INFLUENCE OF CROSS-CORRELATION BETWEEN NORMAL STRESSES ON BIAXIAL FATIGUE

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The paper contains an analysis of cross-correlations between stresses under biaxial tension-compression and its influence on the fatigue life. The analysis is based on the data from literature and the results obtained by the authors during the tests of cruciform specimens of 10HNAP steel for various cross-correlation coefficients of loadings. The authors compared permissible amplitudes of stresses under non-proportional and proportional loadings. Fatigue tests were the basis for calculations of life according to several criteria. The best results were obtained with the modified criteria of maximum normal stress and of maximum shear and normal stresses in the critical planes. The fatigue lives calculated as geometric means of the life obtained with the stress and strain criteria are also close to the data from experiments.

INTRODUCTION

Up to now none experimental results have been used for analysis of influence of the cross-correlation between the random stress or strain state components on fatigue life of the materials. The first attempts of theoretical approach to the problem were proposed by Łagoda and Macha (1). There are, however, some results for the plane stress state under cyclic out-of-phase loadings which can be treated as a case where a specific cross-correlation between stresses occurs. These data and the results obtained by the authors under biaxial random tension-compression were used for analysis of influence of cross-correlation between normal stresses on fatigue life.

SIMULATION TESTS

Basing on the data from literature the authors analysed changes of the ratio of amplitudes of non-proportional stresses to the proportional ones for the same fatigue life versus the coefficient of stress cross-correlation. From this analysis it appears that under biaxial cyclic tension-compression the permissible ratio of non-proportional stress amplitudes to those proportional ones decreases as the stress

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cross-correlation coefficient decreases (from 1 to -1). It means that at the constant amplitudes of stresses, σ_{axx} and σ_{ayy}, a change of the stress cross-correlation coefficient from r = 1 (in-phase) to r = -1 (out-of-phase at π) causes drop of the fatigue life. There is one exception -cast iron. For the correlation coefficient r = 0.5 we can observe a little increase of the ratio of amplitudes.

EXPERIMENTS

The cruciform specimens of 10HNAP steel were tested under biaxial stress state. Their shape is shown by Będkowski (2). The central part of each specimen had a spherical notch, its radius of curvature was 150 mm. The specimen thickness in its thinnest part was 1 mm. The following constants of the $\sigma_a - N$ curves were determined under uniaxial cyclic tension - compression

$$\lg N = A - m \lg \sigma_a = 29.69 - 9.82 \lg \sigma_a \qquad (1)$$

$\sigma_{af} = 252.3 \pm 18.7$ MPa, $N_o = 1.28 \times 10^6$ cycles; $A \in (22.39; 36.98)$; $m \in (6.89; 12.75)$.
The tests under uniaxial random loadings with zero expected values were carried on for medium- and long life time. Random histories of loadings with non-Gaussian probability distribution and wide-band frequency spectra were generated with the matrix method. It was found that under the tested loadings the experimental life could be efficiently estimated with the rain flow algorithm and Palmgren-Miner hypothesis, taking into account amplitudes less than the fatigue limit by a half, i.e. for $\sigma_{ai} \geq 0.5\sigma_{af}$. Let us substitute the histogram of amplitudes by the weighed average stress amplitude

$$\sigma_{aw} = \left(\frac{1}{N_b} \sum_{i=1}^{k} n_i \sigma_{ai}^{m} \right)^{1/m} \qquad (2)$$

Thus we can determine constants of the fatigue curve $\sigma_{aw} - N$ under uniaxial random tension-compression

$$\lg N = A_r - m_r \lg \sigma_{aw} = 27.51 - 8.99 \lg \sigma_{aw} \qquad (3)$$

$A_r \in (21.34; 33.69)$; $m_r \in (6.46; 11.52)$.
It can be seen that parameters A, m of the cyclic curve (1) and A_r, m_r of the random curve (3) are close together respectively. We can say the same about their confidence intervals, determined at probability of 0.95.
Fatigue tests of the cruciform specimens under biaxial stress state were done for the loading cross-correlation coefficients close to $r_{F_{xx},F_{yy}}$ = -1, 0, +1. The probabilistic

characteristics of loadings, on X-axis and Y-axis were similar to those ones applied during uniaxial tests. The detailed results of the tests are given in reference (2).

THE ALGORITHM OF FATIGUE LIFE CALCULATIONS

The biaxial stress state is reduced to the uniaxial one with a chosen fatigue criterion. Next, cycles and half-cycles are counted from the history of the equivalent stress, $\sigma_{eq}(t)$, with use of the rain-flow algorithm and damages are cumulated according to Palmgren-Miner hypothesis. The reduction takes place in the critical plane determined with the variance method by Będkowski and Macha (3).

