LIMIT LOAD OF AN AXIALLY CRACKED PIPE UNDER COMBINED PRESSURE BENDING AND TENSION

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The use of limit loads of structures containing defects is widely spread in flaw assessment. In the case of an axially cracked pipe, tension and bending do not contribute to the elastic opening of the crack lips, but increase the level of plasticity, and a safe J estimation requires a full expression of the limit load, including all applied loadings, even when they have no influence on the elastic J estimation. The aim of the present paper is to propose a lower bound estimation method for the global collapse of an axially cracked cylinder, based on the shell Von Mises yield criterion, under pressure tension and bending.

INTRODUCTION

The collapse mechanism is developed from the study proposed by Kitching (1) for slots under pressure. The global collapse expression gives accurate J estimations for surface cracks and over conservative for long through-wall cracks.

The cracked cylinder is presented on figure 1. The limit analysis will be first carried out for two shells, one with a through-wall defect and the other one without defect. The yield criterion is the usual shell approximation of the Von Mises criterion. In our case, it may be expressed in a non-dimensional form as:

$$n_\xi^2 - n_\xi n_\theta + n_\theta^2 + m_\xi^2 \leq 1 \qquad (1)$$

This relation will be satisfied in each of the two shells.

CYLINDER WITH A THROUGH-WALL DEFECT

The thickness of this cylinder is a, the applied load is $\overline{Q}_a$ and the limit load is

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$\overline{Q}_{La}$. Equilibrium equations are given in the following non-dimensional form:

$$\frac{1}{4\rho^2}\frac{\partial^2 m_\xi}{\partial \xi^2} + n_\theta = P_a^* \quad (2) \qquad \frac{\partial n_\theta}{\partial \theta} = 0 \quad (3) \qquad \frac{\partial n_\xi}{\partial \xi} = 0 \quad (4) \qquad \frac{a}{4c}\frac{\partial m_\xi}{\partial \xi} = q_\xi \quad (5)$$

static boundary conditions :

- $n_\theta = 0$ for $\theta = 0$ and $\xi \in [0,1]$ (6)
- $q_\xi(\xi = 0) = 0$, symetry condition (7)
- n_ξ equilibrates axial loads m_{ga} and n_{ga} (8)

collapse mechanism :

Two principal circumferential regions are created to equilibrate bending, and three axial crowns for pressure. The collapse mechanism of table 1 is based on a simple linear distribution of shears and satisfies conditions (2) to (7) mentioned above. A quadrant of the pipe shell has been divided into six regions.

The last boundary condition (8) to satisfy is:

$$m_{ga} = \frac{n_\xi^+ - n_\xi^-}{2}\sin\gamma_a, \text{ with } \gamma_a = \pi\frac{n_{ga} - n_\xi^-}{n_\xi^+ - n_\xi^-} \text{ and } \gamma_a \in [0,\pi] \qquad (9)$$

For the lower bound analysis the yield criterion (1) must be complied at any point of the structure. So this criterion will be satisfied in the most heavily loaded sections where the collapse occurs.

	$\theta \in [0,\gamma_a]$	$\theta \in [\gamma_a,\pi]$
$\xi \in [0,1]$	$n_\theta = 0,\ n_\xi = n_\xi^+,\ q_\xi = (**)$ $m_\xi = \left[m_0 + 2\rho^2(\xi^2 - 1)\right]P_a^*$	$n_\theta = 0,\ n_\xi = n_\xi^-,\ q_\xi = (**)$ $m_\xi = \left[m_0 + 2\rho^2(\xi^2 - 1)\right]P_a^*$
$\xi \in [1,B]$	$n_\theta = B/(B-1)P_a^*,\ n_\xi = n_\xi^+,\ q_\xi = (**)$ $m_\xi = \left[m_0 + 4\rho^2\frac{B(\xi-1) - 0.5(\xi^2-1)}{B-1}\right]P_a^*$	$n_\theta = B/(B-1)P_a^*,\ n_\xi = n_\xi^-,\ q_\xi = (**)$ $m_\xi = \left[m_0 + 4\rho^2\frac{B(\xi-1) - 0.5(\xi^2-1)}{B-1}\right]P_a^*$
$\xi \geq B$	$n_\theta = P_a^*,\ n_\xi = n_\xi^+,\ m_\xi = (*),\ q_\xi = (**)$	$n_\theta = P_a^*,\ n_\xi = n_\xi^-,\ m_\xi = (*),\ q_\xi = (**)$

(*) m_ξ is evanescent in this region and disappears when ξ increases.
(**)q_ξ is linked to m_ξ according to equation (5).

