NUMERICAL APPROACH IN VISCOELASTIC FRACTURE MECHANICS

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In the fracture theory applied to linear viscoelastic material, the stress intensity factor is the primary characterising parameter for fracture initiation time and crack speed. But, the principle of Boltzmann which makes up the relationship between strain and stress tensor must be introduced in the different formulations of fracture characteristics. This paper presents a numerical model, implemented in Castem 2000, a finite element code, which evaluates the energy release rate and the stress intensity factors for complex plane structures.

INTRODUCTION

For viscoelastic media, energy release rate is often employed in the initiation crack growth. For reason of mathematical simplicity, the material in a small neighbourhood surrounding the crack tip is described by a failure zone size which is defined by the stress intensity factors. To realise fracture simulation for complex structures and any loading, it is necessary to developed numerical models.

The first part presents, rapidly, the incremental method based in a discrete creep spectrum decomposition described, in details, by other publishes.

In order to define different fracture characteristics, the second part concerns the singular stress and displacement fields around the crack tip domain, for a linear viscolelastic media.

In order to obtain fracture and viscoelastic coupling, the third part treats an incremental modelling to evaluate stress intensity factors and the energy release rate.

The last part presents a numerical application with a CTT specimen in a creep simulation, with an opening mode loading.

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LINEAR VISCOELASTIC BEHAVIOUR MODEL

Linear viscoelasticity and especially the model of linear viscoelastic solid can be traced back to Boltzmann who considered an elastic material with memory. At any point of the body, the stress tensor whenever t, depends on the complete past history of strain tensor. This method adapted for plane stress problems with time-independence of Poisson ratio. If the material is assumeding, the stress-strain linear relationship is given by:

$$\begin{pmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ 2\cdot\varepsilon_{12} \end{pmatrix} = \begin{bmatrix} 1 & -\nu & 0 \\ -\nu & 1 & 0 \\ 0 & 0 & 2\cdot(1+\nu) \end{bmatrix} \cdot \int_0^t \frac{1}{E(t-\tau)} \cdot \frac{\partial}{\partial \tau} \begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{pmatrix} d\tau \qquad (1)$$

$\varepsilon_{\alpha\beta}$ and $\sigma_{\alpha\beta}$ represent respectively the strain and stress tensor components. According to the discrete creep spectrum, presented by Mandel, and response of the generalised Kelvin model, shown in Figure 2, the Young modules $E(t)$ can be written in terms:

$$\frac{1}{E(t)} = \frac{1}{k^0} + \frac{t}{\eta^\infty} + \sum_{m=1}^{M} \frac{1}{k^m} \cdot \left(1 - e^{-\frac{k^m}{\eta^m} t} \right)$$

were k^0 and η^∞ represent the spring and dashpot constants of the Maxwell model and k^m and η^m is the values of spring and dashpot composing the mth Kelvin Voigt element. The entire development and validations of this model, introduced in the software Castem 2000, are presented in different papers (1) and (2).

FRACTURE VISCOELASTIC CHARACTERISTICS

This parts defined stress and displacement fields for a viscoelastic media around the crack tip. According to the linear elasticity theory, these fields can be expressed as:

$$\sigma_{\alpha\beta}(\theta, r) = \frac{f_{\alpha\beta}(\theta)}{\sqrt{2\cdot\pi\cdot r}} \cdot K_\beta^{(\sigma)}$$

$$u_\alpha(r,\theta) = \sqrt{\frac{r}{2\cdot\pi}} \cdot \left(\frac{1}{2\cdot\mu} \cdot g_{\alpha\beta}(\theta) \cdot K_\beta^{(\sigma)} + \frac{\lambda}{2\cdot\mu} \cdot h_{\alpha\beta}(\theta) \cdot K_\beta^{(\sigma)} \right) \qquad (2)$$

where (r, θ) is the polar co-ordinates in the crack tip. $f_{\alpha\beta}$, $g_{\alpha\beta}$ and $h_{\alpha\beta}$ are well known angular functions, μ is the shear modules and $\lambda=(3-\nu)/(1+\nu)$ for plane stress. With a viscoelastic body, these stress intensity factors do not uniquely define the crack tip fields, since in the viscoelastic case there is not a one to one relationship between the stress field and the strain. Equation (2) can be transformed as, Brincker (3):

$$u_\alpha(r,\theta,p) = \sqrt{\frac{r}{2\cdot\pi}} \cdot \left[g_{\alpha\beta} \cdot C_\beta(p) + h_{\alpha\beta} \cdot D_\beta(p) \right] \qquad (3)$$

where C_β and D_β represent the strain intensity factors. With a viscoelastic behaviour, in the time domain, these factors can be defined as:

$$C_\beta(t)=\int_0^t \frac{1}{2\cdot\mu(t-\tau)}\cdot\frac{\partial K_\beta^{(\sigma)}(\tau)}{\partial\tau}d\tau \text{ and } D_\beta(t)=\int_0^t \frac{\lambda(t-\tau)}{2\cdot\mu(t-\tau)}\cdot\frac{\partial K_\beta^{(\sigma)}}{\partial\tau}d\tau \qquad (4)$$

