INFLUENCE OF LOCAL STRESS AND STRAIN CONCENTRATORS ON THE RELIABILITY AND SAFETY OF STRUCTURES ILSSCRSS*

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In the frame of a Copernicus program a cooperative research is made on the effect of stress and strain concentrators on the reliability and safety of structures. In the limited volume of this paper only 4 aspects are presented.

- stresses at notch tip,
- local stress fracture criteria for notched structures,
- influence of local stress and strain concentrators on fatigue life of randomly loaded structures,
- effect of stress concentration on damage localisation.

INTRODUCTION

The main goal of the network created in the frame of COPERNICUS CIPA CT 094 0194 program is to join the specialists who are working in the field of "notch effects in engineering practices".

The importance of this topic is proved by the fact that each engineering structure has stress or strain concentrators. These places are the weakest areas of structural elements where fracture or fatigue cracks could be initiated. One of the advantages to consider all the stress concentrators as equivalent cracks is due to the fact that the stress distribution is less complex than for a notch. Irwin (1) has shown the existence of a stress singularities at notch tip and a $1/\sqrt{r}$ dependance. Cracks are also the most severe defect and it is a conservative point of view to treat each of them as an equivalent crack. On contrary, this leads to overestimate the nocivity and severity and increases safety factors, weight and cost of the structure. Fracture tougness is usually measured with precracked specimens. This gives a minimum value of the fracture resistance. It is well-known that for very brittle materials (ceramics, glasses, high strength steels), it is difficult to precrack the specimens without any risk of fracture and notched specimens are generally specimens prefered. At the present time, fatigue crack initiation time emanating from notched particulary in the low cycle fatigue and endurance regimes is evaluated with relatively old criteria which dont take into account the real stress gradient at notch tip.

These problems are the basis of a cooperative programm entitled " Influence of Stress and Strain Concentratorson the Reliability and Safety of Structures" (2) and performed by 27 laboratories from 12 european countries. In the present paper few aspects are presented involving stress distribution at notch tip, local fracture criteria,influence of local stress and strain concentrators on fatigue life or randomly loaded structures and effect of stress concentrators upon damage localisation.

STRESSES AT NOTCH TIP (2)

Several descriptions of the stress distribution at notch tip have been proposed (3,4,5,6,7,8,9).The solution proposed by Creager (4) consists in adding a geometrical correction factor to the Irwin's solution. This correction factor is a function of the square root of the notch radius. The stress distribution normal to the crack plane in mode I can then be written as follows :

$$\sigma_{yy} = \sigma_N + \frac{K_I}{\sqrt{2\pi r}}\cos(\theta/2)[1+\sin(\theta/2)\sin(3\theta/2)] + \frac{K_I}{\sqrt{2\pi r}}\left(\frac{\rho}{2r}\right)\cos(3\theta/2) \quad (1)$$

The stress gradient is always characterised by a $\sqrt{r}$ dependance and by the stress intensity factor K_I. Williams (3) has considered a "sharp notch" where the notch radius is equal to zero but the notch angle is $\psi \neq 0$. The stress distribution normal to the notch plane is given by

$$\sigma_{yy} = \frac{K^*_I}{(2\pi r)^{\alpha}} \quad (2)$$

The stress gradient has a $r^{-\alpha}$ dependance where a is less than 0.5 and depends on the notch angle ψ.

$$\alpha(\psi) = 0.5 - 0.089\left(\frac{\psi}{\pi}\right) + 0.442\left(\frac{\psi}{\pi}\right)^2 - 0.853\left(\frac{\psi}{\pi}\right)^3 \quad (3)$$

k^*_I is the so-called "notch stress intensity factor". Other elastic solutions of the crack tip distribution have been proposed by Neuber (5), Chen (6), Kujawski (7), Usami (8) and Glinka (9). The solution given by Chen [6] is close to that got by finite elements method (2).

