J ESTIMATION SCHEME FOR SURFACE CRACKED PIPINGS UNDER COMPLEX LOADING: PART I: THEORETICAL BASIS.

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This paper proposes a J-estimation scheme, the KJ95 rule , for surface cracked cylinders and elbows under combined pressure and bending. The approach is developed within the frame of the deformation plasticity theory and attempts to generalise the ideas of the GE-EPRI method and of the British R6 rule. The results summarise a large study supported by EDF and achieved by Framatome. More than thirty three-dimensional elasto-plastic finite element computations of cracked components have been completed for developing and checking the formulations. This first part presents the theoretical foundations of the scheme and solutions for circumferentially or axially cracked cylinders under tension, pressure, bending or combined loadings.

INTRODUCTION

Since 1990, the French utility EDF, atomic energy commission CEA and Framatome are joining their efforts to develop and validate J estimation schemes, namely for thermomechanical loadings (1). All these schemes are based on the reference stress technique initially applied to a defect assessment procedure by R.A. Ainsworth (2). The objective of the present work, initiated in 1994 (3), is to examine the significance of reference stresses in cracked structures and to develop a coherent and accurate set of formulae for surface cracked cylinders and elbows subjected to pressure and bending. The resulting scheme, called KJ95, does not consider displacement and strain controlled loadings.

The Option 2 of the R6 rule (4) is a general purpose defect assessment route in which conservative assumptions are made in order to make safe predictions for crack initiation and growth. The KJ95 rule is merely a J estimation scheme, limited to mode I primary mechanical loadings, but built to deliver values as accurate as those obtained via the finite element solutions of the GE-EPRI handbook (5, 6).

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The KJ rule expresses J as a product of J^e by a yield function γ which has the same expression than in the R6 Option 2 approach. Je may be obtained via tabulated or fitted influence functions, from the elastic stress field computed on the uncracked structure. The main basic difference between R6 Option 2 and KJ95 consists in the definition of the reference load considered here to differ from "the plastic yield load of the flawed structure" (4).

THEORETICAL FOUNDATIONS

Under monotonically increasing proportional loading, the crack driving force J defines, within the plastic zone, the amplitude of the crack tip singularity. This has been demonstrated for strain hardening infinite cracked bodies subjected to tension fields (7,8), and from the local stress-strain fields expression we define the following averaged strain energy density expression:

$$\overline{\omega} = \overline{\varepsilon_V \sigma_V} = \frac{J}{I_n r}\tilde{\sigma}(n,\theta)\tilde{\varepsilon}(n,\theta) \qquad \text{for a stress-strain law } \varepsilon_V = k\,\sigma_V^n \quad (1)$$

where I_n is a function of the strain hardening exponent n and $\tilde{\sigma}$ and $\tilde{\varepsilon}$ are dimensionless function of the polar angle and n.

At least, for the HRR field, the angular dependence of the adimensional strain energy $\overline{\omega}$ is quite low for strain hardening exponent greater than 3. We therefore approximate the γ function by the non singular ratio:

$$\gamma = \frac{J}{J^e} = \frac{E\overline{\varepsilon_v}}{\overline{\sigma_v}}H(n) \qquad (2)$$

From this relationship, we deduce that:

- The γ function may be obtained without considering the stress singularity.
- The prediction of J from Je requires to select for determining the function γ a plastically admissible field, the reference stress being an equivalent stress.
- The reference stress exists only if H does not depend on the strain hardening or if this dependence is identical whatever the load level (as for the power law curve). In this case, we have: $\gamma = E\varepsilon_{ref}/\sigma_{ref}$

For cracks in infinite bodies, the H function depends on the strain hardening and may not be finite for a perfectly plastic material (8). For finite bodies, if the material is perfectly plastic, it is obvious that the γ function is given by a limit load expression. For strain hardening materials, if the field is of tensile type, the stress and strain distributions remain uniform in the ligament but in three dimensional cases, the extent of yielding depends on the strain hardening properties. In bending, the stress distribution is always strain hardening dependent. Thus in most cases, the reference stress technique is not exact since the relationship between the local yield and the load may depend of the load level. This is particularly true in the small scale yielding range where R6 proposes a correction independent of the stress-strain law, which is not consistent with the principles of the deformation plasticity theory. As in the R6 approach, we use the concept of yielding index Lr, but defined here as the ratio of the primary load to a reference load. In bending the reference load depends on strain hardening properties (9) and in three dimensional models the yielding may begin at other locations than in the cracked section. Figure 1 shows, for a cylinder under bending, the influence of the crack location in the section. The difference is due to strain hardening and is well predicted by KJ95.

