GENERALIZED PATH INDEPENDENT INTEGRAL IN LINEAR FRACTURE MECHANICS

Ph. Bressolette*, Ch. Petit†, A. Vergne*

In linear fracture mechanics, the bilinear integral A allows direct computation of stress intensity factors K_I and K_{II} with mechanical and thermal loadings. This paper deals with a generalization of this integral to overcome the original limitation on the thermal flux and to take into account pressures on the crack lips. These developments are illustrated through two numerical mixed mode examples with reference solutions. The values obtained with this approach are compared to results of classical methods and prove the accuracy and stability of this method.

INTRODUCTION

The prediction of the behaviour of cracked structures under various kinds of loadings is an important problem for their safety. A lot of methods have been developed to determine the cracking parameters of such structures (Petit (1)). Among many global methods, one of the most used is the path independent integral G_θ proposed by Suo and Combescure (2) :

$$G_\theta = \int_\Omega \left(-w\theta_{k,k} + \sigma^u_{ij} u_{i,k}\theta_{k,j}\right) dV + \int_V \beta u_{i,i}\tau_{,k}\theta_k \, dV \tag{1}$$

with $\beta = \alpha(3\lambda + 2\mu)$, λ and μ the Lame coefficients, α the expansion coefficient, u_i the displacement vector, σ^u_{ij} the stress tensor, w the strain energy density, τ the temperature; θ is an arbitrary vector field, continuous and derivable in the crown Ω (see figure 1).

The main drawback of all global methods lies in the fact that they only compute the energy release rate G but not the stress intensity factors K_I and

* LERMES - Université Blaise Pascal, F-63177 Aubière Cedex

† Laboratoire de Génie Civil - Université de Limoges, F-19300 Égletons

K_{II}. To obtain these values, it is possible to separate the fields into symmetric and antisymmetric parts (Bui *and al.* (3)), to compute G_I and G_{II} and then the corresponding values of K_I and K_{II}. Nevertheless, this step requires a symmetric mesh around the crack tip and is very time consuming. In order to obtain directly K_I and K_{II}, Bui *and al.* (3) have developed the bilinear integral A which is theoretically independent of the integration domain :

$$A = \int_\Omega \Big[-\frac{1}{2}(\sigma^v_{ij,1} u_i - \sigma^u_{ij} v_{i,1}) + \gamma\tau(v_{1,j} - \psi_{1,j}) - \gamma\tau_{,j}(v_1 - \psi_1) \Big] \theta_{1,j}\, dV \quad (2)$$

where $\gamma = -\beta\mu/2(\lambda+\mu)$, v_i and σ^v_{ij} are auxiliary elastic fields (Irwin solution) and ψ a biharmonic auxiliary scalar field. This integral can be expressed as a bilinear form of stress intensity factors :

$$A = \frac{1}{E'}(K_I K_I^* + K_{II} K_{II}^*) \quad (3)$$

with $E' = E$ in plane stress and $E' = E/(1-\nu^2)$ in plane strain. In plane strain, the proposed expression of the ψ_1 field is :

$$\psi_1 = \frac{2K_I^v}{\mu}\left(\frac{r}{2\pi}\right)^{1/2}(1-\nu)\cos\frac{\theta}{2} \quad (4)$$

and the following crack tip conditions must be respected :

$$\sigma^u_{ij} n_j = \sigma^v_{ij} n_j = 0 \text{ (hyp1)} \; ; \; v_{k,j} n_j - \psi_{k,j} n_j = 0 \text{ (hyp2)} \; ; \; \tau_{,j} n_j = 0 \text{ (hyp3)}$$

THEORETICAL DEVELOPMENTS

In order to enlarge the application field of integral A to any kind of thermal loadings, Bressolette *and all* (4) propose a generalized vector field $\vec{\psi} = (\psi_1\ ,\psi_2)$:

$$\psi_1 = \frac{K_I^v}{2\mu}\left(\frac{r}{2\pi}\right)^{1/2}(k+1)\cos\frac{\alpha}{2} + \frac{K_{II}^v}{2\mu}\left(\frac{r}{2\pi}\right)^{1/2}(k+1)\sin\frac{\alpha}{2} \quad (5)$$

$$\psi_2 = \frac{K_I^v}{2\mu}\left(\frac{r}{2\pi}\right)^{1/2}(k+1)\sin\frac{\alpha}{2} - \frac{K_{II}^v}{2\mu}\left(\frac{r}{2\pi}\right)^{1/2}(k-3)\cos\frac{\alpha}{2} \quad (6)$$

