MODELLING OF CREEP-FATIGUE INTERACTION OF ZIRCONIUM α UNDER CYCLIC LOADING AT 200°C

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The purpose of this paper is to describe a model able to predict damage and initiation under cyclic loading of zirconium α at 200°C. This model is based on an extensive experimental data base obtained at CEA (Mottot (1)). It was shown that the classical rules of creep-fatigue interaction fail, due to the strong apparent interaction in terms of strain based fatigue life and stress based creep life. The model developped here (Vogel et al.(2)) takes into account both cyclic character of the loading and influence of hold times. In a classical damage interaction framework (Lemaitre, Chaboche (3)), a new parameter is used, so that creep damage is controlled by the viscous internal stress calculated by a model with two inelastic deformations (Contesti, Cailletaud (4)), instead of the macroscopic stress as usual.

EXPERIMENTAL OBSERVATIONS

The main experimental observations are summarised in FIGURES 1 to 3. As shown in FIGURE 1, influence of hold time at maximum strain is low (less than a factor 2), in terms of number of cycles to failure, whereas duration of transition periods is more influent (FIGURE 2). It can also be pointed out from stress answers to cyclic loading with different strain rates (FIGURE 3), that a good modeling of stress-strain behaviour is needed first.

MODELLING

Mechanical behaviour

The model used to describe the mechanical viscoplastic behaviour belongs to a group of models called « non unified ». Inelastic flow is built from a plastic part and a viscoplastic part (equation 1). One yield surface is used for each mechanism (J_2 stands for von Mises invariant in the stresses space). The two mechanisms may be coupled (4), but it was not needed in the present study.

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TABLE 1 - Equations of the two inelastic deformations model called DDI (4).

Plastic strain mechanism		Viscous strain mecanism	
Flows			
Plastic strain rate $\dot{\underline{\underline{\varepsilon}}}_p = \dot{\lambda}_p \frac{\partial f_p}{\partial \underline{\underline{\sigma}}} = \frac{3}{2}\dot{\lambda}_p \frac{\underline{\underline{\sigma}} - \underline{\underline{X}}_p}{J_2(\underline{\underline{\sigma}} - \underline{\underline{X}}_p)} = \dot{p}\,\underline{\underline{n}}_p$		Viscous strain rate $\dot{\underline{\underline{\varepsilon}}}_v = \dot{\lambda}_v \frac{\partial f_v}{\partial \underline{\underline{\sigma}}} = \frac{3}{2}\dot{\lambda}_v \frac{\underline{\underline{\sigma}} - \underline{\underline{X}}_v}{J_2(\underline{\underline{\sigma}} - \underline{\underline{X}}_v)} = \dot{v}\,\underline{\underline{n}}_v$	
Isotropic hardening			
$R_p = R_{p0} + Q_p(1 - \exp(-b_p\ p))$ Accumulated plastic strain rate $\dot{p} = \sqrt{\frac{2}{3}\dot{\underline{\underline{\varepsilon}}}_p : \dot{\underline{\underline{\varepsilon}}}_p}$		$R_v = R_{v0} + Q_v(1 - \exp(-b_v\ v))$ Accumulated viscous strain rate $\dot{v} = \sqrt{\frac{2}{3}\dot{\underline{\underline{\varepsilon}}}_v : \dot{\underline{\underline{\varepsilon}}}_v}$	
Non linear kinematic hardening using two variables (i = {1,2})			
$\underline{\underline{X}}_{pi} = \frac{2}{3} c_{pi}\, \underline{\underline{\alpha}}_{pi}$	$\dot{\underline{\underline{\alpha}}}_{pi} = \dot{\underline{\underline{\varepsilon}}}_p - d_{pi}\, \underline{\underline{\alpha}}_{pi}\, \dot{p}$	$\underline{\underline{X}}_{vi} = \frac{2}{3} c_{vi}\, \underline{\underline{\alpha}}_{vi}$	$\dot{\underline{\underline{\alpha}}}_{vi} = \dot{\underline{\underline{\varepsilon}}}_v - d_{vi}\, \underline{\underline{\alpha}}_{vi}\, \dot{v}$

$$\underline{\underline{\varepsilon}}_{in} = \underline{\underline{\varepsilon}}_p + \underline{\underline{\varepsilon}}_v \qquad f_p = J_2\left(\underline{\underline{\sigma}} - \underline{\underline{X}}_p\right) - R_p \qquad f_v = J_2\left(\underline{\underline{\sigma}} - \underline{\underline{X}}_v\right) - R_v \tag{1}$$

