MULTIFRACTALITY OF TRANSVERSE CRACKING AND CRACKS' LENGTH DISTRIBUTION IN A CROSS-PLY LAMINATE UNDER FATIGUE

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Matrix cracking in cross-ply laminates is analysed with an utilization of multifractal formalism in order to describe more precisely the stochastic character of a crack set evolution in a specimen under fatigue. In a study each transverse crack for a given loading history (number of cycles) is characterised by the crack position - co-ordinate along the specimen's axis - and by its length. Two sets of matrix cracks are described - for both edges of a single specimen under increasing number of loading cycles.

INTRODUCTION

Fiber-reinforced cross-ply laminates demonstrate a wide spectrum of failure phenomena under fatigue. The first stage of fatigue loading in $[0_m/90_n]_s$ carbon-epoxy laminates is characterised by the multiple generation and development of transverse cracks in 90° layers. The spatio-temporal evolution of this process depends both on composite's structure and loading history, but even the twin specimens under the same loading conditions demonstrate sufficiently different matrix cracks' sets. Traditional description of this process in terms of a representative unit cell, i.e. supposing the equality of all intercrack distances, can be hardly called an adequate one, as the scatter of this distances can reach hundreds of percents. This presupposes the necessity for introduction of new characteristics in a description of such kind of behaviour. In this paper the utilisation of multifractal analysis for treatment of experimental data on matrix cracking characterisation is discussed.

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MULTIFRACTAL ANALYSIS

Experimental Data

Analysis of failure mechanisms in carbon-epoxy laminates is in the centre of interest during last years (see Hénaff-Gardin and Lafarie-Frenot (1), Lafarie-Frenot and Hénaff-Gardin (2), Hénaff-Gardin et al (3) and references there). This paper deals with experimental observations, which were carried out on a $[0_7/90]_s$ cross-ply carbon-epoxy T300/914 laminate. The fatigue tests consist of a sinusoidal tension under load control, at a 10 Hz frequency and a 0.1 load ratio. The applied maximum stress was equal to 60 per cent of the static failure stress for such laminates. The damage evolution is observed throughout tests by means of X-ray radiography. After a certain initiation delay, transverse cracks appear on the specimen's free edges and grow in number with the loading history up to a saturation value. In the same time, they propagate slowly inside the specimen width (1), (2). The study of the X-radiographs for different fatigue cycle numbers provide with information on the evolution of both crack distribution (along the specimen's free edges) and crack length.

This process is characterised by the high level of randomness, the latter being caused by the stochasticity of the spatial material properties' distribution, which results in a non-uniform generation and growth of microscopic defects. Figure 1 presents the evolution of matrix cracks, forming the set at the right edge of a specimen (the region under study occupies a 20-mm portion of a specimen). It is obvious, that crack generation and growth begin at the early stages of loading history. Both the moment of generation and dynamics of growth of various cracks are different. The intercrack distances at the developed - saturated - stage of fatigue are in the interval from 0,3 till 1 mm, thus having a sufficient scatter (the mean value being 0.6 mm). None of the cracks span the entire width of the specimen (15 mm), and only few cracks overcome - and insufficiently - the middle of it. Analysis of the second set of transverse cracks of this specimen - forming from its opposite (left) edge shows, that neither number nor geometric characteristics (position and length) of these cracks coincide. So, even the various portions of the same specimen are characterized by sufficiently different pictures of transverse cracking.

Multifractal Formalism

Still, even such a complicated behaviour can be characterised by means of modern tools of analysis of sophisticated dynamic processes, using, for instance, fractal approaches (Silberschmidt (4), (5)). It was shown, that the macroscopic result of a highly random process of evolution of microscopic defects at various scale levels can be characterised by non-trivial temporal and spatial scaling, thus necessitating in the utilization of adequate description methods. Investigation of general properties of matrix cracking distribution in cross-ply laminates by Silberschmidt (6) approved its multifractality. Generally speaking, multifractal character of the distribution means that all its moments scale as a power low with an infinite set of exponents. The latter are linked by the Legendre transform with the function $f(\alpha)$, called 'a multifractal spectrum' of distribution (for more details on the method and its application to characterisation of cracking see (4), (6) and respective references there). Let note some important properties of f - α function for a multifractal distribution: a) it is a cup convex, lying under a bisector $f = \alpha$ and having only one point of connection with this bisector; b) the maximum value of the function is equal to the dimension of a geometric support of the distribution (in other words, to 1 for a distribution along the length, to 2 for a 2*d*-distribution, etc.).