TABLE 1- Collapse mechanism for the through cracked cylinder.

The two parameters m_0 and B affect mainly P_a^*, then under pure pressure conditions we solved the associated optimisation problem:

$$P_a^{*opt} = \left\{ \underset{(m_0 \in R, B>1)}{\text{Max}} P_a^* \,/\, \forall \xi \in \{0, 1^-, 1^+, B^-\}\ n_\theta^2 + m_\xi^2 \leq 1 \right\}$$

The values of m_0 and B that led to this optimal value were interpolated with an important accuracy for a parameted pipe geometry ρ :

$$m_0 = 0{,}0186\rho^4 - 0{,}2325\rho^3 + 1{,}9979\rho^2 - 2{,}2701\rho \qquad \rho \in [0,6] \qquad (10)$$

$$B = \begin{cases} 1{,}826\rho^{-1{,}0599} & \rho \in]0,1] \\ 0{,}0054\rho^4 - 0{,}0943\rho^3 + 0{,}6044\rho^2 - 1{,}724\rho + 3{,}0434 & \rho \in [1,6] \end{cases} \qquad (11)$$

m_0 and B will subsequently be taken as the results of (10) and (11).

For the combined loading the yield criterion (1) is a binomial expression of n_ξ and implies that at any point of the structure:

$\varphi^+(1, m_\xi, n_\theta) \le n_\xi \le \varphi^-(1, m_\xi, n_\theta)$

As n_ξ is not a function of ξ we have :

$$n_\xi^+ = \underset{\xi\in\{0,1^-,1^+,B^-\}}{\mathrm{Min}} \varphi^+(1, m_\xi, n_\theta) \quad (12) \qquad n_\xi^- = \underset{\xi\in\{0,1^-,1^+,B^-\}}{\mathrm{Max}} \varphi^-(1, m_\xi, n_\theta) \quad (13)$$

If n_{ga} and P_a^* are given, n_ξ^+ and n_ξ^- are obtained through (12) and (13) and m_{ga} through (9). The yield surface is then entirely defined.

CYLINDER WITHOUT DEFECT

The thickness of this cylinder is e=T-a, it is loaded with $\overline{Q}_e$ and the corresponding limit load is $\overline{Q}_{Le}$. The yield surface of this cylinder is defined by:

$$m_{ge} = \sqrt{1 - 0.75P_e^{*2}}\,\sin\gamma_e, \text{ with } \gamma_e = \frac{\pi}{2}\left(1 + \frac{n_{ge} - P_e^*/2}{\sqrt{1 - 0.75P_e^{*2}}}\right) \text{ and } \gamma_e \in [0, \pi] \quad (14)$$

CYLINDER WITH A PART-THROUGH DEFECT

We will assume that axial strengths are divided in both shells, proportionally to the surface of their section: $m_{gT} = m_{ga} = m_{ge}$ and $n_{gT} = n_{ga} = n_{ge}$.

Internal pressure is: $p_e = (1-\alpha)p_T$, $p_a = \alpha p_T$, $\alpha \in [0,1]$.

The α parameter is unknown, the consequences on the limit pressure parameters are: $P_e^* = A_e P_T^*$ and $P_a^* = A_a P_T^*$.