Equations of TABLE 1 are the classical ones giving rise to non linear kinematic and isotropic hardenings. The accumulated plastic strain p ($\dot{p}$ its rate) is determined with the consistency condition $\dot{f}_p = 0$, coming from the fact that the plasticity criterion $f_p \le 0$ cannot be violated. The accumulated viscous strain v ($\dot{v}$ its rate) is defined as a power function of the viscoplastic yield function f_v.

$$\dot{p} = \frac{\underline{\underline{n}}_p : \dot{\underline{\underline{\sigma}}}}{H} \text{ with } H = b_p Q \exp(-b_p\ p) + \sum_{i=1,2} (c_{pi} - d_i\ \underline{\underline{n}}_p : \underline{\underline{X}}_{pi}) \qquad \dot{v} = \left\langle \frac{f_v}{K} \right\rangle^n \tag{2}$$

With this model, the material can yield either in plasticity or viscoplasticity, according to the respective values of f_p and f_v, but plastic and viscoplastic strain rates can also be both present.

Crack initiation prediction model

Because we need to find critical mechanical parameters in order to discriminate cyclic and creep phenomena, we used macroscopic stress answers given by DDI model to take into account « fatigue » effects pointed out on FIGURE 3 and values of viscoplastic kinematic hardening variable X_v to take into account « creep » effects. This variable is chosen because it gives a good representation of the importance of strain rate on the experimental failure data observed in FIGURES 2 and 3. The « creep » contribution in the crack initiation is directly related to X_v values (FIGURES 4 and 5). To represent creep-fatigue interaction, we used a model based on non linear cumulation (3). For the creep part, macroscopic stress σ is replaced by X_v. The interaction between creep and fatigue damages D_C and D_F is expressed using the total damage $D = D_C + D_F$. Evolution laws for D_C and D_F are functions of total damage D. One dimensional formulation of the model is given here, but a 3D expression is also available, using the pertinent invariants (Vogel (5)).

$$dD_F = \left[1-(1-D)^{\beta+1}\right]^{\alpha}\left[\frac{\sigma_M-\overline{\sigma}}{M_0(1-D)}\right]^{\beta} dN \tag{3}$$

$$\alpha = 1-a\left\langle\frac{\sigma_M - S_L}{\sigma_U-\sigma_M}\right\rangle \qquad S_L = \overline{\sigma}+(1-b\,\overline{\sigma})\,\sigma_L \qquad M_0 = M(1-c\,\overline{\sigma}) \tag{4}$$

Functions α, S_L and M_0 are used to take into account respectively non-linear cumulation of D_F damage, influence of fatigue limit σ_L and effect of mean stress $\overline{\sigma}$.

$$dD_C = C\left[\frac{|X_V|}{A}\right]^{r}(1-D)^{-k}dt \qquad \begin{cases} C=1 & \text{si } X_V>0 \\ C=C_0 & \text{si } X_V<0 \end{cases} \qquad 0\le C_0 \le 1 \tag{5}$$

Coefficient C allows the model to consider or not that a compressive loading has a damaging effect.

When each damage is considered to be influent alone on the material, expressions (3) and (5) may be integrated for D between 0 and 1 and we determine the number of cycles to failure for either pure « creep » or pure « fatigue » :

$$N_F = \frac{1}{(1+\beta)(1-\alpha)}\left(\frac{\sigma_M-\overline{\sigma}}{M_0}\right)^{-\beta} \tag{6}$$

$$(k+1)\int_0^{t_C} C\left(\frac{|X_V|}{A}\right)^{r} dt = 1 \qquad N_C = t_C/T \tag{7}$$

When creep and fatigue damages are both present, the interaction scheme produces non linear interaction (FIGURE 6), the number of cycles to failure N_{sim} being obtained for D equal to 1. The strength of interaction depends on the distance between the curves in FIGURE 6, thus, from the values of macroscopic stress σ, mean stress $\overline{\sigma}$, maximum stress σ_M and internal viscous stress X_v obtained with DDI model for a stabilised cycle. For each cycle, damage starts from D_0 and reaches D_2, according according to the following expressions :

$$\left[1-(1-D_2)^{\beta+1}\right]^{1-\alpha}-\left[1-(1-D_1)^{\beta+1}\right]^{1-\alpha} = \frac{\alpha(\beta+1)\langle\sigma_M-S_L\rangle}{\sigma_U-\sigma_M}\left[\frac{\sigma_M-\overline{\sigma}}{M_0}\right]^{\beta} \tag{8}$$

$$(1-D_0)^{(k+1)}-(1-D_1)^{(k+1)} = (k+1)\int_{cycle}\left(\frac{|X_V|}{A}\right)^{r} dt \tag{9}$$

Calculation is therefore needed at two levels, first to determine integration of X_v for the stabilised cycle considered, and second, to build cumulation and evaluate the number of cycles to failure proposed by the model.