LOCAL STRESS FRACTURE CRITERIA FOR NOTCHED STRUCTURES

The logarithm of the ratio of the stress normal to notch plane over the nominal stress is plotted versus the logarithm of the standardized distance. A schematic representation of this distribution is presented in (2).
* In the first part the stress is practically constant over a distance X_c called the characteristic distance.

$$\sigma_{yy} \approx cst = k_t . \sigma_N \quad for \quad 0 \leq x \leq X_c \tag{4}$$

*In the second part, the stress distribution follows a power law.

$$\sigma_{yy} = \frac{K_\rho}{(2\pi r)^\alpha} \tag{5}$$

This second part represents the so called notch stress (or strain) singularity.This singularitiy is represented by two parameters : the notch stress intensity factor K_r and the coefficient. The α coefficient is a function of the notch angle ψ and also a function of the mechanical behaviour. In the elastic case $\alpha = 0.43$ In every cases, α is always lower than the corresponding values for a crack which is equal to 0.5.

*In the third part, the stress distribution reaches asymptotically the value of the nominal stress σ_N. The local stress fracture criteria has the following expression (10) :

$$\frac{1}{X_c}\int_0^{X_c} \sigma_{yy} dx \leq \sigma_c^* \tag{6}$$

$$\frac{1}{X_c}\int_0^{X_c} \frac{K_I^*}{(2\pi . x)^\alpha} dx \leq \sigma_c^* \tag{7}$$

Where σ_c* is the critical stress.

INFLUENCE OF LOCAL STRESS AND STRAIN CONCENTRATORS ON FATIGUE LIFE OF RANDOMLY LOADED STRUCTURES (11)

Structural components are always notched (holes, keyways, grooves, shoulders etc ...). This phenomenon conditions the local strain increase, accelerates the fatigue crack initiation in the root of the notch and consequently shortens the fatigue life. Considering that the process is always a plain process at the notch vicinity, is of practical importance to elaborate a practical method of transformation of the plain random process to its notch pairs considering that it may concern both strain as well as stress. Such a transformation is not problematic for static cases using stress or strain concentration factor mathematically derived or experimentally. The local strain ε_l or stress σ_l at notch tip are :

$$\varepsilon_l = K_\varepsilon \varepsilon \quad ; \quad \sigma_l = K_\sigma \sigma_g \tag{8}$$

Where K_ε and K_σ are the plastic notch strain and stress concentration factor. The same approach can be analogically used also for sinusoidal loading althought not in a so straightforward way.In other words the relation between the plain stress and strain amplitude σ_a ans ε_a and their corresponding local amplitudes $\varepsilon_{a,l}$, $\sigma_{a,l}$ is expressed by the Neuber formula :

$$\sigma_{a,l} \cdot \varepsilon_{a,l} = k_f^2 \cdot \sigma_a \cdot \varepsilon_a \tag{9}$$

Where k_f is the fatigue reduction factor. The Basquin relationship is :

$$\sigma_a = \sigma'_f (2N_R)^b \tag{10}$$

where $\sigma'f$ is the fatigue strength, b exponent and N_R the number of cycles to failure.The Manson-Coffin relationship is expressed by :

$$\varepsilon_a = \varepsilon'_f (2N_R)^c + \frac{\sigma'_f}{E}(2N_R)^b \tag{11}$$

By combining these relationships, we get the Manson-Coffin relationship for notched specimens in the form :

$$\sqrt{E\varepsilon_a \cdot \sigma_a} = \frac{1}{k_f}\sqrt{\left((\sigma'_f)^2 (2N_R)^{2b} + E\,\sigma'_f \cdot \varepsilon'_f (2N_R)^{b+c}\right)} \tag{12}$$

When the applied load is random, the situation is even more complicated and one can propose few possible approaches with one criterion of their suitability: the resulting fatigue life. The plain random process transformation can be realized in the following ways :

a) The global random process σ_{gi} and ε_{gi} are multiplied by the stress or strain concentration factors $K_{\sigma,i}$ or $K_{\varepsilon,i}$ respectively which yields the corresponding notch pair of ordinates i,e.

$$\sigma_{e,i} = k_{\sigma,i}\,\sigma_{g,i} \text{ and } \varepsilon_{e,i} = k_{\varepsilon,i}\,\varepsilon_{g,i} \tag{13}$$

