

FRACTURE OF RUBBERS: EXPERIMENTAL AND NUMERICAL  
DETERMINATION OF THE J INTEGRAL

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Fracture tests on two kinds of rubbers are performed using S.E.N.T specimens and the evolution of the energy parameter J as a function of the remote strain energy density has been obtained. The J integral values issued from a finite element analysis are in good agreement with the experimental data when considering the deeply cracked specimen while a significant divergence is highlighted in the range of  $a/w < 0.5$ . It is attributed to a lack of accuracy in the determination of the experimental calibration factor. Concerning the fracture behaviour of our materials, as observed for many rubbers in the literature, crack initiation is governed by a critical value of J.

INTRODUCTION

Global approaches are of a common use when dealing with fracture of rubber-like materials and the fracture surface energy issued from such an analysis can be considered as an intrinsic property of this kind of material. For a given specimen geometry, this property can be measured using a single specimen (1,2). In a recent work (3), the expression of the energy parameter J proposed by Andrews (2) for cracked panels was re-examined and the influence of finite dimensions was introduced, leading to express J as:

$$J = 2 \cdot k(W_0) \cdot W_0 \cdot a \cdot F(a/w, a/h, \dots) \quad (1)$$

In this expression, k is only function of the uniform strain energy density,  $W_0$ , and F is related to the dimensional characteristics of the specimen. In order to evaluate the different terms of expression (1), Naït Abdelaziz et al (4) have developed an original technique issued from an energy separation procedure.

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Our purpose is to confirm the validity of this method. Experimental results obtained from relation (1) have been compared to those numerically computed using the finite elements method.

The numerical determination of this parameter has been achieved using two techniques: the integral of Rice (5) and the calculation proposed by Watwood (6) which is an energy rate interpretation of J and needs two entire analyses to be performed. The energy parameter J is expressed as:

$$J = \lim_{\Delta a \rightarrow 0} \left( \frac{-\Delta U}{B \cdot \Delta a} \right) \quad (2)$$

### EXPERIMENTAL AND NUMERICAL STUDY

The fracture tests have been performed using the S.E.N.T (Single Edge Notch in Tension) specimen of 200 mm length, 40 mm width with different crack lengths. Two kinds of rubber have been chosen in this investigation:

- an ethylene-propylene-diene (E.P.D.M) of 3.4 mm thickness
- a polybutadiene ( B= 4.5 mm)

The numerical part of this work has been achieved using the finite elements code "ANSYS. V 50". Plane stress conditions and non-linear elastic behaviour of the materials have been assumed. The standard tensile test allows the determination of the materials stress-strain relationships. Both triangular elements with six nodes and quadrilaterals elements with eight nodes have been chosen to perform the specimen meshing (figure 1).

### RESULTS-DISCUSSION

#### Energy parameter J

J has been numerically computed for each crack length value corresponding to those we have checked in the experiments. The numerical load-displacements solutions have been compared with the experimental curves, for each crack length. The results, shown for two particular crack length values in figure 2, are in good agreement.

The J integral has been found path independent provided the contour is sufficiently far from the crack tip.

The calculation proposed by Watwood is in good agreement with the results issued from the direct computation of the J integral (figure 3).

The separation method allows the determination of  $k(W_0)$  and  $F(a/w)$  from the experimental load-displacement records. The experimental parameter J has been then computed using expression (1). A nice concordance has been pointed out

between the experimental and numerical J values when considering the deeply cracked specimen ( $a/w \geq 0.5$ ), while the results diverge below this limit (figure 4).

k(W<sub>0</sub>) and F(a/w) data

In order to find the fount of the above-mentioned divergence, the potential function k(W<sub>0</sub>) and the calibration factor F(a/w) have been determined using the J integral numerical values and compared to those issued from the experimental results. The separation technique has been adjusted to the J values since a multiplicative form of this parameter is expected (relation 1). Thus, considering a crack length reference a<sub>i</sub>, the separation parameter S<sub>ij</sub> defined as:

$$S_{ij}|_{W_0} = \frac{J(a_i)}{J(a_j)} = \frac{a_i \cdot F\left(\frac{a_i}{w}\right)}{a_j \cdot F\left(\frac{a_j}{w}\right)} \quad (3)$$

is a constant for a given crack length a<sub>i</sub>, as shown for a particular value of the reference in figure 5. As the same trends have been observed for all references, the multiplicative form of J is therefore valid and k and F can be determined as follows:

$$F\left(\frac{a_i}{w}\right) = S_{ij} \cdot \frac{a_j}{a_i} \cdot F\left(\frac{a_j}{w}\right) \quad (4)$$

$$k(W_0) = \frac{J(a)}{2 \cdot W_0 \cdot a \cdot F\left(\frac{a}{w}\right)} \quad (5)$$

For each material, a good agreement has been highlighted between the experimental and numerical data when considering k(W<sub>0</sub>) (figure 6). But divergent results in terms of the calibration factor have been obtained (figure 7) in the range of  $a/w < 0.5$ . This later point confirm that the above-mentioned divergence between numerical and experimental J values has to be attributed to this calibration factor.

Fracture surface energy

The critical value of J corresponding to crack initiation is plotted as a function of the crack length in figure 8. A constant fracture surface energy is obtained when considering the values issued from the finite elements analysis. The results confirms that the crack initiation is here governed by a critical value of J. A good agreement between experimental and numerical results is once again highlighted in the range

of  $a/w \geq 0.5$  while the values issued from the experimental procedure are over-estimated below this limit. It indicates that applying the separation procedure and compute  $J$  from experimental results is only valid for deeply cracked specimen.

