FRACTURE OF PMMA DUE TO V-NOTCHES

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Brittle fracture of polymethylmethacrylate due to V notches is examined in this work. A fracture criterion is proposed to explain and predict this kind of failure under mode I loading. The criterion is based on a combination of the physical mechanisms that produces the fracture of PMMA and the basic principles of LEFM. Theoretical predictions and experimental results corresponding to fracture of V-notched bend specimens of PMMA are compared and show good agreement.

INTRODUCTION

As cracks, V notches in linear elastic bodies produce stress singularities at the notch tip and this favours brittle failure. For mode I loading, Williams (1) has shown that the singular term of the stress and strain fields has the same functional form as that of a crack, but the order of the singularity depends on the notch angle. As a consequence, the generalization of the LEFM criterion based on the stress intensity factor would lead to a critical value of the V notch stress intensity factor which would be a function of the notch angle rather than a material constant. A more general criterion would be desirable, but now the physical mechanisms of fracture should be taken into account explicitly since the logical argument incorporating them through fracture toughness fails.

In the case of polymers, and specifically of polymethylmethacrylate (PMMA), reported research work indicates that fracture is preceded by crazing (2), so a crazed zone would appear ahead of the V notch tip. This zone behaves as a crack able to transmit normal forces between its two faces due to the fibrils of material that join them. According to this, a crazed zone has been modelled as a crack whose faces attract each other with a constant stress which is a material property (2).

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This paper examines theoretically and experimentally the brittle failure of PMMA bend specimens with V notches under mode I loading. The crazing phenomenon is modelled as aforementioned and incorporated into a fracture criterion which links the basic approach of LEFM and the stress concentration due to a V notch.

FRACTURE CRITERION

The crazed zone ahead of the notch edge is modelled as a crack subjected to the constant stress σ_c shown in Figure 1. By virtue of the superposition principle, the stress intensity factor K_I^e produced by the external loads is reduced by these internal forces by an amount K_I^i, so that the resultant stress intensity factor K_I at the crack tip is the difference $K_I^e - K_I^i$. According to LEFM, brittle failure will occur if K_I attains the fracture toughness of the material, K_{IC}, but the application of this criterion requires the crack size a and the expressions of K_I^e and K_I^i to be known. Since the crack represents the effect of crazing, its size must be determined from an evolution law relating crazing extension to loading and geometry, but no such law is available, so it has been substituted for the purpose of the criterion by a limit condition: the crack is assumed to have the size that maximizes the stress intensity factor K_I. Then the formulation of the criterion is as follows:

$$\underset{a}{\mathrm{Max}}\{K_I\} = K_{IC} \qquad \text{namely} \qquad \begin{cases} K_I = K_{IC} \\ \dfrac{\partial K_I}{\partial a} = 0 \end{cases} \tag{1}$$

For small cracks such as must be expected to model crazing, Gallagher (4) determined the stress intensity factor K_I^e as a function of the parameters characterizing the singular term of Williams' solution for the stress field produced by the V notch in the absence of a crack. As an intermediate result, Gallagher found the stress intensity factor for the following stress distribution acting on the crack faces:

$$\sigma = Ax^p \tag{2}$$

where x is the distance to the edge notch and A and p are constants. The solution according to Gallagher is:

$$K_I = \frac{A\sqrt{2}a^{p+1/2}}{p+1}\exp\left\{\frac{p+1}{\pi}\int_0^\infty \frac{1}{x^2+(p+1)^2}\ln\frac{x^2\sin(2\alpha)+x\sinh(2x\alpha)}{-2x^2\sin^2\alpha-1+\cosh(2x\alpha)}dx\right\} \tag{3}$$

