MIXED MODE (I+II) MODEL OF FATIGUE CRACK GROWTH

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A mechanical model is proposed to describe fatigue crack growth rate when the crack is subjected to mixed mode (I+II) loading. In the crack tip stress-strain distribution the modified Hutchinson solution is adopted. The model is based on maximum plastic strain energy density release rates at the tip of the crack. The required data for predicting fatigue crack growth rate can be found in standard material handbooks where cyclic and fatigue properties of the materials are presented.

INTRODUCTION

Structures and elements in service are often subjected to mixed mode loading conditions. In order to develop proper design methodology for combined - mode fatigue fracture, it is necessary to gain insights into the nature of near-tip fields under conditions of load mixture. In the first theoretical analysis of the mixed mode fracture problem, the basic assumption was that linear elastic fracture mechanics (LEFM) was applicable. For plane stress problem, the fracture criterion can be expressible in terms of stress intensity factors for mode I (K_I) and mode II (K_{II}) and material properties.

In the literature different models top describe mixed mode crack growth problem have been proposed (Bold, Brown and Allen (1)). They were initially developed for brittle fracture, but they have been used for mixed mode fatigue as well. The justification for this is that under proportional loading, the maximum value of the stress around a crack tip which drivesa brittle fracture, is directly proportional to the maximum range of that stress.

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The universal functions $\sigma_{ij}(\theta,n)$, $\varepsilon_{ij}(\theta,n)$ and $u_j(\theta,n)$ in Eqs. (2) vary with polar angle θ, the strain hardening exponent n and the state of stress, i.e. plane stress or plane strain. The factor I_n depends minly on the strain hardening exponent n.
The Hutchinson singular fields, (2), can be extended to provide the near-tip fields for mode I-mode II crack problems. Under conditions of small - scale yielding, the far-field stress components for a crack subjected remotely to tensile opening and sliding stress intesity factors, K_I and K_{II}, respectively, are given by

$$\sigma_{ij} = \frac{1}{\sqrt{2\pi x}}\left[K_I \tilde{\sigma}_{ij}^{I}(\theta) + K_{II}\tilde{\sigma}_{ij}^{II}(\theta)\right] \tag{3}$$

where x and θ are the polar coordinates centered at the crack tip and $\tilde{\sigma}_{ij}^{I}$ and $\tilde{\sigma}_{ij}^{II}$ are the dimensionless universal functions. For small-scale yielding, we introduce equivalent stress intensity factor related to mixed -mode stress intesity factors by

$$K_{eq} = f(K_I, K_{II}) \tag{4}$$

The relative strengths of K_I and K_{II} can by characterized in terms of an elastic mixity parameter, M^e, which is defined as:

$$M^e = \frac{2}{\pi}\tan^{-1}\left|\lim_{r\to\infty}\frac{\sigma_{\theta\theta}(r,\theta=0)}{\sigma_{r\theta}(r,\theta=0)}\right| = \frac{2}{\pi}\tan^{-1}\left|\frac{K_I}{K_{II}}\right|. \tag{5}$$

In this charasterization , $M^e=0$ for pure mode II, $M^e=1$ for pure mode I, and $0<M^e<1$ for different mixities of modes I and II.
In terms of the plastic strain energy density distribution, equ. (2) reduced to

$$\Delta W^p = \frac{1-n'}{1+n'}\sigma_{ij}\varepsilon_{ij}^{p} = D\left(\frac{K^2}{IEx}\right) \tag{6}$$

where

$$D = \frac{1-n'}{1+n'}\tilde{\sigma}_{ij}(\theta,n)\tilde{\varepsilon}_{ij}(\theta,n)$$

For cyclic loading let us now introduce a equivalent stress intensity range ΔK_{eq}, and crack blunting radius, r_c, thus

$$\Delta W^p = D\frac{\Delta K_{eq}}{IE(x+r_c)} \tag{7}$$

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Analysis of the fatigue crack growth during cyclic loading requires a fatigue failure criterion and specification of the zone where such a criterion can be applied. The finite element calculations of stress and strain ahead ofthe crack tip have shown, that there exists a small region-damage process zone, denoted herein by δ^* where the stress and strain have a finite magnitude.

