J ESTIMATION SCHEME FOR SURFACE CRACKED PIPINGS UNDER COMPLEX LOADING: PART II COMPLEX SHAPED ELBOW SOLUTIONS

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This part presents the KJ95 J-estimation scheme for circumferentially or longitudinally surface cracked elbows under pressure and in plane bending. The formulation is limited to cracks having a short length compared to the diameter. The plastic function J/J^e is derived from limit load solutions for elbows with two planes of symmetry, constant mean radius and wall thickness. These solutions have been computed using special beam elements and non-linear mathematical programming. Closed form formulae for reference pressure and bending moment are established from these results. Non symmetric boundary conditions and wall thickness variations are taken into account using appropriate stress indices. Finally, the load interaction function obtained for the cylinder is generalised to the elbow case.

INTRODUCTION

The theoretical bases of the KJ95 J-estimation scheme, straight pipe formulae, and symbol definitions have been presented in the first part of the paper. This second part extents the method to pipe bents for circumferential and longitudinal cracks with a length 2c limited to the Rm/3 value. Two types of configurations are considered: elementary models and representative ones. The elementary pipings consist of elbows connected to straight pipes, having a same circular section of constant thickness and mean radius with a defect located in the middle section of the elbow. The loading is applied at the end of a straight pipe. The influence of elbow length, end effects, location of the crack along the circumference are examined. For combined loadings, a global yield function for the cracked section is derived and validated on the basis of numerically computed limit loads.

In a second step, the extension of this J estimation scheme to any section and any type of elbow is investigated. These effects are accounted for using a linear

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elastic stress index type of approach. Additional elastoplastic finite element analyses were conducted on complex shaped cracked elbows and used as a basis to validate the J estimation scheme.

FINITE ELEMENT ANALYSES

The programme overview is presented in the first part of the paper. The computations have been conducted in a same way as for cylinders. However, the mesh around the circumference has been refined for the highly curved elbows. In some cases, the finite element code SYSTUS (1) has been used instead of CASTEM2000. Two types of thick elbow geometries are considered: one with a large radius of curvature, the other with a short radius. Four types of loadings are applied to these pipings: pressure, in plane bending, out-of-plane bending and combined pressure and in-plane bending. No pressure is applied on the crack faces. The effect of the out-of-plane moment appears to be, in our case, a little more severe than the in-plane bending moment. In the present version of KJ, this type of loading is not considered.

LIMIT LOAD COMPUTATIONS

The computation of the limit loads is performed by the ELSA software (Elbow Limit and Shakedown Analysis). This software uses the direct method, namely mathematical programming, as a basic tool for solving the non-linear problem of limit or shakedown analyses. Upper bounds of the load multiplier corresponding to the plastic failure of the structure are obtained via its discretisation into finite beam elements. As regards to the general formulation, ELSA adopts the one proposed by Nguyen Dang Hung & al. (2). It appears that this direct calculation offers a considerable time saving in comparison with the conventional step-by-step method. The beam element accounts for ovalisation, extension and warping, but also for the non-symmetric deformation of the cross section. The major numerical difficulty caused by the non differentiability of the plastic dissipation is overcome by regularising the operation in accordance with a mathematical artifice. It consists in taking a fictitious elastic-plastic material having a Young's modulus tending towards infinity such that the fictitious plastic strain energy identifies in the limit to the plastic dissipation of the rigid perfectly plastic material (3).

The interaction surfaces for elbows of three different curvatures subjected to pressure, in plane and out-of plane moment are obtained. The influences of other parameters like the length of curvature and the boundary conditions were analysed in a second step. For this study, forty geometry and loading cases were treated and results compared with the literature.

REFERENCE LOADS FOR SYMMETRIC CONSTANT THICKNESS ELBOWS

In straight pipes and elbows subjected to mechanical loads, stress relaxation is negligible and local and general yielding condition are dependant. Therefore the interaction equation from which Lr is computed, is easily derived in terms of applied loads. For elbows, the reference loads are given by the following formulae:

$$p_{ref} = p_{yPnc}\, g\left(R_c, \beta, A, S_p\right) \qquad (1)$$

$$M_{ref} = \mu_{Eh}\, \mu_{Eg}\, \mu_{Ec}\, M_{yPnc} \qquad (2)$$

These expressions have the same form as for cylinders (see part I), but in the elbow case, the functions g and μ_{Eg} ,which stand for component geometry effects, depend strongly on stress indices. The function μ_{Eh} takes into account strain-hardening dependence, g and μ_{Ec} the influence of crack size, location and orientation.

