FRACTURE-INDUCED SCALE EFFECT ON THE MODULUS OF RUPTURE IN QUASI-BRITTLE MATERIALS

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This paper analyzes the approaches of various authors to the size effect on the rupture modulus and compares the predictions of the models to each other and to some of the available experimental results. Closed form analytical expressions are given for the modulus of rupture predicted by Hillerborg's cohesive crack model, the Bazant-Li boundary layer model and the Jenq-Shah two parameter model; the multifractal scaling law of Carpinteri, Chiaia and Ferro is also analyzed.

1 INTRODUCTION

It is widely accepted that the modulus of rupture of brittle or quasi-brittle materials such as concrete, rocks and ceramics is size dependent when measured for beams in either three or four point bending (Fig.1) (Reagel (1), Wright (2), Nielsen (3), Lindner (4), Walker (5), Petersson (6), Alexander (7) and Elices (8)).

This paper reviews various deterministic theories developed to explain the size effect on the modulus of rupture, all based on more or less simplified nonlinear fracture models. We start by describing the cohesive crack model. Next, we summarize the predictions of the rupture modulus by the boundary layer model of Bazant and Li (9), the two parameter model of Jenq and Shah (10) and the recent multifractal scaling law of Carpinteri Chiaia and Ferro (11). The paper closes by comparing the various approaches with some experimental results.

2 THE COHESIVE CRACK MODEL

Fracture of quasi-brittle materials can be conveniently described by the cohesive or fictitious crack model introduced by Hillerborg and co-workers (12). A review of the relevant properties of this model was made by the authors in some recent papers (13, 14). In its simplest formulation, the model assumes that a crack initiates where the maximum principal stress reaches the tensile strength f_t.

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The crack opens transferring stress from one face to another and satisfying, for pure mode I, a material function called the softening curve $\sigma = f(w)$ (Fig.2).

The authors have shown that the softening curve influences the modulus of rupture only through its initial part, that can, in most cases, be approximated by a straight line, as depicted in Fig. 2 (15). This is because the peak load occurs before any point on the cohesive zone softens very much, and thus only two parameters of the softening curve are relevant: the tensile strength f_t and the horizontal intercept of the initial tangent w_1 (Fig. 2). Then, from the basic equations governing the crack growth, a closed form expression for the modulus of rupture can be sought:

$$\frac{f_r}{f_t} = \beta + \frac{3-\beta+99D^*}{(1+2.44D^*)(1+87D^*)} \quad \text{with} \quad D^* = \frac{D}{l_1} \quad \text{and} \quad l_1 = \frac{Ew_1}{2f_t} \qquad (1)$$

E being the elastic modulus. The factor β stands for the slight difference between four and three point bending due to the different stress distribution along the central cross-section. According to Timoshenko(16), β is equal to $(1-0.1773D/s)^{-1}$ for three point bending and $\beta = 1$ for four point bending. Equation (11) satisfies the two asymptotic limits: $f_r \rightarrow 3\, f_t$ for $D \rightarrow 0$ (plastic limit solution), and $f_r = \beta\, f_t$ for $D \rightarrow \infty$ (elastic-brittle behavior) and is consistent with other proposals for medium and large sizes (Guftasson (17)).

3 MODULUS OF RUPTURE ACCORDING TO THE BAZANT-LI MODEL

The approximation of Bazant and Li (9) is based on the assumption that prior to the peak load, the cracking in concrete is distributed rather than localized. The peak load is assumed to occur when the greatest depth of the microcracked zone reaches a certain critical value l_f. Bazant and Li propose the following size effect for relatively large sizes ($D >> l_f$):

$$\frac{f_r}{f_t} = \beta\left(1 + 2\,\frac{l_f}{D}\right) \qquad (2)$$

where we have introduced the factor β to take into account the effect of the concentrated load.

Fig.3 compares the predictions of the Bazant-Li and the cohesive crack model for three point bending beams with $s/D=4$ ($\beta=1.046$). The models have been adjusted for identical asymptotic behavior setting $l_f = 4.5\, l_1$.