Since the process of damage in the process zone is controlled by plastic strain range the fatigue criterion can be expressed in the form (Golos (5), Golos and Ellyin (6)):

$$\Delta W^p = \zeta(2N_f)^\alpha \tag{8}$$

where ζ and α are material parameters.

The plastic strain energy density distribution within the process zone is given by

$$\Delta W^p = D\frac{\Delta K_{eq}^{\;2}}{IE(\delta^* + r_c)} \tag{9}$$

The plastic strain energy density within the process zone may be calculated from equation (7) setting by $x=\delta^*$.

The corresponding number of cycles, ΔN required for the crack to penetrate through δ^* can be determined from equation (8).

Substituting from equation (8) into (7) the crack growth rate per cycle, da/dN is therefore can be estimated as follows:

$$\frac{da}{dN} = \frac{\delta^*}{\Delta N} = \left(D\frac{\Delta K_{eq}^{\;2}}{4IE\sigma_f'\varepsilon_f'} \right) \tag{10}$$

The r_c can be calculated, assuming that for $\Delta K_{eq}=\Delta K_{eq,th}$, da/dN=0. The experiments show unstable crack growth when the range of the stress intensity range approaches the critical value,i.e. ΔK_{eq}-$\Delta K_{eq,c}$ .Thus translates to the crack growth through the process zone at the instant loading. Then we can calculate the value of δ^*, as follows:

$$\delta^* = D\frac{(\Delta K_{eq}^{\;2} - \Delta K_{eq,th}^{\;2})}{4IE\sigma_f'\varepsilon_f'} \tag{11}$$

Rearranging equation (11), the FCGR can be described as:

$$\frac{da}{dN} = 2\delta^* \left[\frac{\Delta K_{eq}{}^2 - \Delta K_{eq,th}{}^2}{(4IE\sigma_f{}'\varepsilon_f{}'\delta^*)/D} \right]^{-1/\alpha} \tag{12}$$

In the case when $\Delta K_{eq,th} << \Delta K_{eq}$, we get

$$\frac{da}{dN} = 2\delta^* \left[\frac{\Delta K_{eq}{}^2}{(4IE\sigma_f{}'\varepsilon_f{}'\delta^*)/D} \right]^{-1/\alpha} \tag{13}$$

COMPARISON WITH EXPERIMENT AND DISCUSSION

In the present study the experimental data for 1045 steel (7) were used.
For the analysed model ΔK_{eq} has been set as $\Delta K_{eq} = \left(K_I^4 + 8K_{II}^4\right)^{0,25}$ (8).

The experimental and theoretical results for this material in full line is shown in Fig.1. Predictions of the proposed model are in good agreement with the FCGR. Forthermore, the required data can be found in the material handbooks where fatigue properties of materials are listed.

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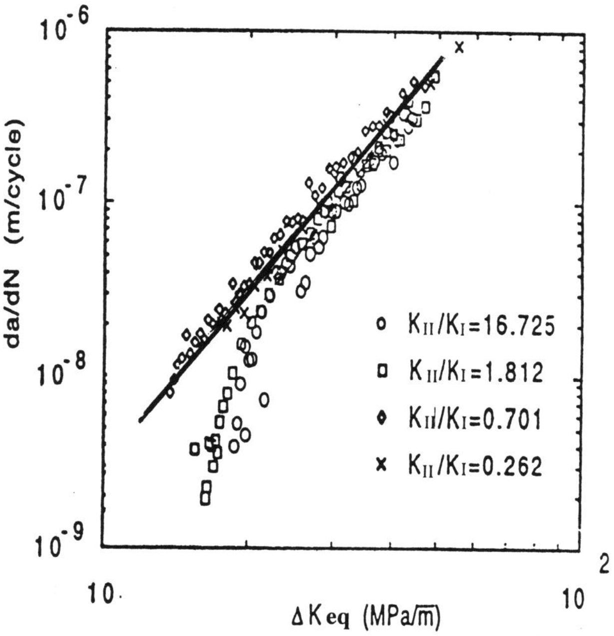

Fig. 1. Theoretical predictions and experimental data of fatigue crack growth rate for 1045 steel (7).