INFLUENCE OF THE CRACK TIP THREE-DIMENSIONALITY ON THE EVALUATION OF SIFs BY CAUSTICS

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The extent of the three-dimensional region surrounding the tip of an opening-mode crack is studied experimentally by the optical method of caustics. It is shown that the state of stress changes from plane strain at the tip to plane stress at a critical distance from the tip. For the correct evaluation of stress intensity factors (SIFs) the generatrix curve of the caustic should lie in the region where the state of stress is plane stress. Use of optically anisotropic materials to compensate for the three-dimensional effect in the determination of SIFs is proposed.

INTRODUCTION

Evaluation of stress intensity factors in fracture mechanics problems by the method of caustics is met with difficulties due to the changing state of stress in the neighborhood of the crack tip. Near the tip plane strain conditions dominate, while further away from the tip the stress state approaches plane stress. Between the two regions the stress field is three-dimensional. The caustic formed on a reference screen placed at some distance from the specimen is the image of the so-called initial curve on the specimen. The initial curve for linear elastic crack problems is a circle centered at the crack tip. Evaluation of the caustic to determine stress intensity factors is influenced by the condition of the stress field along the initial curve. When the initial curve lies in the region where the state of stress is plane strain, three-dimensional or plane stress, the stress-optical coefficients related to the evaluation of SIFs are quite different. The difference in the values of the stress-optical coefficients depends on the material of the specimen. It is of the order of 30 percent or 40 percent for

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Plexiglas or Araldite B, respectively, for the extreme cases of plane stress and plane strain. Thus, the values of stress intensity factors obtained from caustics can vary significantly.

Before any evaluation of SIFs from caustics, the state of stress along the initial curve of that particular caustic should be known in advance. The effect of the changing state of stress on the evaluation of SIFs by caustics and its implications on the applied load level, specimen size and dimension of the optical arrangement has been studied by Konsta-Gdoutos and Gdoutos in a series of publications (1-3).

In the present paper experimental results concerning the extent of the three-dimensional region surrounding the crack tip are presented. Furthermore, the use of optically anisotropic materials to compensate for the effect of the triaxiality of the state of stress in the determination of SIFs is proposed.

EXPERIMENTAL

A series of experiments were performed in single edge notched specimens made of plexiglas. The thickness t and the width w of the specimens took the values t=3.0, 4.5, 9.5 and 12.5 mm, and w=42.5, 47.5, 51.5 and 63.5 mm. The crack length was a=15.5 mm. The specimens were subjected to a progressively increasing tensile load in an Instron testing machine. A Ne-He laser was used to illuminate the specimens. The experimental arrangement used is shown in Figure 1. The specimen was illuminated by a divergent light beam. The dimensions of the optical arrangement expressed in terms of the magnification factor and the distance between the specimen and the screen where the caustic was formed were changed in the experiment. In this way, the radius of the generatrix curve of the caustic, the so-called initial curve took a host values.

THREE-DIMENSIONAL REGION AT CRACK-TIP

The experimental value of mode-I stress intensity factor K_{exp} was calculated by (4)

$$K_{exp} = 0.0934 \frac{D^{5/2}}{z_0 c t m^{3/2}} \qquad (1)$$

where z_0 is the distance between the specimen and the viewing screen, c is the stress-optical constant of the specimen under conditions of plane stress, t is the specimen thickness, m is the magnification factor and D is the transverse diameter of the caustic at the crack tip.

The values of K_{exp} were compared with their theoretical counterparts calculated by (5)

$$K_{th} = \sigma\sqrt{\pi a}\left[1.12 - 0.23\left(\frac{a}{w}\right) + 10.55\left(\frac{a}{w}\right)^2 - 21.72\left(\frac{a}{w}\right)^3 + 30.95\left(\frac{a}{w}\right)^4\right] \quad (2)$$

where a is the crack length, w is the specimen width and σ is the applied stress.