4 MODULUS OF RUPTURE PREDICTED BY JENQ-SHAH MODEL

The Jenq-Shah model (10) assumes that starting from a preexisting crack, which may be taken to be vanishingly small, a macro crack grows until the peak load is reached, at which moment both the stress intensity factor K_I and the crack tip opening displacement w_T reach their critical values K_{Ic} and w_{Tc}. To determine the peak load and thus the rupture modulus for any given size we make use of the LEFM expressions for K_I and w_T for the three or four point bending geometry particularized for the peak load condition and assuming that the two material parameters K_{Ic} and w_{Tc} have been determined by other experiments. An analytical expression for three point bending beams with s/D=4 has been fitted by the authors to describe the Jenq-Shah prediction for sizes $D \geq 0.15\ l_0$. The expression is as follows:

$$\frac{f_r}{f_0} = 1.049\left(1+\frac{6.1D/l_0}{(1+6.1D/l_0)(1+5.3D/l_0)}\right) \tag{3}$$

with $f_0 = 1.5\,\frac{K_{Ic}^2}{E\,w_{Tc}}$ and $l_0 = \frac{E^2\,w_{Tc}{}^2}{K_{Ic}{}^2}$.

This formula is compared with the cohesive model in Fig. 3, forcing again equal asymptotic behavior ($f_0 = 0.997f_t \approx f_t$ and $l_0 = 2.3\ l_1$).

5 CARPINTERI'S MULTIFRACTAL SCALING LAW

Recently, a scaling law for strength based on the consideration of the fractal nature of the fracture process has been put forward by Carpinteri, Chiaia and Ferro (11). The multifractal scaling law can be written in the following way:

$$\frac{f_r}{f_t} = \beta\sqrt{1+\frac{l_M}{D}} \tag{4}$$

where f_t is the tensile strength in the macroscopic limit (large size) and l_M is a constant length characteristic of the material and of the geometry. We introduce the factor β to provide consistency with the other theories.

In order to compare with the other theories, we again make the asymptotic expressions coincide ($l_M = 0.87\ l_1$). The result is shown in Fig. 3. The multifractal law lies between the size effect curves deduced from the cohesive model and that corresponding to the Bazant-Li model.

6 COMPARISON WITH EXPERIMENTS AND FINAL REMARKS

In the foregoing analysis, the various models for size effect on the modulus of rupture were analyzed, assuming that for large sizes they must predict identical behavior. This is possible because all models display the same asymptotic structure.

However, in practice the asymptotic limit is never reached and what is usually sought is the value of the parameters of the model based on specimen results. Then the problems arise about the ability of the models to describe the results, on one hand, and of the ability of the results to select the best model.

Fig. 4 shows the experimental results from 9 experimental series and the theoretical curves found by nonlinear correlation. It appears that the Jenq-Shah model is the one experiencing most difficulty in describing the experimental results. The Bazant-Li and Multifractal fittings are essentially coincident for all experimental series. When all circumstances are taken into account it is difficult to conclude that any of the models is clearly superior to the others. The cohesive model has the advantage of being a very general fracture model that can be verified by independent tests, which is not possible for the Bazant-Li and Multifractal models.

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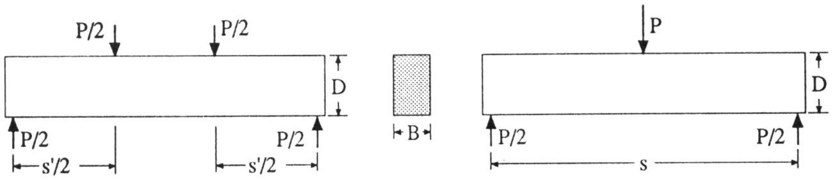

Fig. 1. Four and three-point bending beams.

