSING OF THE IMPACT FRACTURE TEST DATA

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To evaluate precisely the dynamic fracture toughness of a brittle material in the tests with small time-to-fracture, both tup and anvil forces have to be known. Unfortunately, the anvil force is rarely registered by the standard equipment. The method for numerical evaluation of the support reactions by using registered tup force and the specimen modal parameters is proposed. It assumes that the contact between the specimen and the supports can be described by Hertz's law. Two methods of linearization of the specimen-support contact stiffness are considered. It has been found that selection of linearization method has insignificant effect on the final result - dynamic stress intensity factor variation with time. The efficiency of the method has been verified by processing three-point-bend data reported by W.Böhme and J.D.Kalthoff.

INTRODUCTION

Instrumented impact fracture testing of precracked beam specimens is widely used for determination of dynamic fracture toughness of brittle materials. Only the tup force $F(t)$ is usually registered during a test. To complete the boundary conditions for further theoretical analysis, permanent contact between the specimen and supports is assumed. Unfortunately, this simplistic assumption is invalid (Böhme and Kalthoff (1)) and leads to noticeable errors in determination of $K_I(t)$. When the time-to-fracture is comparable with the period of the specimen oscillation according to the first mode, the registration of anvil force $R(t)$ is necessary to compute $K_I(t)$ with reasonable accuracy (Rokach (2,3)).

Unfortunately, simultaneous registration of $F(t)$ and $R(t)$ is often impossible with the help of standard equipment. An alternative method for $R(t)$ determination is also needed when one wants to reanalyze more precisely previously obtained data for tests in which only $F(t)$ has been recorded. For these situations, the method for $R(t)$ calculation using specimen modal parameters and specimen-support contact stiffness is proposed below.

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MODEL ASSUMPTIONS AND THEORY

The following assumptions were used to model the specimen interaction with supports:

- specimen material is linearly elastic;
- specimen-striker and specimen-supports interaction is modeled by three point forces: $F(t)$ and two equal $R(t)$;
- supports are perfectly stiff;
- contact between the specimen and supports is described by Hertz's law;
- both registered loading $F(t)$ and calculated one $R(t)$ are approximated piecewise linearly with the constant time step.

Taking into account symmetry of the problem, we consider a half of the specimen (Fig.1). Let the point force $P(t)$ acts in the x-direction at the (x_P,y_P) point of the specimen. Vertical (in the Fig.1 convention) displacements in arbitrary point (x,y) can be represented as the sum of rigid body movement of the specimen and its deflection in this point

$$u(P(t),x_P,y_P,x,y,t)=\int_0^t \frac{P(\tau)}{m}(t-\tau)d\tau+ \\ +\sum_{i=1}^{\infty}\frac{\phi_i(x,y)\phi_i(x_P,y_P)}{\omega_i}\int_0^t P(\tau)\sin(\omega_i(t-\tau))d\tau \quad (1)$$

Utilizing classic Timoshenko approach, one can obtain the following equations for vertical displacements in the contact point R between the specimen and the support

$$u_R=u(\frac{1}{2}F(t),x_F,y_F,x_R,y_R,t), \qquad u_R\le 0, \quad (2)$$

$$u_R=u(\frac{1}{2}F(t),x_F,y_F,x_R,y_R,t)-u(R(t),x_R,y_R,x_R,y_R,t), \quad u_R>0 \quad (3)$$

On the other hand, for $u_R>0$, the displacement in the point R is related to the contact force $R(t)$ by Hertz's law $R(t)=ku_R^{3/2}$. After substitution of this relation into Eqn. (3) one can obtain the nonlinear equation for $R(t)$ determination. For small u_R values this equation can be linearized assuming $R(t)=k^*u_R$. After substituting piecewise linear approximations of $F(t)$ and $R(t)$ into linearized Eqn.(3), the simple recurrent formula can be obtained to determine the $R(t)$ values with the constant time step.

CALCULATIONS

Modal parameters ϕ_i and ω_i have been computed for Euler-Bernoulli beam and two-dimensional (2D) plane stress models of the specimen. For the first model the existence of the crack has been modeled by an elastic hinge with suitable stiffness situated in the specimen midspan. Plane model data were obtained using finite element program ADINA 6.1.

To check the accuracy of the method proposed, results of calculations were compared with experimental data obtained by Böhme (4). Due to the lack of space, results only for one three-point-bend impact test will be considered below.

Araldite B specimen with length 412 mm, width 100 mm, thickness 10 mm and crack length 30 mm was loaded by 4.9 kg free-falling tup. Distance between supports was 400 mm, r=10 mm. During the test $F(t)$ and $R(t)$ were registered using strain gages pasted on the striker and the supports, $K_I(t)$ was registered by caustics method.

Contact stiffness between the specimen and the support can be computed from the relation $k=(4/3)r^{1/2}E/(1-\nu)$. Two methods were used to determine linearized contact stiffness. In the first method it was assumed that $k_1^* = \max(R(t))/\max(u_R) = \max(R(t))^{1/3}k^{2/3}$, where as the first approximation the relation: $\max(R(t)) = 1/2 \max(F(t))$ was used. This method overestimates the Hertzian contact force (Fig. 2). In the second method linearized contact stiffness k_2^* was calculated from the assumption of equality of the works done by Hertzian and linearized contact forces when u_R increase statically from 0 to $\max(u_R)$. Using this assumption one can obtain $k_2^* = (4/5)\max(R(t))^{1/3}k^{2/3} = (4/5)\,k_1^*$.