This expression can be used in plane strain ($k = 3-4\nu$) and plane stress ($k = (3-\nu)/(1+\nu)$). Furthermore, to take into account pressure vector field $\vec{P}$ along the crack lips (L), we propose the generalized A integral as follow :

$$A = \int_\Omega \Big[-\frac{1}{2}(\sigma^v_{ij,k} u_i - \sigma^u_{ij} v_{i,k}) + \gamma\tau(v_{k,j} - \psi_{k,j}) - \gamma\tau_{,j}(v_k - \psi_k) \Big] \theta_{k,j}\, dV - \frac{1}{2}\int_L P_i\, v_{i,j}\, \theta_j\, dx_1 \quad (7)$$

NUMERICAL RESULTS

In order to assess the presented method, a comparative analysis is proposed on two mixed mode cracks with reference solutions. We use the finite element code Castem2000 for all the computations presented here and all the meshes use quadratic isoparametric elements with radiating mesh around the crack tip in order to produce regular crowns. For these examples the results obtained with the A-integral are compared for the 5 first crowns (called C_i, $i = 1,5$) with the results obtained with the G_θ-method.

First example

This application concerns a plate with inclined crack under uniform tensile loading (Kitagawa and Yuuki (5)). The computation is made with the hypothesis of plane stress. The numerical data are the following (see Figure 2) :

- Geometry : $2l = 100\ mm, 2w = 50\ mm, 2a = 20\ mm\ (a/w = 0.4)$
- Material properties and loading : $E = 2\ 10^4\ MPa, \nu = 0.3$, $\sigma = 1\ MPa$
- Reference solution : $K_I = 4.74\ MPa.mm^{1/2}$, $K_{II} = 2.52\ MPa.mm^{1/2}$

TABLE 1 – Plate with inclined crack (K in $MPa.mm^{1/2}$, G in $MPa.mm$)

		C_1	C_2	C_3	C_4	C_5
A	G	$1.44\,10^{-3}$	$1.44\,10^{-3}$	$1.44\,10^{-3}$	$1.44\,10^{-3}$	$1.44\,10^{-3}$
	K_I	4.74	4.74	4.74	4.74	4.74
	K_{II}	2.53	2.52	2.52	2.52	2.52
G_θ	G	$1.44\,10^{-3}$	$1.44\,10^{-3}$	$1.44\,10^{-3}$	$1.44\,10^{-3}$	$1.44\,10^{-3}$
	K_I	4.73	4.74	4.74	4.74	4.74
	K_{II}	2.52	2.52	2.52	2.52	2.52

The results obtained with A-integral and G_θ-method are presented in table 1 (in order to obtain the values of K_I and K_{II} with the G_θ-method, the displacement field is separated into symmetric and antisymmetric parts). These values can be compared with those obtained using the classical crack opening displacement method (COD) : $K_I = 4.61, K_{II} = 2.45\ (MPa.mm^{1/2})$. From table 1, we can conclude that both A-integral and G_θ-method give very accurate results (specially when compared with COD). Furthermore, the values are independent of the integration crown.

Second example

The second example is a plate with horizontal crack submitted to uniform pressures on the crack lips (Rooke and Cartwright (6)). Plane strain assumption is considered. In order to apply accurately the pressure on the crack lips, the mesh is refined along the crack. The numerical data are (see Figure 3) :

- Geometry : $2b = 1600\ mm, 2h = 1400\ mm, 2a = 200\ mm$
- Material properties, loading : $E = 2\,10^4\ MPa, \nu = 0.3, P_x = 1, P_y = 2\,(MPa)$
- Reference solution : $K_I = 35.45\ MPa.mm^{1/2}$ $, K_{II} = 17.72\ MPa.mm^{1/2}$

TABLE 2 – Plate with horizontal crack (K in $MPa.mm^{1/2}$, G in $MPa.mm$)