Figure 2 shows the variation of K_{th}/K_{exp} versus r/t, where r is the radius of the initial curve for t=12.5 mm The experimental points correspond to a host of applied stress levels, while the specimen width took the values w=42.5, 51.5 and 63.5mm. Note that K_{th}/K_{exp} increases with r/t up to a limiting value of 1, at which the state of stress is plane stress. This occurs for r/t=0.45. The results of Figure 2 indicate that for $0<r/t<0.45$ the state of stress in the neighborhood of the crack tip is three-dimensional, while for r/t>0.45 the state of stress is plane stress. Thus, for the correct evaluation of opening-mode stress intensity factor the radius of the initial curve r of the caustic should be such that r>0.45t. For values of r smaller than the above value the stress intensity factor obtained by experiment K_{exp} is much lower than its theoretical value K_{th}. Thus, application of the method of caustics to determine SIFs when r<0.45t gives erroneous results.

COMPENSATION OF THREE-DIMENSIONAL EFFECTS

Use of optically anisotropic materials allows evaluation of SIFs by caustics, even though the initial curve of the caustic lies in the region where three-dimensional effects dominate. For optically anisotropic materials the variation of the optical path takes different values along the two principal stress directions. This is in contrary to optically isotropic materials, where the variation of the optical path along the two principal stress axes is the same. This results to a double caustic for optically anisotropic materials (Figure 3), whereas for optically isotropic materials only a single caustic is obtained. The double caustic generated in optically anisotropic materials contains more information than the single caustic obtained in optically isotropic materials. This information can properly be used to obtain the correct value of stress intensity factor even though the initial curve of the caustic lies in the region where three-dimensional effects dominate. A methodology to obtain SIF follows.

The three-dimensional state of stress in the neighborhood of the crack tip can be characterized by a triaxiality factor, such that

$$\sigma_z = k(\sigma_x + \sigma_y) \tag{3}$$

where σ_x and σ_y are the in-plane stresses and σ_z is the out-of-plane stress. k takes the values 0 and 1 for conditions of plane stress ($\sigma_z=0$) and plane strain ($\sigma_z=\nu(\sigma_x+\sigma_y)$), respectively, where ν is Poisson's ratio. Values of k between 0 and 1 identify different degrees of stress triaxiality.

In optically anisotropic materials, the degree of optical anisotropy is expressed by the coefficient of stress-optical anisotropy ξ. This coefficient depends on the triaxiality of the state of stress, and therefore, on k. The variation of ξ_t related to the light rays traversing the specimen versus k for Araldide B is shown in Figure 4. The curves of the figure correspond to different values of the index of refraction n_0 of the medium surrounding the specimen. Note that for n_0=1.0, 1.1, 1.2 and 1.3 ξ_t is larger for plane strain (k=1) than for plane stress(k=0), while the contrary occurs for k=1.4 and 1.5.

The split-up of the double caustic expressed by the quantity $(D_0-D_i)/D_0$, where D_0 and D_i are the transverse diameters of the outer and inner caustic, respectively, depends on the coefficient of anisotropy ξ. The variation $(D_o-D_i)/D_o$ versus ξ is shown in Figure 5. From Figures 4 and 5, we obtain in Figure 6 the variation of $(D_0-D_i)/D_0$ versus k. Figure 6 allows determination of the coefficient of stress triaxiality k by measuring the values of D_0 and D_i from experiment. Once k is known the correct values of the stress-optical constant c_t can be determined from equation

$$c_t = \frac{\alpha_t + \beta_t}{2} \tag{4}$$

where

$$\begin{aligned} \alpha_t &= \frac{1}{E}\left[(1-k\nu^2)b_1 - \nu[2-k(1-\nu)]b_2 + (n-n_0)\nu(k-1)\right] \\ \beta_t &= \frac{1}{E}\left[-\nu(1+k\nu)b_1 + (1-\nu)(1+\nu k)b_2 + (n-n_0)\nu(k-1)\right] \end{aligned} \tag{5}$$

Once the value of c_t is known for the particular state of stress triaxiality valid along the initial curve of the caustic, the correct value of SIF is determined from Equation (1).