Results and Discussion

The unique opportunity for a detailed study of transverse cracking in cross-ply laminates provide experimental information on position (along the specimen edge) and length of each matrix crack for a set of consequently increasing numbers of load cycles. At the first stage of investigation it was proved that cracks' distribution along the specimen axis is of multifractal character. Obtained multifractal spectra (Figure 2) possess all the properties, mentioned in the previous section. Multifractal spectra evolve with the growth in number of load cycles: their widths decrease, that witnesses the increase of the uniformity extent: a homogeneous distribution is characterized - for our case - by a degenerated to a point $f = \alpha = 1$ function. This can be naturally explained: first transverse cracks nucleate in the most 'weak' places, but this process with the decrease in an averaged intercrack distance results in a specific stress redistribution linked with shielding effect of cracks, and at the developed stage of fatigue the position of a newly nucleated crack is determined by its surrounding (nearest cracks). Transition to a so-called "characteristic distribution state" - saturation regime - is reflected in spectra: for a developed fatigue stage even a ten-fold increase in number of cycles only slightly influenced $f(\alpha)$ function, in contrast to a short loading histories (Figure 2).

Figure 3 demonstrates the closeness of multifractal spectra of crack distribution for both edges of the specimen, though neither number of cracks, nor cracks' co-ordinates coincide. The extent of coincidence increases with the growth of number of loading cycles. This is linked with the increase of the number of cracks in each set (for each edge): the sets at the initial stages of loading history are not only more random, but are also far from being a representative set. So, the obtained multifractal spectra for matrix cracks approves the possibility to use them as the characteristic of the process.

One more parameter, characterizing matrix cracking, is the total cracked surface (2). For a given stacking order this parameter is proportional to the total length of all cracks, as far as a matrix crack after generation crosses all the thickness of 90°-layers. So, multifractal analysis was implemented also for distribution of this parameter (again - for both edges). Results of calculations are presented in Figure 4. It is vividly seen, that distributions are also multifractal ones, though the level of randomness is here higher, as for distribution of cracks. The same trend of diminishment of spectrum's width with increase in number of cycles is observed.

Thus, both distribution of cracks and of cracks' lengths are of multifractal character and can be used as additional quantitative characteristic of matrix cracking. Such an evidence allows also to elaborate new methods for numerical simulation of composite failure, using, for instance, fractal approaches (5).

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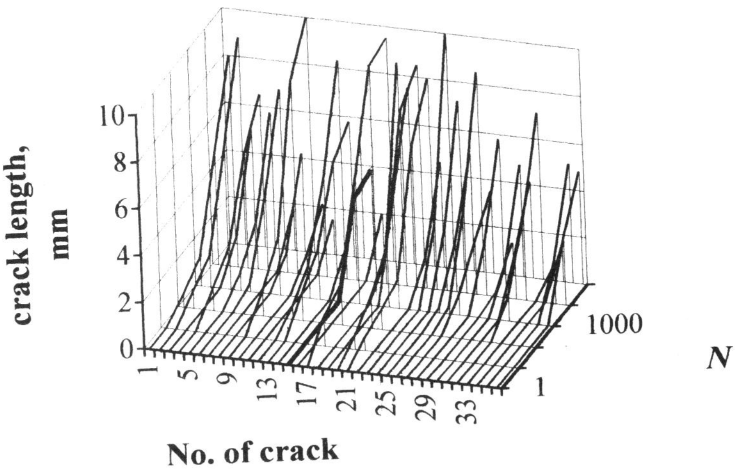

Figure 1 Dynamics of cracks' growth in laminate under fatigue (right edge)