In the generalized criterion of multiaxial random fatigue it is assumed that

1. Fatigue occurs under the influence of a combination of the normal stress, $\sigma_\eta(t)$ and the shear stress, $\tau_{\eta s}(t)$ in $\bar{s}$ direction in the fracture plane with the normal $\bar{\eta}$,
2. Direction $\bar{s}$ coincides with the mean direction of maximum shear stress, $\tau_{\eta s\,max}$,
3. The maximum value of linear combination of stresses $\tau_{\eta s}(t)$ and $\sigma_\eta(t)$ under multiaxial random loadings satisfies the following equation (see reference (1))

$$\max_t \{ B\, \tau_{\eta s}(t) + K\, \sigma_\eta(t) \} = F \tag{4}$$

In specific cases for the biaxial stress state we obtain criteria of

-the maximum normal stress in the fracture plane (B = 0, K = 1)

CI - $$\sigma_{eqCI}(t) = [\hat{l}_\eta^2 \sigma_{xx}(t) + \hat{m}_\eta^2 \sigma_{yy}(t)] \tag{5}$$

-the maximum strain in the fracture plane (B = 0, K = 1)

CII - $$\sigma_{eqCII}(t) = [\hat{l}_\eta^2 (1+\upsilon) - \upsilon]\sigma_{xx}(t) + [\hat{m}_\eta^2 (1+\upsilon) - \upsilon]\sigma_{yy}(t)] \tag{6}$$

-the maximum shear stress in the fracture plane (B = 2, K = 0)

CIII - $$\sigma_{eqCIII}(t) = 2[\hat{l}_\eta \hat{l}_s \sigma_{xx}(t) + \hat{m}_\eta \hat{m}_s \sigma_{yy}(t)] \tag{7}$$

-the maximum shear and normal stresses in the fracture plane (B = 2, K ≠ 1)*

CIV - $$\sigma_{eqCIV} = (1+K)^{-1}\left\{\left[K\hat{l}_\eta^2 + 2\hat{l}_\eta \hat{l}_s\right]\sigma_{xx}(t) + \left[K\hat{m}_\eta^2 + 2\hat{m}_\eta \hat{m}_s\right]\sigma_{yy}(t)\right\} \tag{8}$$

In the plane stress state a position of the vector normal to the fracture plane can be expressed with one angle α in relation to 0x-axis. Hence, the direction cosines of the stress axis are

$$\hat{l}_\eta = \cos\alpha, \qquad \hat{m}_\eta = \sin\alpha, \qquad \hat{l}_s = -\sin\alpha, \qquad \hat{m}_s = \cos\alpha \tag{9}$$

THE ANALYSIS OF TEST RESULTS

The fatigue life was calculated according to criteria (CI÷CIV). In criterion IV constant K assumed various numbers. Unfortunately, none of the analysed criteria gave satisfactory results. The results obtained according to CII seem to be the relatively best. Thus, the authors tried to modify the considered criteria (5)-(8). The first favourable results were obtained while modifying criterion CI in the following way

CIM -
$$\sigma_{eqCIM}(t) = \left(0.5\sigma_{af}/\tau_{af}\right)^{r/2}\sigma_{eqCI}(t) \tag{10}$$

The modification consists in multiplication of instantaneous values of the equivalent stress, $\sigma_{eqCI}(t)$, by the constant depending on the ratio of the fatigue limits under tension-compression, σ_{af} and torsion, τ_{af} and the correlation coefficient of normal stresses, r. A greater part of the results is included in the scatter band of the coefficient 3 (the scatter was greater only in two specimens with the loading correlation -1). From the analysis of Eq.(10) it results that under constant variances of normal stresses decrease of the cross-correlation coefficient, r, causes increase of variance of the modified equivalent stress ($0.5\ \sigma_{af}\ /\ \tau_{af} < 1$) and it means decrease of the fatigue life. In the case of criterion CIV the modification consists in assumption of the following equation expressing the equivalent stress

CIVM -
$$\sigma_{eqCIVM}(t) = K\sigma_\eta(t) + B\sigma_{\eta s}(t) \tag{11}$$

K=0.8490 and B=0.7752 were assumed from experiments. A good agreement with the experimental results was obtained in the scatter band of the factor 3 (a greater part of the results was included in the scatter band of the factor 2); only two specimens were exceptions. In all the cases we assumed τ_{af} = 182.2MPa.