We will express the whole problem in terms of applied load $\overline{Q}_T$ and L_r. For ε =+ or - we have the following values of $v_\xi^\varepsilon = L_r n_\xi^\varepsilon$ for ξ in $\{0,1^-,1^+,B^-\}$:

$$\begin{cases} v_\xi^\varepsilon(0) = \varphi^\varepsilon(L_r, (m_0 - 2\rho^2)A_a\pi_T^*, 0) \\ v_\xi^\varepsilon(1^-) = \varphi^\varepsilon(L_r, m_0 A_a \pi_T^*, 0) \\ v_\xi^\varepsilon(1^+) = \varphi^\varepsilon(L_r, m_0 A_a \pi_T^*, B/(B-1)A_a\pi_T^*) \\ v_\xi^\varepsilon(B^-) = \varphi^\varepsilon(L_r, (m_0 + 2\rho^2(B-1))A_a\pi_T^*, B/(B-1)A_a\pi_T^*) \end{cases} \qquad (15)$$

Equations (12) and (13) are multiplied by L_r :

$$v_\xi^+ = \underset{\xi\in\{0,1^-,1^+,B^-\}}{\mathrm{Min}} v_\xi^+(\xi) \quad (16) \qquad v_\xi^- = \underset{\xi\in\{0,1^-,1^+,B^-\}}{\mathrm{Max}} v_\xi^-(\xi) \quad (17)$$

Equations (9) and (14) are written in terms of applied load :

$$\begin{cases} \mu_{gT} = \dfrac{v_\xi^+ - v_\xi^-}{2} \sin\gamma_a \\ \gamma_a = \pi \dfrac{v_{gT} - v_\xi^-}{v_\xi^+ - v_\xi^-} \quad \text{with } \gamma_a \in [0,\pi] \end{cases} \quad (18) \qquad \begin{cases} \mu_{gT} = \sqrt{1-0.75(A_e\pi_T^*)^2}\,\sin\gamma_e \\ \gamma_e = \dfrac{\pi}{2}\left(1+\dfrac{v_{gT} - A_e\pi_T^*/2}{\sqrt{1-0.75(A_e\pi_T^*)^2}}\right) \quad \text{with } \gamma_e \in [0,\pi] \end{cases} \quad (19)$$

The solution of this problem is the minimal value of L_r such as: for a given $\overline{Q}_T$ and a given geometry L_r and $\alpha \in [0,1]$(A_e and A_a depend on α)satisfy both (18) and (19).Equation (18) depends on (10), (11), (15), (16) and (17). A numerical solution of this problem can be easily obtained.

APPLICATION TO SIMPLIFIED J ESTIMATION

Ainsworth (2) simplified J estimation , is derived through the expression:

$$J_s = J^{el}\left(\frac{\varepsilon(\sigma_y L_r)}{\sigma_y L_r / E} + \frac{1}{2}\frac{L_r^2}{1+L_r^2}\right)$$

We will analyse an internal elliptical crack submitted to a combination of wall pressure and bending, there is no axial tension and no lips pressure. The tensile curve is a Ramberg-Osgood law with n=6. The internal radius is 300 mm, the half axial length of the elliptical crack is 45 mm (c is taken as π*45/4 mm), the crack depth is 15 mm, the thickness is 60 mm, the length of the mesh is 810 mm.

Two loads were applied:

$\overline{Q}_{T1}(\lambda) = \lambda(0 \quad 0 \quad 1{,}104)$ and $\overline{Q}_{T2}(\lambda) = \lambda(1{,}136 \quad 0 \quad 1{,}104)$

Limit analysis gives : $L_r(\overline{Q}_{T1}) = 1{,}222\ \lambda$ and $L_r(\overline{Q}_{T2}) = 1{,}792\ \lambda$

All calculated J values were maximum at the deepest point of the crack. We will then consider these J values. Figure 2 compares finite element results obtained under $\overline{Q}_{T1}$ and $\overline{Q}_{T2}$ at the same level of pressure and clearly proves the influence of bending on J integral. Figure 3 compares for $\overline{Q}_{T1}$, finite element results and simplified J estimations, the accuracy of the simplified method with pure pressure is very good. Figure 4 compares under $\overline{Q}_{T2}$ the simplified and numerical calculation of J, the simplified method remains satisfactory.

CONCLUSION

This study shows that the global limit load of a part-through wall axially cracked pipe under wall pressure, bending and tension can be obtained by solving a non linear system of two equations depending on two parameters. The proposed collapse mechanism is based on Kitchings (1) one for slots under pure pressure, with the shell Von Mises criterion instead of the two moment limited criterion.

Some extensions were necessary to take into account axial loadings. It would be possible to introduce more optimisation parameters to improve the present limit analysis. The proposed method leads to accurate and conservative J estimations.