Closed form limit load formulae

Pressure. In order to derive the g function, we define the following reduced admissible stress field:

$$\hat{\sigma}_r = -\beta\left(1 - \frac{\beta}{3}\right)$$

$$\hat{\sigma}_\theta = (1-\beta)\, C_{1\theta} / (\delta_{is} q_s + \delta_{i\theta}) \qquad (3)$$

$$\hat{\sigma}_s = C_{1s} / (\delta_{i\theta} q_\theta + \delta_{is})$$

where δ_{is} and $\delta_{i\theta}$ are Kronecker symbols. $C_{1\theta}$ is defined as the ratio of the membrane circumferential stress in a pressurised elbow to the same stress in the cylinder having the same nominal cross section (constant thickness and mean radius). C_{1s} is the corresponding index for longitudinal membrane stress. The values of these indices are relative to the crack location (defined by the abscissa s of the cross section and the θ_c angle of the crack centroïd). They are obtained from a linear elastic finite element computation of the sound pressurised elbow (LESPE results). For short cracks, the q_θ and q_s factors are assumed to have the same value as in a cylinder. The q_θ formula is given in reference (4) and q_s is derived from the g_{sP} expression of the cylinder (see part I) through the following expression:

$$q_s = \frac{4(1-\beta)}{1-4\beta + 2\sqrt{3\left(\frac{1}{g_{Ps}^2} - \frac{1}{4}\right)}} \approx g_{Ps} \qquad (4)$$

Then, from the field (3) we get $g_{Es} = \frac{\sqrt{3}}{2\hat{\sigma}_{eq}}$ and $g_{E\theta} = \frac{\sqrt{3}}{2\hat{\sigma}_{eq}} S_p$ (5)

In highly curved elbows ($\lambda \leq 0.5$), end effects reduces slightly the C1c value, which is taken into account in the LESPE computation. Figure 1 shows a good agreement between finite element computed and KJ95 predicted J/J^e variations with increasing pressure in a slightly curved elbow.

Pure in-plane bending.

Spence & Findlay's (5) analytical formula (6) gives the best predictions of the toroidal solution obtained by numerical computations. For $\lambda \leq 0.5$, Calladine's formula (6) is also in very close agreement with the computed results. This means that, at least for a torus, the ELSA code delivers upper bounds which are almost exact solutions.

$$SF = \min\left[0.8\ \lambda^{0.6}\ ,\ 1\right] \qquad (6)$$

In order to derive a limit load formula for elbows of angle of curvature less than 180 degrees, an investigation on the C_2 index was conducted. Series of linear elastic finite element computations of elbows of different angles α lengthened by straight portion of pipes was performed. A stress index $C_{2\theta}$, defined as the ratio of maximum circumferential stress in the elbow to the maximum bending stress in the nominal cylinder appeared to be linearly varying with the bend angle. This index measures the level of ovalisation and is closely related to the limit load index (7). Therefore, we propose the following interpolation formula for the coefficient μ_{Eg}:

$$\mu_{Eg} = \max\left[SF\ ,\ 1 + (SF-1)\omega\right] \qquad (8)$$

where $\omega = \dfrac{\alpha}{135}$ and α the bend angle in degree.

An excellent correlation with the numerically computed limit loads is shown on Figure 2, since the numerical results represent an upper bound.

An upper bound of the strain hardening correction, derived from the analysis of a non linear beam under bending, gives: $\mu_{Eh} \cong \frac{2}{3}\left(\frac{3n}{2n+1}\right)^{\frac{n+1}{n-1}}$ This function should depend also on the characteristic factor λ, and for the sake of simplicity, the approximation $\mu_{Eh} = \mu_{Ph}(\theta_c = \pi/2)$ (9) has been retained.

For short cracked elbows, we assume that the defect is subjected to a local tensile field (which should be conservative for deep cracks) and that the defect influence on local yielding is the same as for a circumferentially cylinder of the same cross section. $\mu_{Ec\theta} = \mu_{Ecs} = q_\theta \cdot S_t$ (10) where q_θ and S_t are defined in (4). Using a technique presented in (8), reference moment values have been derived from finite element J computations. Figure 3 shows for two circumferentially cracked elbows under bending, that formula (2) gives reasonable approximations of reference moments.

The ovalisation makes the circumferential distribution of the equivalent stress almost uniform when local hinges appear at the crown and the intrados, allowing to disregard the effect of crack location in the cross section when the elbow is subjected to pure bending. For highly curved elbows, this assumption may be conservative for extrados cracks, but slightly unconservative for crown cracks.

Combined loading

The interaction formula is considered the same as defined for the straight pipe. In the sound elbow case, this assumption is verified by the limit load results. A method for deriving the interaction surface from finite element results on cracked structures has been presented in (8). A good correlation is obtained with the theoretical interaction surface for cracked cylinders, similar results have been obtained in this study for cracked elbows. In Figure 4, KJ95 predicted J/J^e variations are slightly lower than the finite element results in a circumferentially cracked elbow under constant pressure and increasing bending.