Particularization of this equation for $p = 0$ and $A = \sigma_c$ provides K_I^i and particularization of the values C and $\lambda - 1$ corresponding to the singular term of William's solution provides K_I^e. The exponent $\lambda - 1$ is an implicit function of the notch angle $\pi - \alpha$ defined in Figure 1, since:

$$\lambda \sin(2\alpha) = -\sin(2\lambda\alpha) \tag{4}$$

whereas the factor C is a different function of loads and geometry for each particular notch problem, analogous to the stress intensity factor of a crack problem. Then, particularization of Eq (3) as indicated yields:

$$K_I = K_I^e - K_I^i = Ca^{\lambda-1/2} f_e(\alpha) - \sigma_c a^{1/2} f_i(\alpha) \tag{5}$$

where $f_e(\alpha)$ and $f_i(\alpha)$ are the functions plotted in Figure 2. Substitution of this expression in Equations (1) and subsequent elimination of the crack size between them lead to:

$$\frac{C}{\sigma_c} = \frac{f_i(\alpha)}{(2\lambda-1)f_e(\alpha)} \left[\frac{\sigma_c f_i(\alpha)}{K_{IC}} \frac{2(1-\lambda)}{2\lambda-1} \right]^{2\lambda-2} \tag{6}$$

which is a failure criterion based on two material constants (the fracture toughness K_{IC} and the crazing stress σ_c) and predicts the value at fracture of the V notch stress intensity factor as a function of the notch angle.

EXPERIMENTAL AND NUMERICAL ANALYSIS

To check the fracture criterion given by Eq (6), a numerical and experimental study was performed involving fracture testing and finite element modelling and computing of V–notched bend specimens of PMMA. The mechanical properties of the material are summarized in Table 1 and the shape and dimensions of the specimens are shown in Figure 1.

TABLE 1– Mechanical properties of the PMMA used in the experiments

Elastic modulus	Poisson ratio	Yield stress	Fracture Toughness	Crazing Stress
2.7 GPa	0.4	69 MPa	1.04 ± 0.02 MPam$^{1/2}$	80 MPa *

* According to reference (2)

Eight fracture tests were carried out for each of the following notch angles: 15°, 30°, 60°, 90°, 120° and 150°. The mean value and the standard deviation of the failure load P measured in the tests is given in Table 2 as a function of the notch angle. There is no available expression of the V notch stress intensity factor C for the bending configuration chosen as fracture test, so a finite element modelling and a subsequent linear elastic calculation of each specimen with a different notch angle were required to determinate this factor. A method developed by Carpenter (4) allows the factor C to be calculated by means of a path independent curvilinear integral defined from the displacement and stress fields, since these fields are obtained as a result of the finite element calculation. The expression for C is:

$$C = \frac{G\sqrt{2\pi\lambda}}{h}[1 - \lambda\cos(2\alpha) - \cos(2\alpha\lambda) + \lambda]\int_{\Gamma}(\boldsymbol{\sigma}\vec{n}\cdot\vec{u}* - \boldsymbol{\sigma}*\vec{n}\cdot\vec{u})ds \quad (7)$$

where the integration curve Γ must go from one side to another of the V notch, $\vec{n}$ is the outward normal to Γ, $\boldsymbol{\sigma}$ is the stress tensor field, $\vec{u}$ is the displacement field, G is the shear modulus and $\boldsymbol{\sigma}^*$, $\vec{u}^*$ and h are a tensor field, a vector field and a scalar field dependent on the notch angle and defined explicitly in reference (4). Using this method, the values of the V notch stress intensity factor C at fracture were obtained for each test performed. The average and the standard deviation are given in Table 2 as a function of the notch angle.

TABLE 2– Test results.

Notch angle	P(N)	C/σ_c ($10^3 m^{\lambda-1}$)
15°	193 ± 1%	4.65 ± 0.05
30°	241 ± 2%	5.8 ± 0.1
60°	207 ± 3%	5.50 ± 0.16
90°	317 ± 2%	10.7 ± 0.2
122°	490 ± 5%	29 ± 2
150°	880 ± 2%	132 ± 3

The experimental values of C/σ_c given in Table 2 are plotted in Figure 4 versus the notch angle together with the theoretical ones obtained from Equation (6) and the mechanical properties of Table I. The plot shows good agreement between theory and experiments.

The crack size at fracture is given in Figure 5, where Eq (5) is plotted for the notch angles and the fracture values of the factor C of Table 2. As might be expected from the agreement of Figure 4, the maximum in all the curves hardly differs from the fracture toughness of the material. If these maximums were used for estimating the fracture toughness, they would supply a value of 0.96 ± 0.04 $MPam^{1/2}$, similar to that of Table 1 obtained by testing cracked specimens. As seen in Figure 5, the crack size at the maximums ranges from 0 to 50 µm, increasing with the notch angle. These values are consistent with reported measurements of crazing extension in PMMA (5), which confirms the physical ground of the criterion.