RESULTS AND DISCUSSION

For the test data considered, values of $R(t)$ have been calculated with the 20μs time step for the beam and 2D models of the specimen using two methods for linearization of contact stiffness mentioned above. Number of the specimen eigenmodes taken into account varied from 1 to 12. Computed loading $R(t)$ and registered $F(t)$ were used for $K_I(t)$ determination. The following conclusions may be formulated from the results of calculations:

1. The method proposed allows to compute $R(t)$ with reasonable accuracy (see Fig.3). This accuracy is much better for 2D model of the specimen than for the beam model.
2. The contribution of the higher modes into $R(t)$ is negligible for specimens with the span-to-length ratio close to 1. Thus, only the first mode data may be used in calculations. For relatively longer specimens (e.g., Charpy type) it is recommended to take into account the data for the two lowest modes.

3. The difference of 20 per cent between linearized contact stiffness values (k_1^*=767 MN/m, k_2^*=614 MN/m for the test considered) does not lead to a large difference in $R(t)$. The difference between corresponding $K_I(t)$ curves is yet smaller (see Fig.4).

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SYMBOLS USED

E =	Young modulus of the specimen material
$F(t)$ =	tup or striker force
$K_I(t)$ =	dynamic stress intensity factor
k =	contact stiffness between the specimen and a support according to Hertz law
k_1^*, k_2^*=	linearized contact stiffness
m =	mass of a half of the specimen
$R(t)$ =	force measured on one of the supports
r =	radius of curvature of the contact surface of the support
t =	time
u_R =	depth of penetration of the support into the specimen surface
ϕ_i =	vertical or x-components of normalized eigenmodes of a half of the specimen
ν =	Poisson ratio of the specimen material
ω_i =	nontrivial eigenfrequencies of the half of the specimen (i.e., the symmetric ones for the whole specimen)

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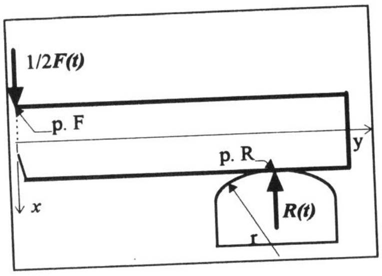

Figure 1. Half of the specimen with forces acting on it

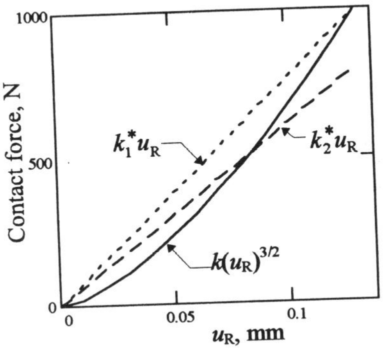

Figure 2. Hertzian contact force and its approximations.

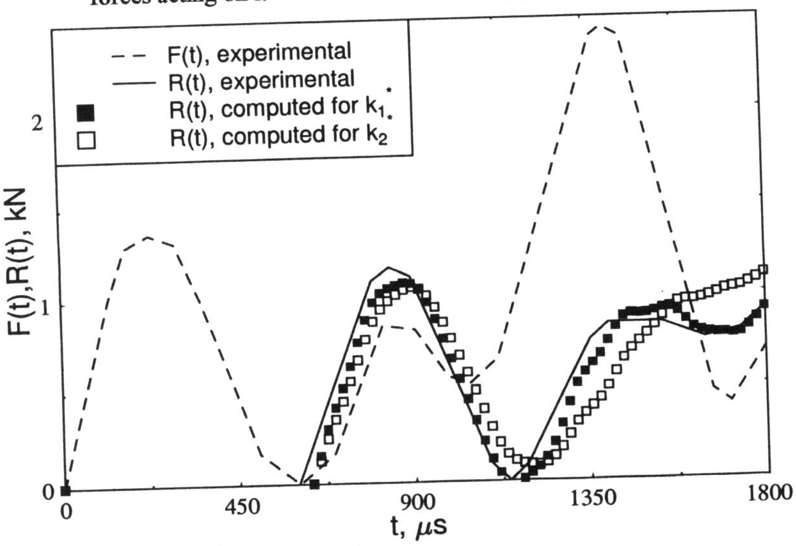

Figure 3. Force-time diagrams registered experimentally and computed for 2D model of the specimen

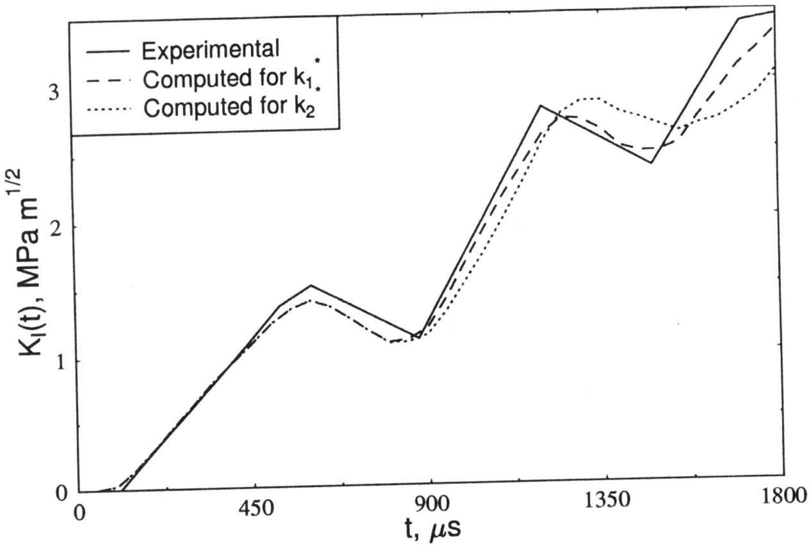

Figure 4. Variation of $K_I(t)$ with time registered experimentally and computed for 2D model of the specimen