Because the stress and strain concentration factors depends on the stress and strain levels, various concepts (e,g) Hardrath-Ohman, Neuber or equivalent energy may produce different results.

b) A global random process (stress and strain) is analysed by the Rain Flow method yielding as plain macroblock of harmonic cycles. Its amplitudes $\sigma_{a,i}$ or $\varepsilon_{a,i}$ are subsequently transformed into a notch macroblock using the same expression as above.

c) A plain harmonic macroblock is correlated with its stress pair using the cyclic stress strain curve equation.

$$\varepsilon_a = \sigma_a/_E \left(\sigma_a/_k\right)^{1/n} \tag{14}$$

Where E is the Young's modulus and n the strain hardening exponent. Knowing the pair amplitudes $\varepsilon_{a,g,i}$ and $\sigma_{a,g,i}$ it is possible to apply the Manson-coffin curve for notched components and directly compute the corresponding numbers of cycle to fracture.

d) Assuming that every plain random process ordinate $\varepsilon_{p,i}$ or $\sigma_{p,i}$ obeys to the Neuber is formula the notch random process ordinate $\sigma_{h,i}$ or $\varepsilon_{h,i}$ obtained in this area then processed by the Rain Flow method yielding again a macroblock of harmonic cycles.

EFFECT OF STRESS CONCENTRATION ON DAMAGE LOCALISATION (12)

The effect of stress concentration damage localisation and subsequent growth in creep conditions can be derived from the basic assumption made according to stress gradient in a crack tip. This was expressed by the following equation :

$$\frac{\partial D}{\partial x} = c\left(\overset{*}{\sigma_c}, \sigma_N\right) \tag{15}$$

Where σ_N is the average net stress in the structure. One can see that for $\sigma_N = \overset{*}{\sigma_c}$, the stress gradients equals to zero i.e there is no stress concentrations.

Therefore $\overset{*}{\sigma_c}$ can be understood as the stress which causes failure in a smooth specimen.

The constant c (dimension m^{-1}) determines a characteristic length. With the help of this assumption and making use of damage D equation for a time dependant process

$$\dot{\varepsilon} = d\left(\frac{\sigma}{1-D}\right)^n . \sigma \tag{16}$$

$$\dot{D} = E\left(\frac{\sigma}{1-D}\right)^m . \sigma \tag{17}$$

Where d and E are contant, m and n are exponents. This set of equation corresponds to the well known theory of brittle failure in creep conditions as described by the Katchanov-Rabotnov coupled damage theory (with slightly newly defined material contants).

REFERENCES

(1) Irwin G.R. Trans ASME Journal of applied mechanics , Vol 24 (1948), pp. 361-364.

(2) Chehimi C, Angot G, Chachrone A and Pluvinage G, First Meeting ILSSCRSS University of Miskolc April (1995), pp. 7-16.

(3) Williams M.L. Int. J. of applied mechanics 74, (1952), pp. 526-528.

(4) Creager M and Paris P.C. International Journal of fracture 3, 1967, pp. 247-252 .

(5) Neuber, Weiss, V. Trans. ASME 1962, paper n°62 - WA-270.(1962)

(6) Chen C, and Pan H, "Collection of Papers on Fracture of Metals"(Chen Ed.), Metallurgy Industry Press, Beijing, (1978) pp. 197-219.

(7) Kujawski D, "Fatigue Fracture Engineering Materials Structures", vol 14, (1991), pp953-965.

(8) Usami S, In "Current Research on Fatigue Cracks' The Society of Material Science, Kyoto, Japan , (1985) pp119.

(9) Glinka G, and Newport A, Int J. Fatigue 9, (1987), pp. 143.

(10) Morozov E.M, First meeting, ILSSCRSS University of Miskolc April (1995), pp. 7-16.

(11) Bily, M. Kliman V, First meeting, ILSSCRSS University of Miskolc April (1995), pp. 149-157.

(12) Chranowski, M. German J, Grabias M, First meeting, ILSSCRSS University of Miskolc, April (1995), pp 122-130.