FRACTURE OF A SEMI-INFINITE MEDIUM CONTAINING A MACROCRACK AND MICROCRACKS

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The boundary conditions are formulated and a solution method is elaborated for the macrocrack- microcrack interaction in an elastic semi-infinite medium by taking into account the possible crack closure. The problem is investigated by assuming the frictionless contact on the crack edges. Evaluated are the macrocrack stress intensity factors. It is determined that crack closure phenomenon influences the magnitude of stress intensity factors and should be taken into account.

INTRODUCTION

In the present work the interaction between a main crack and a field of microcracks in an elastic semi-infinite medium is investigated. Attention has been paid to analyze the possible crack edges contact. The problem is analyzed by assuming a plane stress/ strain state of elasticity. Earlier, the similar problem for macro-microcracks system in an infinite plane subjected to tension at infinity has been solved by Romalis and Tamužs [1] without crack closure consideration. The subsequent investigation (Tamužs and Petrova [2, 3]) is shown that depending on the locations and orientations of the microcracks, the stress intensity factor K_1 for a macrocrack and for microcracks can become negative. That is, the crack surfaces could overlap. Therefore, it is necessary to account for the closure of crack edges. Applying the method similar to [1, 2] the problem for a semi-infinite

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medium is investigated by assuming the frictionless contact on the crack edges.

PROBLEM FORMULATION AND THE METHOD OF SOLUTION

Let an elastic isotropic semi-infinite medium contain a macrocrack of length $2a_0$ and N microcracks of length $2a_k \ll 2a_0$ as shown in Fig.1. It will be assumed that all $a_k = a$ $(k = 1, 2, \ldots, N)$. The main crack can have one contact free portion with the unknown center coordinate z_0^c and unknown length 2c. The coordinate system x_0^c, y_0^c is attached to the macrocrack open portion, which may or may not coincide with the entire crack length. Both subsurface and surface macrocrack have been considered. For the surface crack it is taken $z_0^0 = a_0 h \sin\alpha_0$.

The crack edges are free of tractions while the tensile stress T is applied at infinity parallel to the edge of a semi-infinite medium. The problem can be reduced to solving the problem with boundary conditions on the crack lines:

$$\sigma_n{}^{\pm} - i\tau_n{}^{\pm} = p_n(x) = -(T/2)(1 + e^{-2i\alpha_n}), \qquad |x| < a_n \qquad (1)$$

These conditions are applied on the contact free portion of the main crack or open microcracks. On the closed portions of the macrocrack or on the closed microcracks the shear stresses are known

$$\tau_n{}^{+} = -\mathrm{Im}\{p_n\} = -(T/2)\sin 2\alpha_n \qquad (2)$$

and the transverse displacement discontinuities $[v_n]$ are required to vanish

$$[v_k] = 0 \qquad (3)$$

Singular integral equations for the system of cracks have been derived by Panasyuk et al [4]. The unknowns are expressed as transvers $[v_k]$ and shear $[u_k]$ displacement discontinuities across the cracks,i.e.,

$$v_k' - iu_k' = \frac{2\mu}{i(\kappa+1)}\frac{\partial}{\partial x}([u_k] + i[v_k]) \qquad (4)$$

Plane strain is assumed: $\kappa = 3 - 4\nu$ and the plane stress condition corresponds to $\kappa = (3-\nu)/(1+\nu)$.

For solving the problem with boundary conditions in eqs. (1)-(3) the system of equations should be separated in two: one for the derivative of the transverse displacement jumps and the other for the derivative of the shear displacement jumps

$$\int_{-c}^{c} v_0'(t)\left[\frac{1}{t-x} + R_{00}^{cc}(t,x)\right] dt + \int_{-a_0}^{a_0} u_0'(t)\, S_{00}^{cc}(t,x)\, dt$$

$$+ \sum_{k=1}^{N} \int_{-a}^{a} \left[v_k'(t)\, R_{0k}^{c}(t,x) + u_k'(t)\, S_{0k}^{c}(t,x)\right] dt = \pi\sigma_0(x) \quad |x| < c, \tag{5}$$

where R_{00}^{cc}, S_{00}^{cc} are the regular kernals, containing geometrical parameters of the problem, i.e., $R_{00}^{cc} = f(\alpha_k, c, z_k^0, z_0^0, z_0^c)$. Other equations have the same form.

The solution is obtained in the series form for small parameter $\lambda = a/a_0$, i.e.

$$v_n' = \sum_{p=0}^{\infty} v_{np}\lambda^p, \qquad u_n' = \sum_{p=0}^{\infty} u_{np}\lambda^p \tag{6}$$

For this reason, the kernals R_{nk}^{c}, S_{nk}^{c} should also be expanded in series for λ.

Then, inserting eqs. (6) into eqs. (5), the coefficients of like powers of λ can be equated to yield a recurrent relation for v_{np}, u_{np}. The first equation is the equation for the semi-infinite problem with single macrocrack, the others include microcracks influence.

The numerical method of mechanical quadrature is used for solving the recurrent system of equations (see, for example, [4]). The system of sigular integral equations are replaced by two systems of linear algebraic equations. The systems are distinguished only by their right-sides, so convinient for crack complex geometry calculation. The first system contains applied loading in the right-side, the other includs microcrack influence.