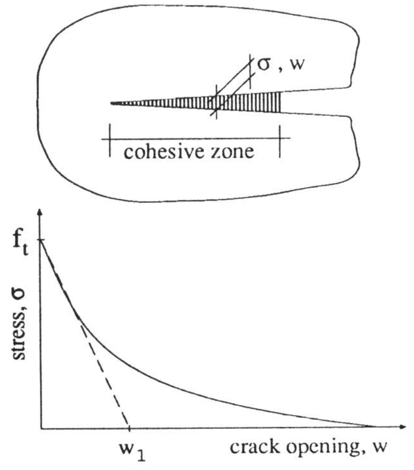

Fig. 2. Cohesive crack model and initial linear approximation

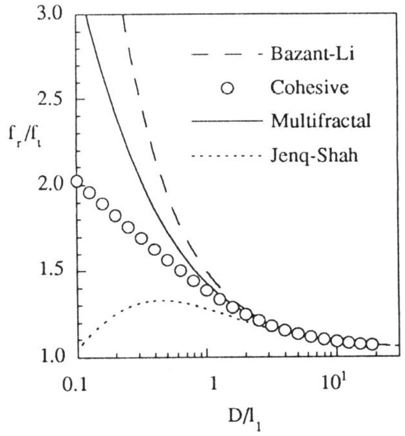

Fig. 3. Comparison of theories for the rupture modulus for identical asymptotic behavior.

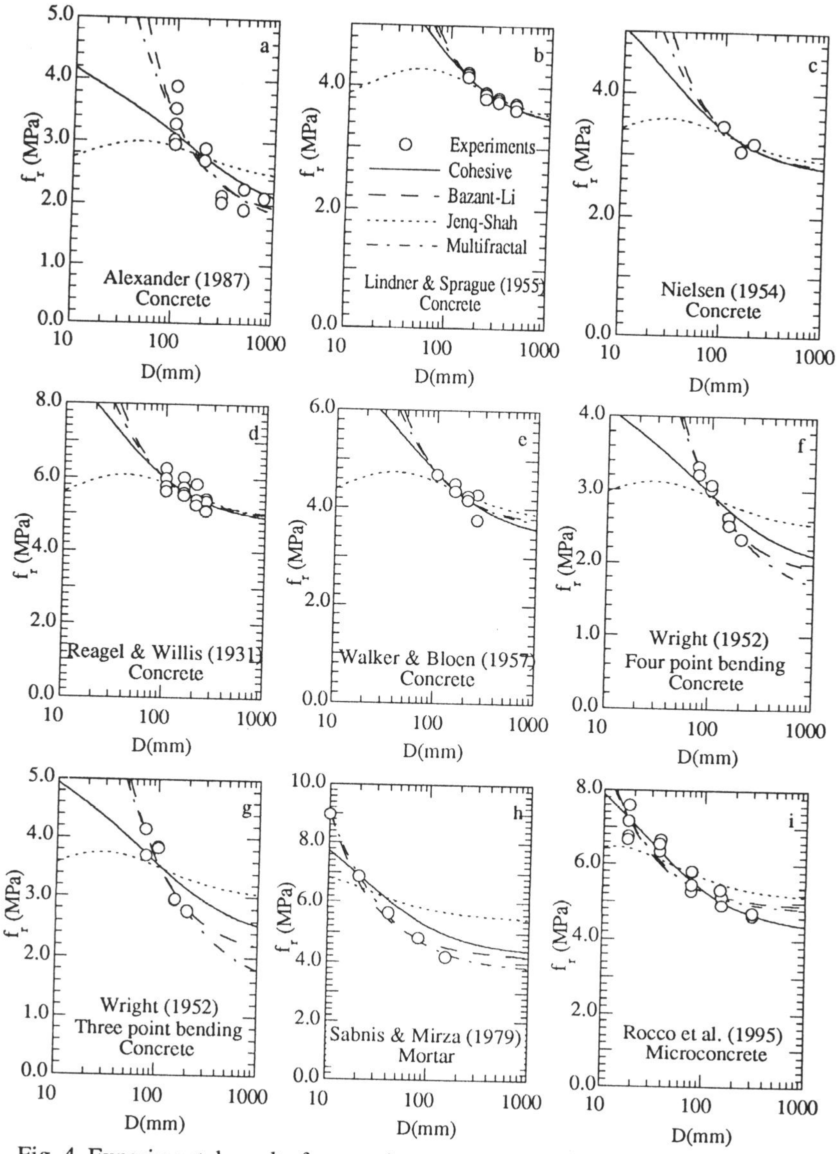

Fig. 4. Experimental results from various sources and best fits for the various models analyzed in the paper.