From displacement field, the crack tip opening displacement is defined by:

$$[\vec{u}](r,t)=\vec{u}(r,\theta=+\pi,t)-\vec{u}(r,\theta=-\pi,t)$$

From equation (3) this crack tip opening displacement components can be write as:

$$[u_1]=2\cdot(C_2+D_2)\cdot\sqrt{\frac{r}{2\cdot\pi}}=K_2^{(\varepsilon)}\cdot\sqrt{\frac{r}{2\cdot\pi}} \text{ and } [u_2]=2\cdot(C_1+D_1)\cdot\sqrt{\frac{r}{2\cdot\pi}}=K_1^{(\varepsilon)}\cdot\sqrt{\frac{r}{2\cdot\pi}}$$

where $K_\beta^{(\varepsilon)}$ represents opening intensity factors which is defined with (4):

$$K_\beta^{(\varepsilon)}(t)=\int_{-\infty}^{+\infty}\frac{1+\lambda}{\mu}(t-\tau)\cdot\frac{\partial K_\beta^{(\sigma)}}{\partial\tau}d\tau \qquad (5)$$

With an elastic media, $K_\beta^{(\sigma)}$ was computed from the energy release rate G using the relation:

$$G=\frac{1+\lambda}{8\cdot\mu}\cdot\left(K_1^{(\sigma)}\right)^2+\frac{1+\lambda}{8\cdot\mu}\cdot\left(K_2^{(\sigma)}\right)^2 \qquad (6)$$

From equations (5) and (6), the G relation can be transformed, for an elastic and viscoelastic behaviour:

$$G=\frac{K_1^{(\sigma)}\cdot K_1^{(\varepsilon)}}{8}+\frac{K_2^{(\sigma)}\cdot K_2^{(\varepsilon)}}{8} \qquad (7)$$

FRACTURE PARAMETERS MODELLING

This part concerns numerical methods to the calculation of stress and opening intensity factors and the energy release rate. To simplify developments, the loading is supposed in an opening mode (mode I). Here, it is necessary to evaluate $K_1^{(\varepsilon)}$ and G; $K_1^{(\sigma)}$ being naturally evaluated with equation (7). The cinematic opening displacement method (COD) is used to find the opening intensity factors. The equation (5) enables:

$$[u_2]^2=\frac{\left(K_1^{(\varepsilon)}\right)^2}{2\cdot\pi}\cdot r \qquad (8)$$

This method is accurate if mesh, around the crack tip, is very refinement. $W\varepsilon$ is the non linear strain density energy, all notations are explicated in Figure 1, the energy release rate can be computed with an independent path $G\theta$-integral, Destuynder (4):

$$G\theta = \int_V \left(-W\varepsilon \cdot \theta_{k,k} + \sigma_{ij} u_{i,k} \theta_{k,j}\right) dV$$

ILLUSTRATIVE EXAMPLES

The numerical method developed in the previous sections can be illustrated with a creep simulation in a CTT specimen with a mode I loading. The specimen and its discretisation are presented in Figure 3. For the validation, the creep simulation and equation (6) permit to write:

$$K_I^{(\sigma)}(t) = 8 \cdot G(t) \cdot E(t) \text{ with } G = G\theta \tag{9}$$

where $K_I^{(\sigma)}(t)$ can be supposed constant. This value of 168MPa.mm$^{1/2}$ is the elastic instantaneous value. $K_I^{(\sigma)}(t)$ and *G(t)* curves are represented in Figure 4 and 5. The results given four different mesh densities, around the crack tip, shows that results are very goods if this mesh is very refined. However, the graphic scale being very small, with the less efficient mesh, the error is inferior to 3%. Now, the $G\theta$ results, in the energy release rate evaluation give good results and they are not depended of the rafinement in the crack tip domain. However, if $G\theta$ gives occurate results in an open or shear mode, it can not be used for a complex loading with mixed mode.

CONCLUSION

Coupled with the incremental formulation of the linear viscoelastic behaviour law, the fracture characteristics modelling is very performant. Indeed, with the determination of the energy release rate and the stress intensity, without the correspondence principle application, the crack initiation time and failure zone size can be easily evaluated for complex geometry's and complex loading. In the future, this approach will be developed for linear viscoleastic and anisotropic media and for a crack growth procedure.