EFFECTS OF NON SYMMETRIC BOUNDARY CONDITIONS AND THICKNESS VARIATIONS

Influence of the location of the cracked section in the longitudinal direction

Under pressure, in highly curved elbows ($\lambda \leq 0.5$), the C_1 index values decrease when the cracked section becomes closer to the end of the elbow. This is accounted for in the LESPE computation.

Under bending, end effects are higher: the location of the section of highest ovalisation as well as the magnitude of this ovalisation depends on the boundary conditions. In our study we do not consider the case of elbows flanged at both ends and we assume that the flange do not reduce the maximum value of the ovalisation, but changes its location along the longitudinal direction. In the section of highest ovalisation, the formulae derived for symmetric elbows are used, and in the other section the reference load is interpolated between straight pipe and middle section of symmetric elbow formulae. The interpolation parameter is the ovalisation index v defined by a ratio of $C_{2\theta}$ indices. Such an approach, based on the variation of linear elastic parameter v with the abscissa s is valid for the characterisation of the non-linear behavior of the elbow under bending since ovalisation induces primary stresses (i.e. necessary to the fulfilment of equilibrium conditions). As for pressure, a finite element computation of the sound elbow under bending is needed (LESB result). The reference bending moment is therefore defined by the following set of formulae:

$$M_{ref} = M_{yPnc}\left[(1-v)\,\mu_P + v\,\mu_E\right]$$

$$\mu_P = \mu_{Ph}\mu_{Pc} \quad \text{(see part I)} \qquad \mu_E = \mu_{Eh}\mu_{Eg}\mu_{Ec} \qquad (11)$$

$$v(s) = \frac{\max\limits_{\theta} \sigma_{\theta\theta}}{\max\limits_{s,\theta}\left[\sigma_{\theta\theta}\right]}$$

Complex shaped elbows

In the primary loop of a Pressurised Water Reactor, elbows are reinforced at the intrados and sometimes the cross section diameter may not be constant. From elastoplastic finite element results obtained on complex shaped cracked elbows, we propose to define in each section a global thickness as the average of the thickness around the circumference. The global mean radius is then obtained by difference of the nominal external radius and half the global thickness. These global parameters have to be used to compute the sound cylinder limit loads and the characteristic factor λ. The local thickness is only used in the computation of stress indices and reduction of area factors. The KJ95 estimation scheme has been applied on an elbow with a longitudinal crack on the intrados subjected to an increasing in-plane bending closing moment. The mesh of the divergent reinforced elbow is represented on Figure 5. The J estimations fit well (Figure 5) with the finite element results up to the collapse moment.

CONCLUSION

The KJ95 J estimation scheme proposes a yield function giving the ratio of the non-linear value of the crack driving force to its linear elastic value. The latter is computed via influence functions established on cracked cylinders. The scheme has been developed mainly using analytical and numerical limit load results, corrections factors accounting for strain hardening effects derived from GE-EPRI results, and a stress index type of approach for geometry and end effects. The KJ95 scheme gives, at least for the 16 finite element computed cracked elbows of the validation programme, good predictions of J even at high load levels. The scheme takes into account the material stress-strain law, the elbow geometry and boundary conditions, the crack location, size and orientation (either longitudinal or circumferential). The application of the scheme requires to complete a linear elastic finite element computation of the uncracked component, but since the γ yield function (J/Je) is given by explicit formulae, the approach allows to make quick estimates for any type of defect. Although not shown in this paper, KJ95 allows to predict the evolution of J along the crack front.

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REFERENCES

(1) SYSTUS233®, Framasoft-CSI.

(2) Save, M. et al., "Limit loads of pipes elbows", WGCS report EUR 15696 EN, European Commission, 1995

(3) Nguyen Dang Hung, Jospin, R.J. and De Saxcé, G., Proc. Plasticity 91 symposium, Grenoble, France, Elsevier, 1991, pp; 623-626

(4) Gilles, Ph. & Bois, C., MF-3c, ASME PVP Conf., Montreal, July 1996.

(5) Spence, J. Findlay, G. E., 2nd ICPVT Conf., San Ant., Tx, USA.

(6) Calladine, C.R., J. Mech. Engng. Sci. 16 (2), 85, 1974.

(7) Rodabaugh, E. C., Battelle report N-0584, Ohio, USA, 1976

(8) Gilles Ph. & Bois C., 2nd Griffith Conference, The Institute of Materials, Sheffield, Sept. 1995.