Evaluated are the macro crack tip stress intensity factors (SIF) K_I, K_2 as well as the SIF for microcracks. To be found in K_I is the noncontacting length 2c. The unknown boundaries of the noncontacting portion are obtained from the absence of singularity at these points $K_I(\pm c) = 0$

RESULTS AND DISCUSSION

The microcracks closing regions have been obtained depending on the locations and orientations of the microcracks (Fig. 2 -a,b). To estimate the summary effect of the microcracks on the macrocrack tip SIF the numerical examples are calculated for field of microcracks ahead a main crack (Fig. 3). A dotted line denotes the value with taking into account the microcrack closure, continuous one - without crack closure consideration. Calculations are carried out for $\lambda = 0.1$. It was obtained that the free boundary increases the SIF magnitude and modifies the configuration of the microcrack closing- opening regions. Account of the microcrack closure leads to changing the values of both SIF K_1 and K_2. In the case of the macrocrack nonorthogonal to the edge of the semi-infinite medium the applied load and the microcracks can cause full or partial closure of the main crack.

CONCLUSIONS

The boundary conditions are formulated and a solution method is elaborated for the macrocrack- microcrack interaction in an elastic semi-infinite medium by taking into account the possible crack closure. The microcrack closure phenomenon influences the magnitude of stress intensity factors K_1 and K_2 and should be taken into account.

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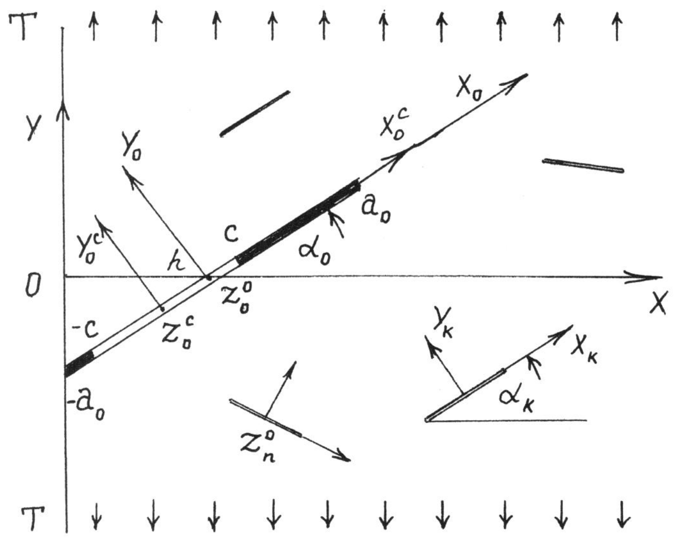

Figure 1 Schematic of microcracks around a partially closed main crack in a semi-infinite medium

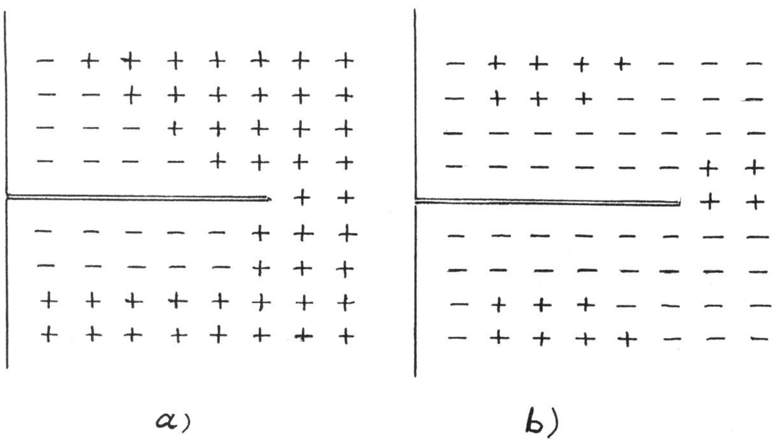

Figure 2 Fully closed (-) and opened (+) microcracks for different orientations around macrocrack. a — $\alpha_k = 45^0$, b — 90^0

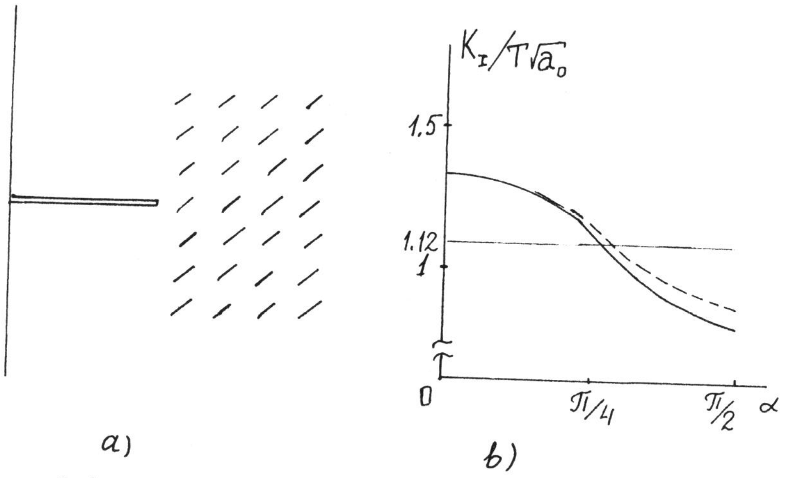

Figure 3 Angular variations of the macrocrack stress intensity factor Mode 1 with one column of microcracks