*N.B: Although the most figures concerns E.P.D.M. rubber, the P.B rubber exhibits the same trends.*

#### SYMBOLS USED

a	= crack length (mm)
B	= specimen thickness (mm)
w	= specimen width (mm)
$W_0$	= strain energy density remote from the crack ( $\text{kJ/m}^3$ )
$k(W_0)$	= potential function
$F(a/w)$	= calibration factor
J	= energy parameter or path independent integral ( $\text{kJ/m}^2$ )
U	= potential energy (kJ)

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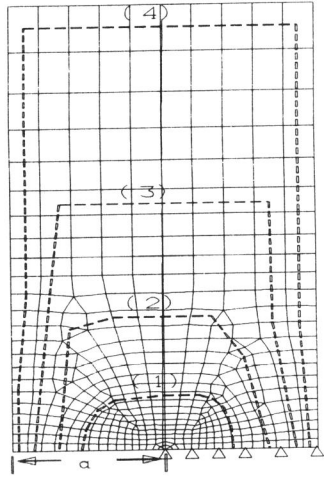


Fig. 1: Meshing of the SENT specimen

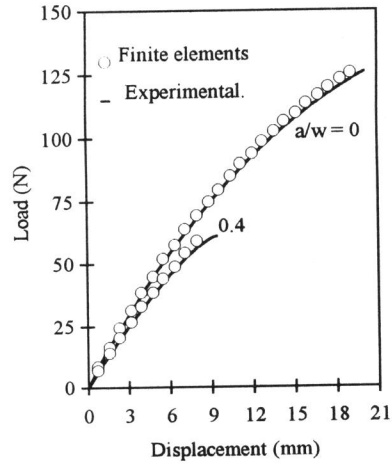


Fig. 2: Load versus displacement (EPDM)

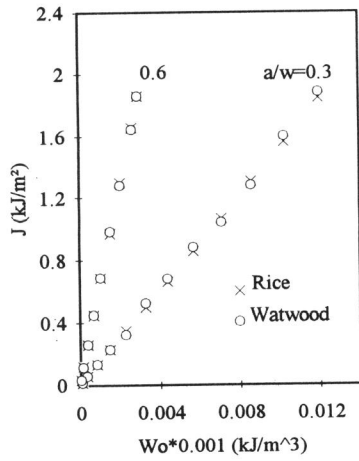


Fig. 3: Numerical J integral as a function of the strain energy density (EPDM)

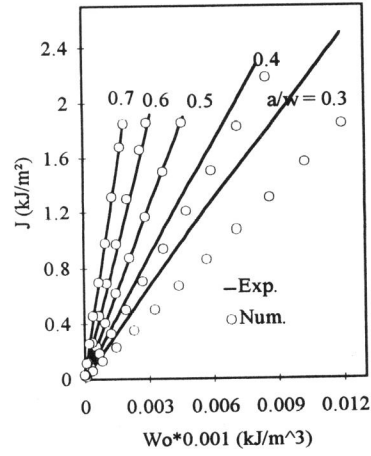


Fig. 4: experimental and numerical J versus  $W_0$  (EPDM)

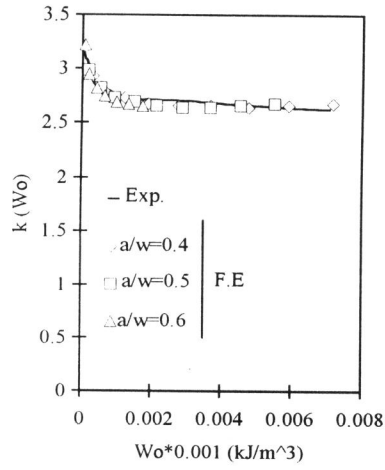
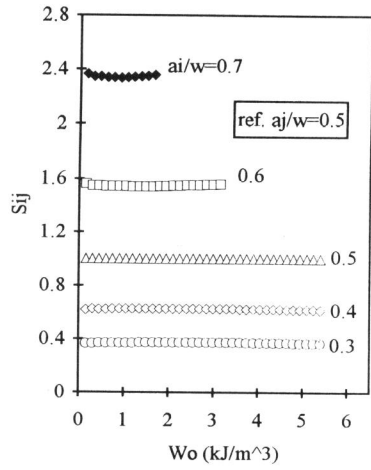


Fig. 5: Separation parameter  $S_{ij}$  versus  $W_0$  (EPDM)

Fig. 6: Experimental and numerical function  $k(W_0)$  (EPDM)

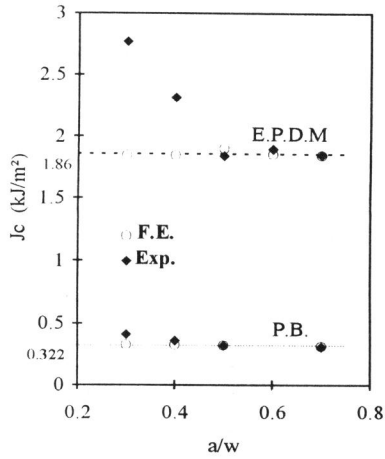
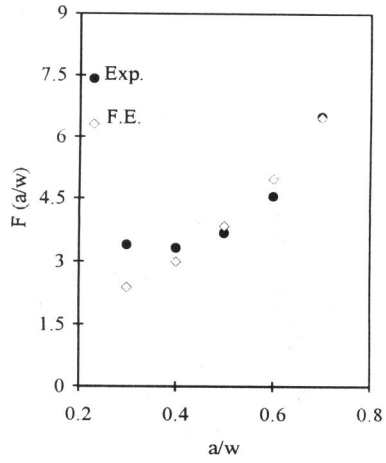


Fig. 7: Calibration factor  $F(a/w)$  (EPDM)

Fig. 8: Fracture surface energy  $J_c$  versus crack length