RESULTS

TABLE 2 gives results of the model compared with experimental data of maximum stress $\sigma_{M\,exp}$ and number of cycles to failure N_{exp}. The good agreement between $\sigma_{M\,exp}$ and $\sigma_{M\,sim}$ shows that identification of DDI model is correct. Values of N_{sim}/N_{exp} place the comparison between experience and modelling into a range of 1/2 and 2 (FIGURE 7). In the present case, compression and tension loadings are assumed to have identical effect. Values of N_F and N_C give interesting indications on strength or weakness of interaction between fatigue and creep damages. When no creep effect is obtained, $N_C > 10^8$. For example, sample 4b shows a strong interaction effect between creep and fatigue damages. As far as values of N_C or N_F decrease, the corresponding damage increases.

TABLE 2 - Experimental data (1) and their predictions at 200°C (units : MPa, s).

Sam-ple	$\Delta\varepsilon$ (%)	$\dot{\varepsilon}$ (s^{-1})	t_m (min)	$\sigma_{M\,exp}$ (MPa)	$\sigma_{M\,sim}$ (MPa)	N_F	N_C	N_{sim}	N_{exp}	$\frac{N_{sim}}{N_{exp}}$
1a	0.6	2.10^{-3}	0	149.7	137.1	61824	$> 10^8$	61824	58199	1.062
1b	0.8	2.10^{-3}	0	152.8	155.1	12728	$> 10^8$	12728	18984	0.670
1c	1	2.10^{-3}	0	163.9	169.7	6355	$> 10^8$	6355	5532	1.149
1d	1.2	2.10^{-3}	0	181.1	185.0	3704	$> 10^8$	3704	3048	1.215
1e	1.6	2.10^{-3}	0	210.5	204.3	2022	71083166	2012	1079	1.965
2a	1	2.10^{-3}	4	165.6	171.2	5674	1126044	4071	6759	0.602
2b	1	2.10^{-3}	10	164.6	168.2	6183	408189	3947	5431	0.727
2c	1	2.10^{-3}	10C	173.9	177.4	5208	$> 10^8$	5208	7616	0.684
2d	0.8	2.10^{-3}	30	150.8	153.7	11697	322552	4925	10979	0.449
2e	1	2.10^{-3}	30	173.9	171.0	5689	108796	3320	3422	0.970
2f	1.6	2.10^{-3}	30	214.6	203.5	2009	10649	1403	1173	1.196
3a	1	8.10^{-5}	0	150.9	155.4	12623	2189761	6040	4723	1.279
3b	1	$3.3.10^{-5}$	0	145.7	150.5	16826	440406	5602	3090	1.813
4a	1	$3.3.10^{-5}$	10	147.7	149.4	16645	36121	3756	4114	0.913
3c	1	$1.1.10^{-5}$	0	140.9	145.1	24523	77733	4671	2817	1.658
4b	1	$1.1.10^{-5}$	30	142.4	144.2	24613	7238	2430	4585	0.530
3d	1	$3.7.10^{-6}$	0	137.4	140.6	36625	17480	3442	2630	1.309

Considering the wide range of experimental conditions ($0.8\% \leq \Delta\varepsilon_T \leq 1.6\%$, $2.10^{-3}s^{-1} \leq \dot{\varepsilon} \leq 3{,}7.10^{-6}s^{-1}$, $0 \leq t_m \leq 90$ min.) and the simplicity of the equations used the proposed model is very efficient to predict initiation for zirconium α at 200°C under cyclic loading. The model is able to explain the strong life reduction in terms of cycles brought by creep on fatigue tests and also the influence of non linear cumulation between « creep » and « fatigue » damages (FIGURE 8). Three dimensional applications of the model are also available (5).

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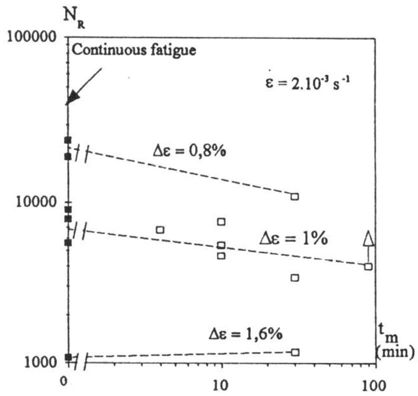
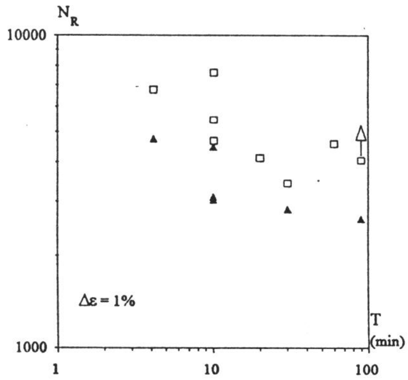