Thus, the use of optically anisotropic materials in conjunction with the above procedure allows the correct determination of SIFs by caustics without paying attention on the location of the generatrix curve of the caustic.

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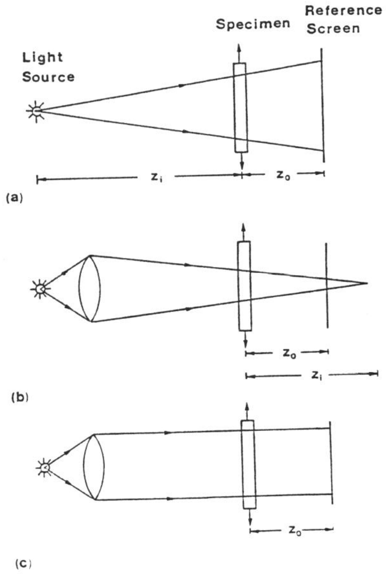

Figure 1 Optical arrangement

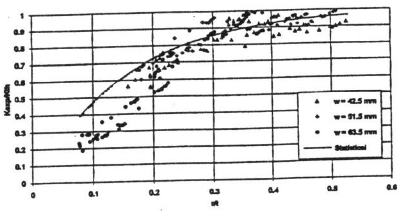

Figure 2 Variation of K_{exp}/K_{th} versus r/t, t=12.5 mm

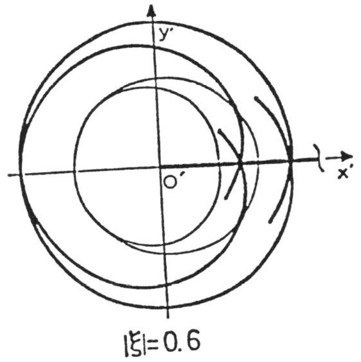

Figure 3 Double caustic created from an optically anisotropic material

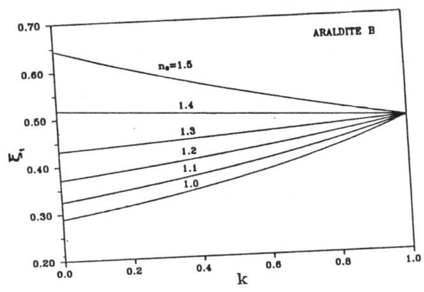

Figure 4 Variation of ξ_t versus k for Araldite B

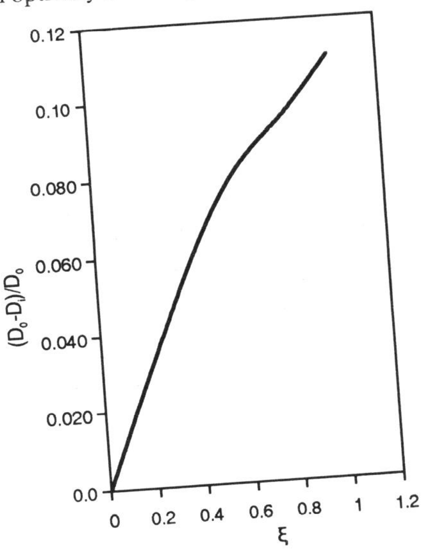

Figure 5 Variation of $(D_0-D_i)/D_0$ versus ξ

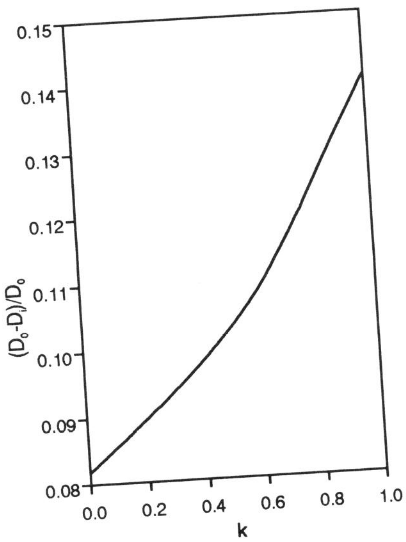

Figure 6 Variarion of $(D_0-D_i)/D_0$ versus k for Araldite B