		C_1	C_2	C_3	C_4	C_5
A	G	$7.14\,10^{-2}$	$7.12\,10^{-2}$	$7.12\,10^{-2}$	$7.12\,10^{-2}$	$7.12\,10^{-2}$
	K_I	35.51	35.51	35.50	35.50	35.50
	K_{II}	17.53	17.46	17.45	17.45	17.45
G_θ	G	$7.09\,10^{-2}$	$7.11\,10^{-2}$	$7.11\,10^{-2}$	$7.11\,10^{-2}$	$7.11\,10^{-2}$
	K_I	35.25	35.31	35.32	35.32	35.32
	K_{II}	14.60	15.00	15.00	15.00	15.00

The results obtained with the two integrals are listed in table 2. With COD method, we obtain $K_I = 36.18, K_{II} = 17.87$ $(MPa.mm^{1/2})$. As previously, in order to obtain the values of K_I and K_{II} with the G_θ-method, the fields are separated into symmetric and antisymmetric parts. From table 2, A-integral give results very close to the reference values, which is not the case for the G_θ-method in particular for K_{II}. This discrepancy is due to the separation of the fields because the value of G is accurate. Nevertheless, both integral methods give results independent of the integration crown.

CONCLUSION

Because of its bilinear form, the A-integral enable to compute directly the stress intensity factors in case of mixed mode of fracture. In this paper, a generalization in order to take into account any thermal loading and pressure on the crack lips is proposed. From the examples presented (and many other (4)), the new formulation proves to be accurate and stable. We try currently to extend this method to the axisymmetric case. In this case, the main difficulty lies in the fact that there is no analytical solution for the auxiliary fields.

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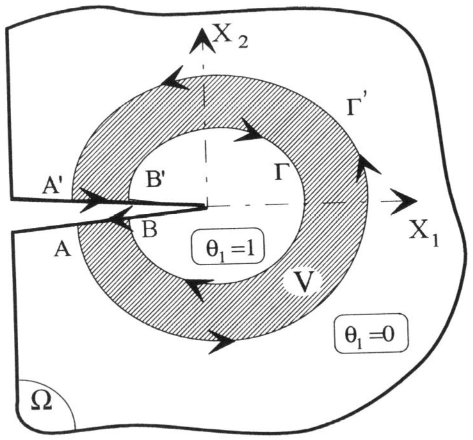
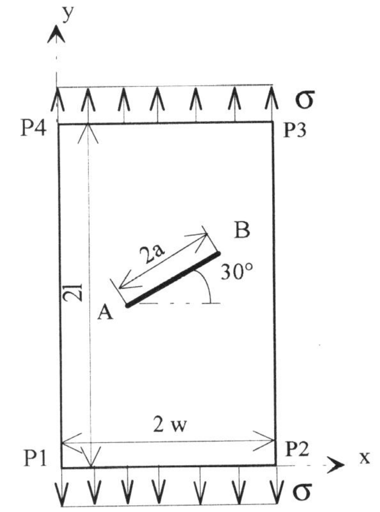


Figure 1 Domain and integration crown



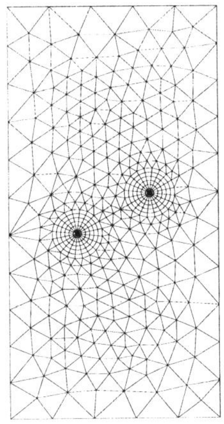
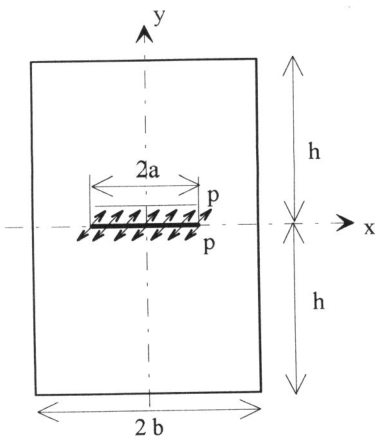


Figure 2 Plate with inclined crack : model and mesh



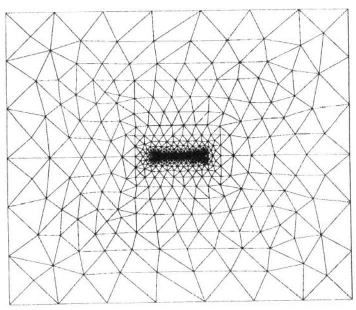


Figure 3 Plate with horizontal crack : model and mesh