If the stress correlation is negative, the fatigue lives calculated according to the criterion of maximum normal stresses are grater and for the criterion of normal strains are less than the experimental ones. If the stress correlation is positive, the opposite situation can be observed. Thus, the geometric means of the lives were calculated according to the both criteria

Cπ -
$$T_{cal\pi} = \sqrt{T_{calCI} \cdot T_{calCII}} \tag{12}$$

Equation (12) gave the fatigue lives close to those obtained according to criterion (11). It can be seen that the fatigue life is influenced by both stresses and strains in a given direction. The results obtained in such a way (Cπ) are only a little worse than those obtained with use of criterion CIVM. In the case of calculations of the

arithmetic means, the scatters of calculation results related to the experimental results are greater than in the case of the geometric means.
In Figure 1 the experimental fatigue lives are compared with the lives calculated according to 4 considered criteria of multiaxial random fatigue. The fatigue lives calculated with the modified criterion of maximum normal stresses in the critical plane (CIM), the mean geometric life (Cπ) and the modified criterion of maximum shear and normal stresses in the fracture plane with the weight participation of the stresses (CIVM) are close to the experimental lives.
From Figure 1 it results that the proposed fatigue model is efficient. The scatter of results is included in the band with the factor of 3.

CONCLUSIONS

1.From the data obtained while the tests under cyclic out-of-phase loadings it appears that in many materials under biaxial tension-compression the permissible amplitude of stress decreases as the cross-correlation coefficient of stresses also decreases.
2.The fatigue life of 10HNAP steel under biaxial random tension-compression strongly decreases when the cross-correlation coefficient of normal stresses decreases from +1 to -1.
3.The estimated fatigue life of 10HNAP steel under the considered loadings is the closest to the experimental one when the modified criterion of maximum shear and normal stresses in the fracture plane is applied.

SYMBOL USED

$\hat{l}_\eta, \hat{m}_\eta, \hat{n}_\eta, \hat{l}_s, \hat{m}_s, \hat{n}_s$ = direction cosines of vectors $\hat{\eta}$ and $\hat{s}$
n_i = number of cycles at the stress amplitude σ_{ai}

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$- \rightarrow r_{F_{xx},F_{yy}} \approx -1$; $O \rightarrow r_{F_{xx},F_{yy}} \approx 0$; $+ \rightarrow r_{F_{xx},F_{yy}} \approx +1$

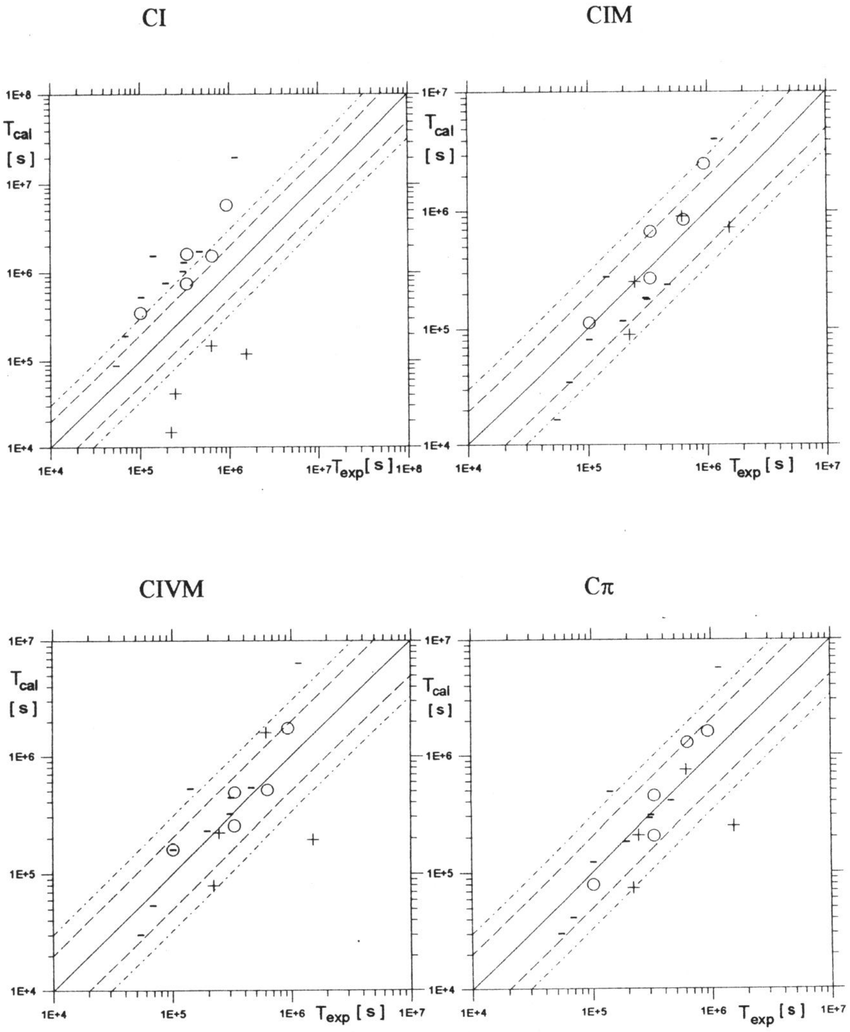

Figure 1 Comparison of the calculated and experiments lifetime for the cross-correlation coefficient of loadings