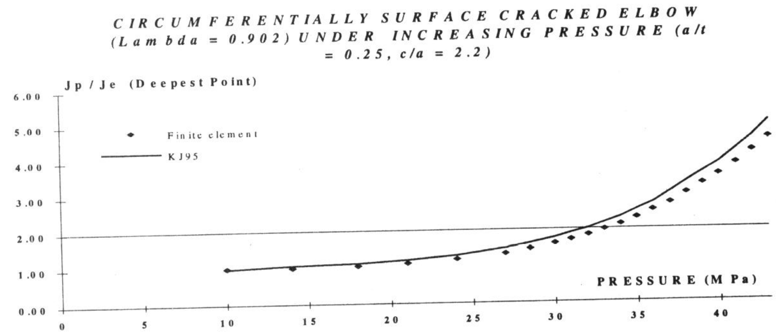

Figure 1: Finite element and KJ95 J/J^e variations in a pressurized elbow.

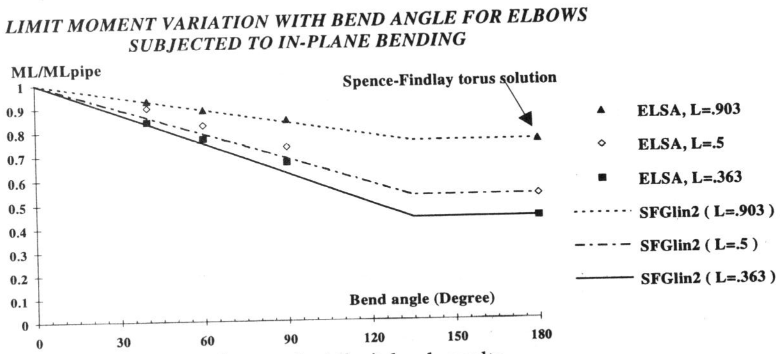

Figure 2: Analytical and numerical limit load results.

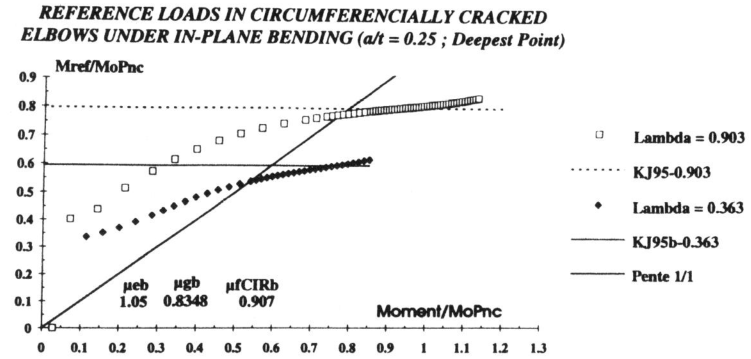

Figure 3: Analytical and numerical reference load results for elbows in bending.

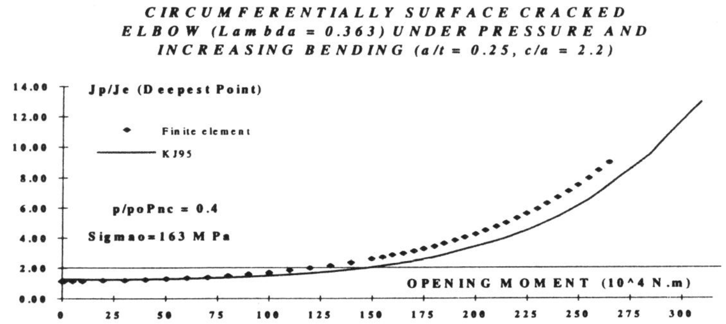

Figure 4: Finite element and KJ95 J/J^{e} in an elbow under pressure and bending.

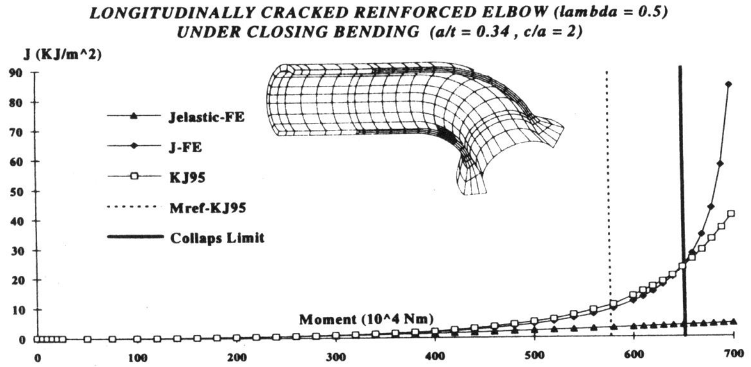

Figure 5: Finite element and KJ95 J variations in a complex shaped elbow.