FINITE ELEMENT ANALYSES

Full three dimensional elasto-plastic finite element computations were conducted on three types of cracked components: one cylinder and two elbows having the same cross section (Rm = 300 mm, t = 60 mm). A basic grid of 17 cases essentially described in Table 1 has been defined to analyse the effect of the type of loading and the crack location. For this grid the material characteristics, the crack shape and size were fixed. The material stress-strain law is described by the following Ramberg-Osgood law:

$$\varepsilon = \frac{\sigma}{E}\left[1 + \left(\frac{\sigma}{\sigma_0}\right)^5\right] \quad \text{with } \sigma_0 = 163 \text{ Mpa} \qquad (3)$$

and by the elastic constants E = 174700 MPa and $\nu = 0.3$.
The crack was semi-elliptical, with a relative depth a/t = 0.25 and an aspect ratio c/a = 2 (for circumferential cracks the ration is c'/a = 2.2), and in all cases was located on the outer wall of the component. Figure 2 shows a typical mesh.

TABLE 1- Basic grid of finite element computations

Component	Cylinder	$\lambda = .902$	$\lambda = .363$
Crack orientation	Circ. / Long.	Circ. Intrados	Circ. Intrados long Intrados
Loading	Pressure / Bending / Pressure then Bending		

The elbows have been opened in their symmetry plane. The circumferentially cracked cylinder was also loaded in pure tension and under bending out-of the plane of symmetry of the cracked cross-section. The bending case was not considered for the axially cracked cylinder. A closing moment was applied on the highly curved elbow with a crack located on the crown. A sensitivity analysis, involving about 15 computations, was conducted on the effects of the strain hardening exponent, on the crack depth, on the level of maximum pressure in combined load cases. No pressure has been applied on the crack faces.

Computations were conducted using CASTEM2000 finite element code. The meshes were constituted with quadratic isoparametic solid elements. The Von-Mises flow rule is selected and calculations are made under the small displacement assumption. The crack driving force J is computed through the use of the G-THETA domain integral method (10).

PHYSICAL SIGNIFICANCE OF THE REFERENCE LOAD

Interaction condition for a circumferentially cracked cylinder under tension, pressure, bending and twisting moments (rigid perfectly plastic material)

In the general case, especially for very large cracks, the interaction surface is given by the following set of equations, valid for $\gamma \leq \dfrac{\pi\,(n_x + 1)}{2 - X}$.

$$\alpha = -\frac{\pi}{2}\,\frac{-\dfrac{n_\theta + n_r}{2} + n_x + \dfrac{X.\gamma}{\pi}\left[\dfrac{n_\theta + n_r}{2} + \sqrt{1 - \dfrac{3}{4}(n_\theta - n_r)^2 - m_t^2}\right]}{\sqrt{1 - \dfrac{3}{4}(n_\theta - n_r)^2 - m_t^2}}$$

$$m_f = \left[\cos\alpha - \frac{X}{2}\sin\gamma\right]\sqrt{1 - \frac{3}{4}(n_\theta - n_r)^2 - m_t^2} - \frac{X}{2}\sin\gamma\left(\frac{n_\theta + n_r}{2}\right) \quad (4)$$

For short in length cracks these equations may be fairly well approximated by a unique equation:

$$\Phi\,(M,p) = \left[\frac{M}{M_{ref}}\right]^2 + \left[\frac{p}{p_{ref}}\right]^2 - 1 = 0 \qquad (5)$$

where M_{ref} and p_{ref} take into account the defect size and orientation and will be defined later for a strain hardening material. The Lr derivation is explained in (9).

Stress triaxiality effects on the limit load

Determining the limit load in a deeply cracked structure is not always straightforward. Even if the structure is under pure tension and the crack front stress singularity is disregarded, the stress state in the ligament may not be uniaxial. The analysis of GE-EPRI results for full circumferential (5) or through-wall cracks (6) in cylinders under tension evidences a strong triaxiality effect on the limit load as shown in Figure 3. In the reference moment, this effect is accounted for by the triaxiality S_t parameter (11). Under pressure, a similar effect exists and the corresponding correction function is denoted S_p.