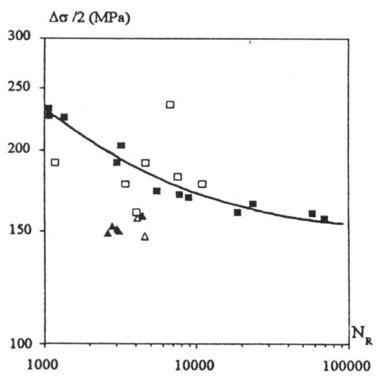
■ Fatigue
□ Fatigue-relaxation

FIGURE 1 : Number of cycles to failure function of hold time.



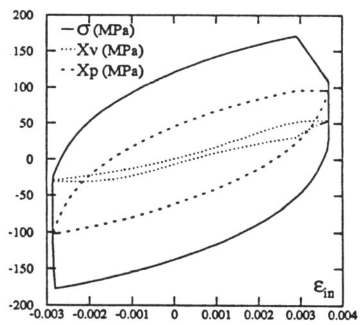
□ Fatigue-relaxation ($\dot{\varepsilon}=2.10^{-3}s^{-1}$)
▲ Fatigue ($8.10^{-5}s^{-1} \leq \dot{\varepsilon} \leq 3{,}7.10^{-6}s^{-1}$)

FIGURE 2 : Number of cycles to failure function of transitions periods.



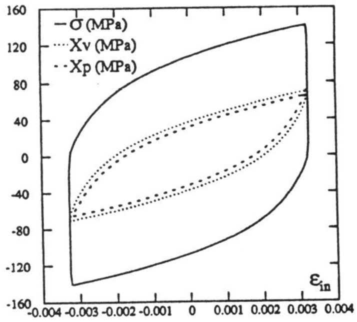
■ Fatigue ($\dot{\varepsilon}=2.10^{-3}s^{-1}$)
□ Fatigue-relaxation ($\dot{\varepsilon}=2.10^{-3}s^{-1}$)
▲ Fatigue ($8.10^{-5}s^{-1} \leq \dot{\varepsilon} \leq 3{,}7.10^{-6}s^{-1}$)
△ Fatigue-relaxation ($\dot{\varepsilon}$ id. ▲)

FIGURE 3 : Variation of stress function of number of cycles to failure.



Fatigue-relaxation test : $\Delta\varepsilon=1\%$, $\dot{\varepsilon}=2.10^{-3}s^{-1}$, $t_m = 30$ min.

FIGURE 4 : Evolution of stress σ and internal variables X_v and X_p.



Fatigue test : $\Delta\varepsilon$=1%, $\dot{\varepsilon}$=3,7.$10^{-6}s^{-1}$

FIGURE 5 : Evolution of stress σ and internal variables X_v and X_p

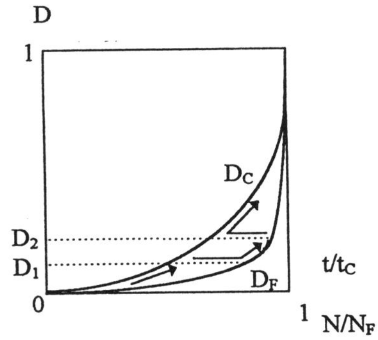


FIGURE 6 : Non-linear interaction between D_C and D_F damages.

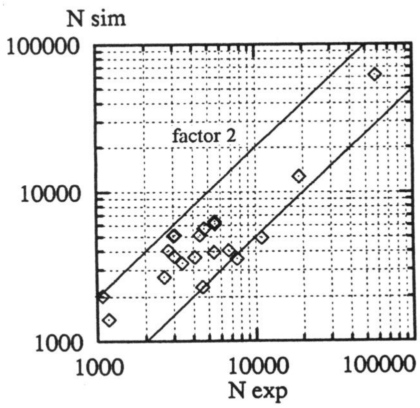


FIGURE 7 : Comparison between experimental and calculated cycles to failure.

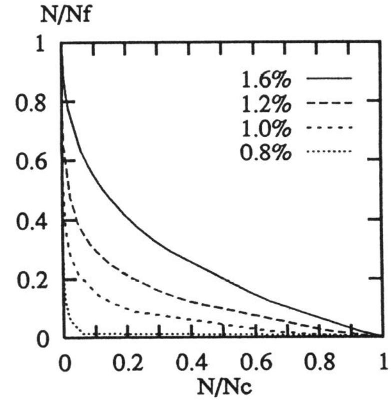


FIGURE 8 : Cumulation of damages in fatigue with different $\Delta\varepsilon$ and $\dot{\varepsilon}$.