SYMBOLS USED

R_m, R_i : mean and internal radius of the cylinder,
x : current thickness of the considered shell,
e =T-a ligament thickness,
$\rho = c/\sqrt{R_i a}$
M_{gx} : beam bending moment applied to the top of the x thickness cylinder,
N_{gx} : tension applied to the top of the x thickness cylinder,
p_x : pressure difference between inner and outer wall of the x thickness cylinder,
σ_y : yield stress limit in a perfectly plastic model,
$\mu_{gx} = \left|M_{gx}/\left(4R_m^2 x\sigma_y\right)\right|$, $\nu_{gx} = N_{gx}/\left(2\pi R_m x\sigma_y\right)$, $\pi_x^* = p_x R_i/\left(\sigma_y x\right)$
m_{gx}, n_{gx}, P_x^* : limit values of μ_{gx}, ν_{gx}, π_x^*,
$\overline{Q}_x = \left(\mu_{gx} \quad \nu_{gx} \quad \pi_x^*\right)$, $\overline{Q}_{Lx} = \left(m_{gx} \quad n_{gx} \quad P_x^*\right)$ applied and limit load of the x thickness cylinder,
Lr : fracture parameter such as $\overline{Q}_T = L_r\overline{Q}_{LT}$,
$\xi = z/c$ relative axial position,
M_z : longitudinal bending moment per unit length,
N_z, N_θ: longitudinal and circumferential direct forces per unit length,
Qz : transverse shear force per unit length,
$m_\xi = 4M_z/\left(\sigma_y x^2\right)$, $n_\xi = N_z/\left(\sigma_y x\right)$, $n_\theta = N_\theta/\left(\sigma_y x\right)$, $q_\xi = Q_z/\left(\sigma_y x\right)$
B : extent of plastic region in ξ direction,
m_o : integration constant,
n_ξ^+, n_ξ^- : upper and lower value of n_ξ,
γ_x : neutral axis angle of axial loads on the x thickness cylinder,
$\varphi^\varepsilon(L_r,u,v) = \frac{v}{2} + \varepsilon\sqrt{L_r^2 - \frac{3}{4}v^2 - u^2}$ $\quad \varepsilon = +$ or -
$A_a = \alpha T/a$, $A_e = (1-\alpha)T/e$, $\alpha \in [0,1]$
$\nu_\xi^+ = L_r n_\xi^+$, $\nu_\xi^- = L_r n_\xi^-$

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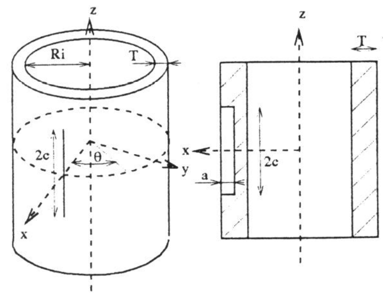


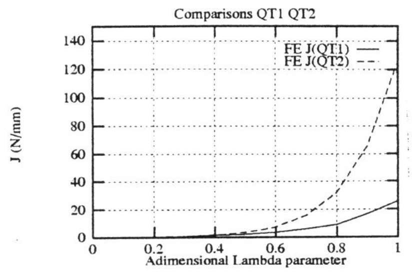
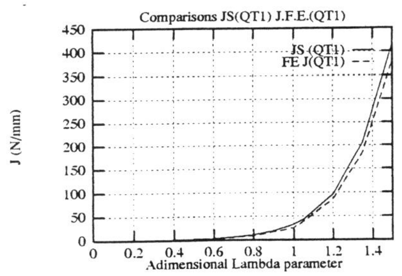


Figure 1: Geometry of the part-through cracked pipe. Figure 2: Influence of bending on J integral.



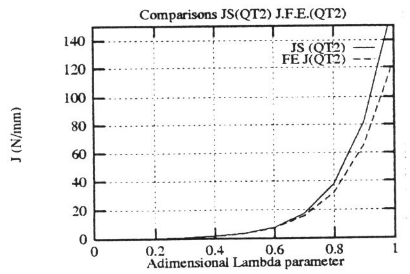


Figure 3: Accuracy of Js under pressure.

Figure 4: Accuracy of Js under pressure and bending