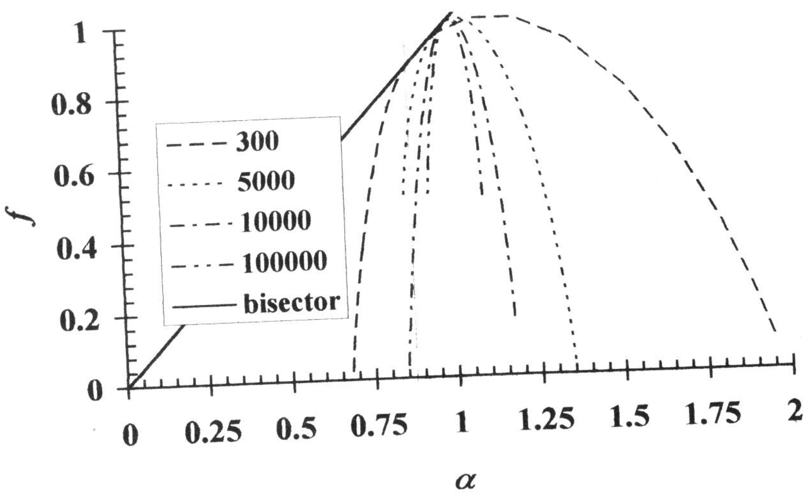

Figure 2 Multifractal spectra of crack distribution in specimen (right edge) for various loading histories

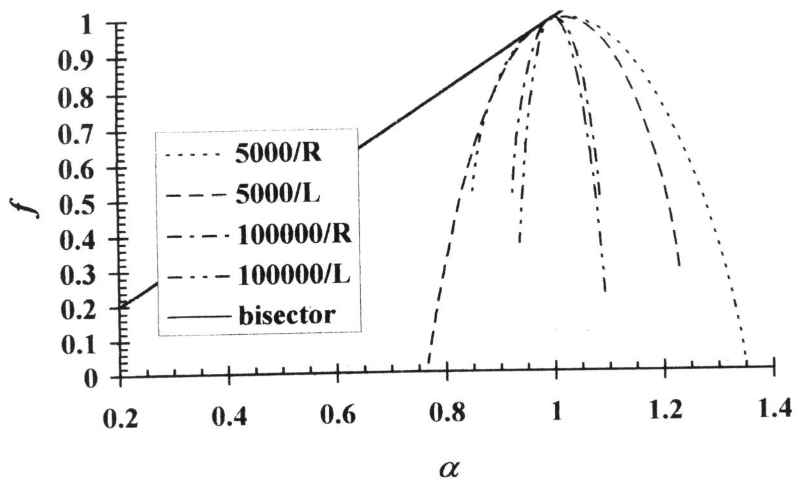

Figure 3 Multifractal spectra of crack distribution in specimen for various loading histories (R -right edge, L - left edge)

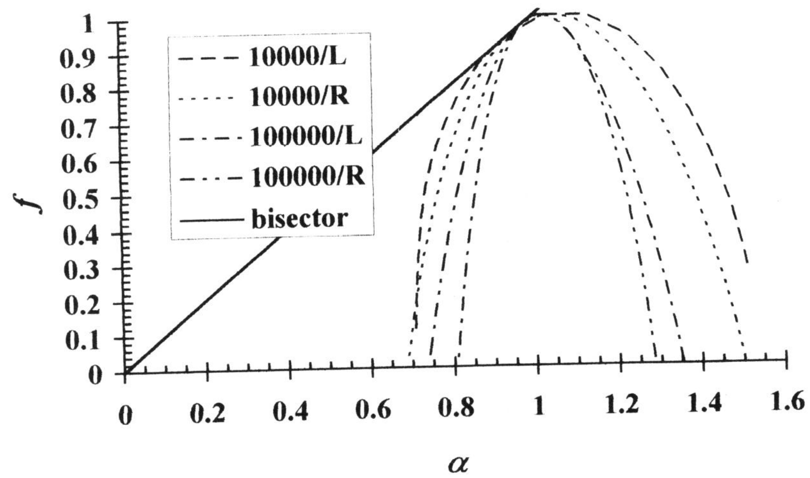

Figure 4 Multifractal spectra of cracks' length distribution in specimen for various loading histories (R -right edge, L - left edge)