CONCLUDING REMARKS

Brittle fracture of PMMA due to V notches can be explained in the context of LEFM by applying basic principles in conjunction with a simple modelling of the physical mechanisms of damage which precede fracture in this material. For fracture testing of V–notched bending specimens the theoretical predictions agree with the experimental results and provide values of crazing extension consistent with the physical scale of the modelled process.

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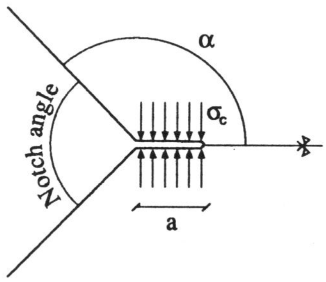

Figure 1 Modelling of crazing ahead of a V notch

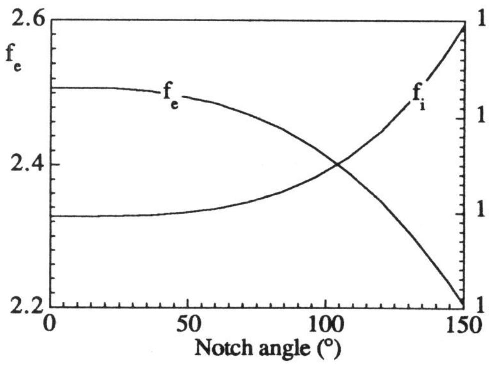

Figure 2 Functions $f_e(\alpha)$ y $f_i(\alpha)$

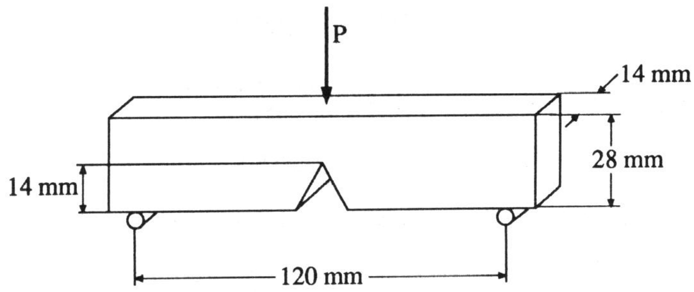

Figure 3 Shape, dimensions and loading of the bend specimens

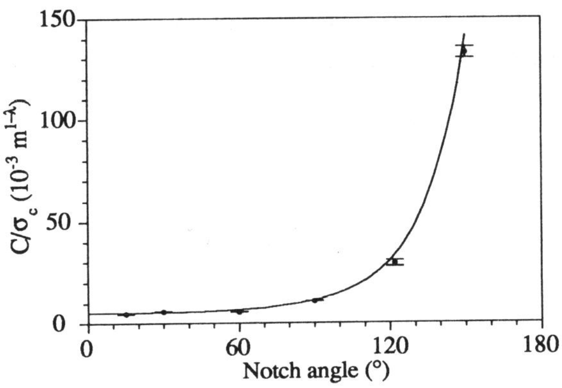

Figure 4 Theoretical predictions and experimental results for fracture of V–notched bend specimens

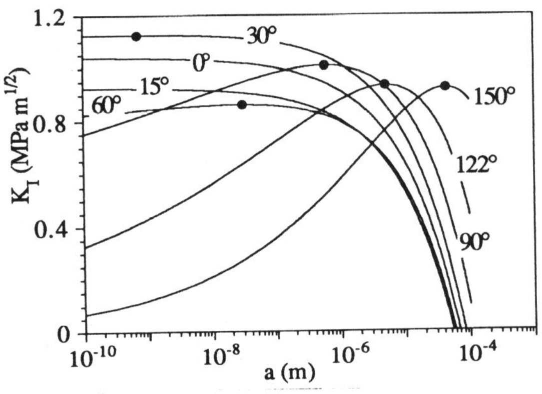

Figure 5 Crazing extension at fracture