From the limit load to the reference load

For small defects in strain-hardening structures, the containment of plasticity around a small surface defect, increases the defect influence on the reference stress. In the R6 route, this is taken into account by a recharacterisation of the defect and the definition of a local yield load. Our proposal gives different results: the effect of shallow defects is amplified and the reference loads are continuous and smooth functions of the crack size. Their expressions are given by:

$$p_{ref} = p_{yPnc} \cdot g\left(R_c, \beta, A, S_p\right) \qquad (6)$$

$$M_{ref} = \mu_{Ph}\ \mu_{Pc}\ M_{yPnc} \qquad (7)$$

$$\mu_{Ph} \cong \left[1 + 0.083 / (n-1)^{0.6}\right] (\sin\theta_c)^{\left(\frac{1}{n}\right)} \qquad (8)$$

<u>For a circumferential crack,</u> $\mu_{P\theta}$ is given in (10) and $g_\theta \approx 1$

<u>For a longitudinal crack,</u> $\mu_{Ps} \approx g_{Ps}$ are given by the following expressions (9):

$$g_{Ps} = \max\left(1 - X^{0.2},\ \frac{1-X}{1+\beta X} S_{ps} + \frac{X}{1 + \frac{\rho}{\sqrt{X(1+\beta X)}}}\right) \qquad S_{ps} = 0.95 + \left(X\frac{\rho}{1+\rho}\right)^2$$

In all the formulae terms of higher order than β have been neglected. The variations of μ_{Ps} and g_{Ps} with the defect size are shown in Figures 4 and 5. An illustration of the efficiency of the scheme is given in Figure 6.

CONCLUSION

This first part has shown that the derivation of an accurate yield function allowing to obtain J from the elastically computed value Je, requires to improve the reference stress definition. This is achieved by expressing a local yield condition at the point of the crack front were J is estimated, on the cracked structure in terms of the applied loads, the crack singularity being disregarded. Thus instead of considering a yield limit surface, a reference interaction surface is derived, accounting for strain-hardening and stress triaxiality effects. The application to surface cracked pipe under simple or combined mechanical loadings appear to be promising.

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SYMBOLS USED

a = crack depth (mm).
c = half length (mm) of the semi-elliptical longitudinal crack
E = Young's modulus (Mpa).
J = crack driving force (kJ/m^2).
M_f, M_h, M_t = In plane and out of plane bending and twisting moments (10 kN.m).
n = Ramberg-Osgood strain hardening exponent.
$p_{yPnc} = 4\beta/\sqrt{3}\ \sigma_y$ uncracked cylinder limit pressure (MPa).

$$M_{yPnc} = 4\,\sigma_y R_m^2\, t \quad m_f = \frac{M_f}{M_{yPnc}} \qquad m_t = \frac{M_t}{2\sqrt{3}\,\pi\,\sigma_y R^2 t}$$

$$n_r = -\frac{p}{\sigma_y}\left(\frac{1}{2}-\frac{\beta}{6}\right), \quad n_\theta = \frac{p}{\sigma_y}\left(\frac{1}{2\beta}-\frac{1}{2}\right), \qquad n_x = \frac{N}{2\pi\sigma_y R t} + \frac{n_r + n_q}{2}$$

$Q_{0\text{-ref}}$ = reference load scaled to σ_0 (the unit is the same as the applied load)
Q_{FE} = Finite Element computed reference load
R_c = elbow curvature radius (mm)
R_m = pipe mean radius (mm)
t = pipe thickness (mm) and X = a/t

Subscripts P and E refers respectively to straight pipe and elbow. The subscript nc refers to the parent uncracked structure, v to Von Mises stress or strain. Subscripts s and θ refers respectively to longitudinal or circumferential orientations. The superscript e refers to elastically computed quantities.

α = Ramberg-Osgood law parameter.
β = pipe curvature parameter ($t/(2.R_m)$ ratio).
γ = half crack length (degree).
λ = Elbow characteristic factor ($\lambda = R_c t/R_m^2$)
ρ = Reduced length ($\rho = c/\sqrt{R_m t}$)
υ = Poisson's ratio.
σ_0 = Ramberg-Osgood stress parameter (MPa).
σ_y = 0.2% proof stress (MPa).