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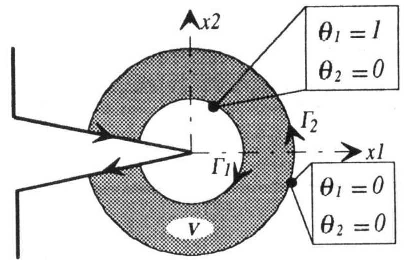

Figure 2 : Path integration for Gθ integral

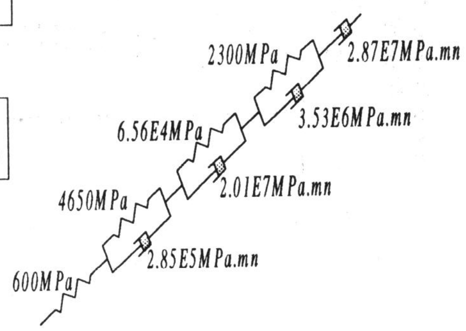

Figure 3 : Spectral decomposition of the Young modules E(t)

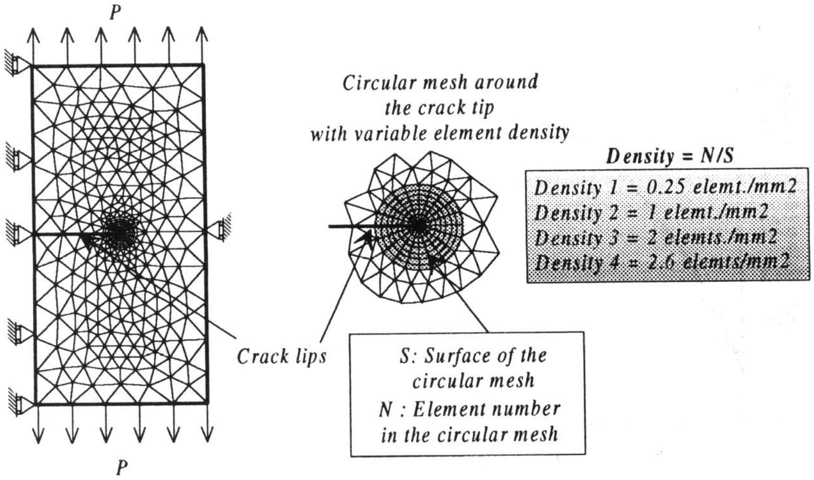

Figure 3 : Mesh of CTT specimen and density specifications

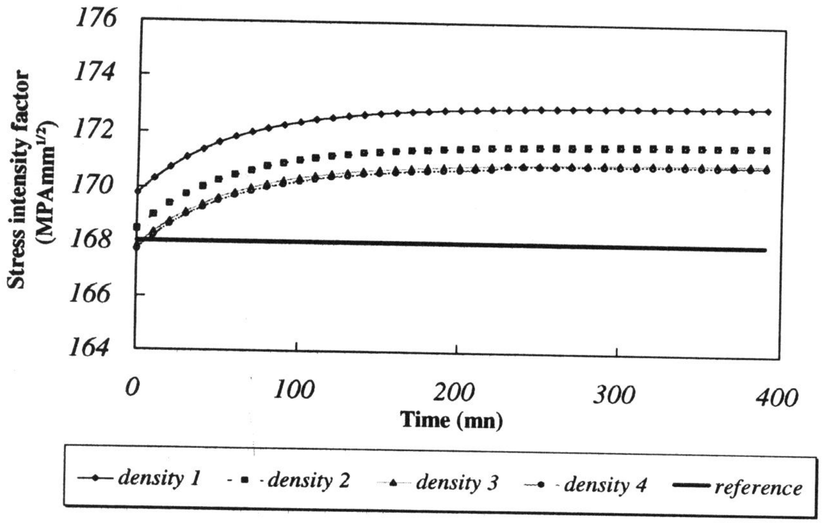

Figure 4 : Stress intensity factor curves

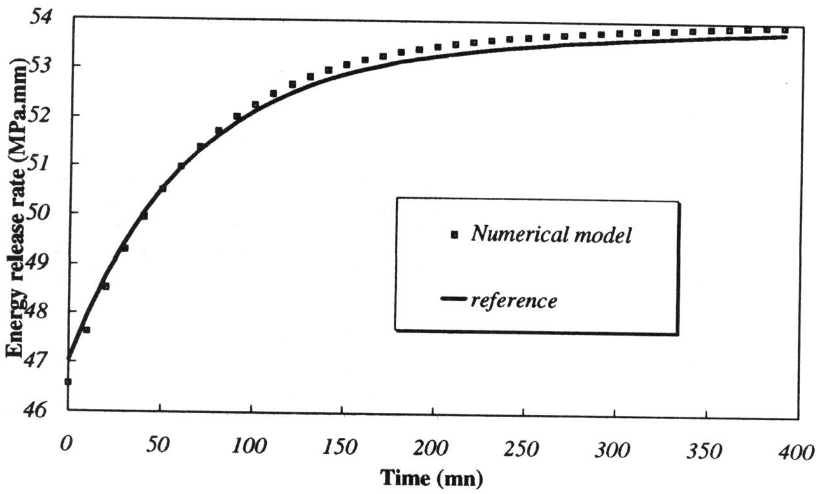

Figure 5 : Energy release rate curves