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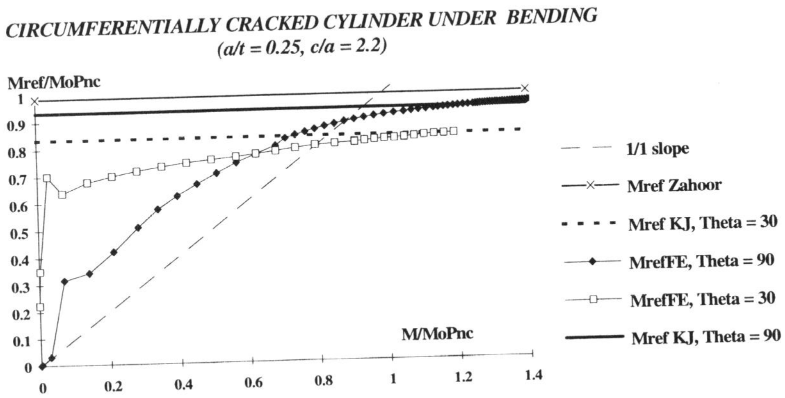


Figure 1: Reference loads derived from FE results and predicted by KJ95

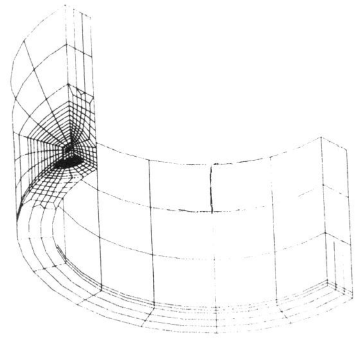


Figure 2: Partial view of the mesh of a cracked cylinder.

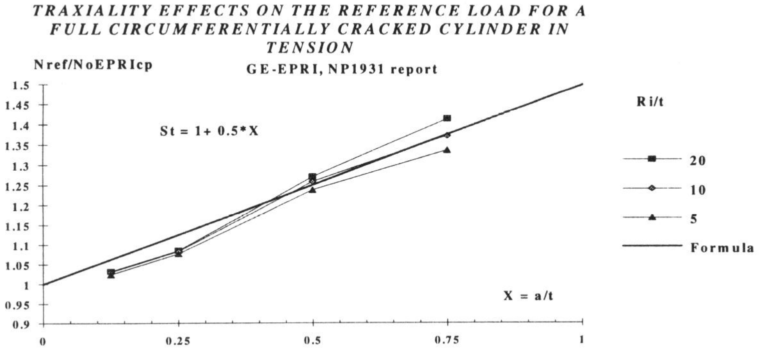
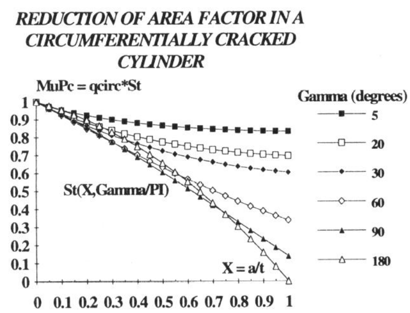


Figure 3: Stress triaxiality effects on the reference load



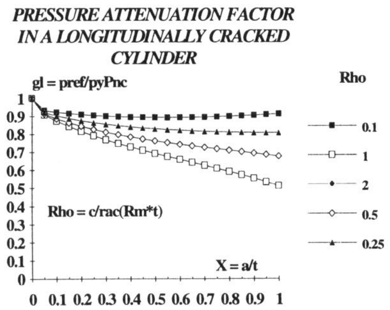


Figure 4: Defect influence on moment

Figure 5: Defect influence on pressure

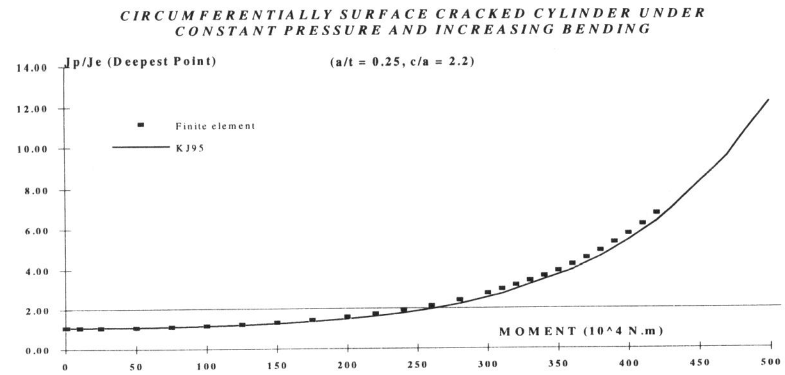


Figure 6: Comparisons of finite element results and KJ95 predictions