FRACTURE EVOLUTION AND SNAP-BACK INSTABILITY IN MULTI-CRACKED FINITE PLATES

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The effects of the crack interaction on the fracture evolution of multi-cracked finite plates in plane strain loading conditions are considered. A modified crack length control scheme is presented in order to analyse such problems depending on one or more independent parameters. The aim is to provide information about a discontinuous response, such as snap-back instability, which can be only highlighted by a deformation controlled process.

With reference to finite plates with one or more rows of evenly spaced collinear cracks, the snap-back branches of the load vs. displacement curve are numerically captured by means of a procedure based on the Boundary Element Method.

INTRODUCTION

Brittle materials, such as mortar, ceramics and rocks, contain numerous microstructural heterogeneities producing local concentrations of tensile stresses, even when the solid is subjected to compressive stresses. An extensive inherent microcracking results even before any load is applied. Nucleation and growth of microcracks can be considered the dominant micromechanisms of the macroscopic failure. Based on such microstructural considerations, brittle solids can be properly modelled as homogeneous linear elastic matrices containing cracks. Many aspects of their nonlinear macroscopic behaviour can be replicated by taking into account their interaction with neighbouring cracks and the consequent growth.

Several analytical models have been developed for crack growth (Horii and Nemat-Nasser (1), Ashby and Hallam (2), Kemeny and Cook (3)) on the solution for a single sliding crack under compression. Du and Kemeny (4) proposed a numerical micromechanical model, based on the Boundary Element Method. Numerical simulations of the nonlinear failure are obtained for uniaxial and biaxial compression tests applying incremental loadings or displacements.

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The global structural behaviour of a cracked homogeneous solid can range from ductile to brittle as a result of crack extension. The brittle behaviour coincides with a snap-back instability in the load vs. displacement curve. The snap-back branch may be captured only if the loading process is controlled by a function always monotonically increasing after the maximum load is reached, that can coincide with the crack mouth opening (or sliding) displacement as well as with the crack length. From an experimental point of view, the crack mouth opening displacement is usually assumed to control the loading process. On the other hand, an increasing function of the crack length is used in numerical procedures: the *indirect displacement control scheme* (5) refers to a displacement norm; the *crack length control scheme* (6) uses the crack length itself.

In the case of a multi-cracked plate, due to the crack interaction, the growth of a specific crack can be arrested and, at the same time, another crack may grow. It follows that the crack controlling the fracture evolution can change at each step of the loading process.

In this paper, a *modified crack length control scheme* is presented in order to analyse the fracture mechanics of brittle materials modelled as multi-cracked solids.

NUMERICAL MODEL

Brittle materials are modelled as elastic matrices containing an initial crack distribution. Plane strain loading conditions are considered. The elastic interaction and the crack growth are taken into account and result in a nonlinear global behaviour. Macroscopic failure coincides with the initiation of macrocracks due to the microcrack coalescence.

In the framework of Linear Elastic Fracture Mechanics, the onset and the direction of crack propagation are determined according to the *Maximum Stress Criterion* (7). The basis for the numerical model is the Displacement Discontinuity Boundary Element technique implemented by Brencich and Carpinteri (8) and suitably developed by the authors. The possibility of controlling the simulation through the crack length has been included in the original algorithm.

The problem deals with the determination of the critical load producing the crack extension. Assigned the initial geometry and the unit load distribution, the superposition principle yields the loading multiplier corresponding to a critical condition of crack growth related to the considered geometry. If plates with a multitude of cracks are considered, the failure strength is associated with the imperfections which are closest to the critical condition: their orientation, length and mutual interaction induce an equivalent Stress Intensity Factor (SIF) K_{eq} at their tips which is greater than the one obtained for the other imperfections. Microcracking evolution is obtained by an incremental procedure: a sequence of incremental straight extensions is considered, each one at those tips which experience the maximum SIF. The related load vs. displacement curve can be easily obtained.

Let us consider the finite square plate with two edge cracks and two further symmetrical central cracks, represented in figure 1. The plate is subjected to traction over the top and bottom sides. In figure 2 the SIFs at the edge crack tips (both pointed out as A because of symmetry) are shown as functions of the relative edge crack length a/b for two assigned values of the relative central crack length a_1/b=0.2 and 0.4. The dashed lines refer to the related SIFs at the central crack tips (each pointed out as B). The SIFs are normalised through the value K_{I0} of the mode I SIF at tips A in the absence of the central cracks.

For a_1/b=0.2 the maximum SIF is always reached at tips A. The edge crack length controls the fracture evolution of the finite plate during the whole loading process. For longer central cracks (a_1/b=0.4) the maximum SIF can be also reached at tips B. The independent parameters controlling the fracture problem are the following two: a_1/b for very small or very long edge cracks, and a/b in the remaining cases.

CRACK INTERACTION AND FRACTURE EVOLUTION

The numerical model described in the previous section is herein applied to analyse the effects of the crack interaction on the mechanical response of multi-cracked finite plates. The reference geometry is a finite square plate with two edge cracks, subjected to traction over the top and bottom sides. The fracture evolution is studied considering a more complex geometry obtained cutting the reference plate with two symmetrical central rows of four evenly spaced collinear cracks (figure 3). Varying the relative vertical spacing h/a between the central rows and the edge cracks, the global failure behaviour is shown to be strongly dependent on the elastic crack interaction.

The greater h/a, the more rapidly the influence of central cracks on the edge cracks vanishes. In figure 4 the normalised stress vs. deformation curve related to h/a_1=4 is compared with that related to the reference plate (σ_p and ε_p are the maximum elastic stress and deformation). The brittle behaviour is characterised by a snap-back instability captured by means of the proposed method. Macroscopic fracture, qualitatively so catastrophic as for the reference geometry, is shown to be due to the unstable edge crack extension.

Decreasing h/a_1 presents a strong influence on the global behaviour. For h/a_1=2, the fracture evolution coincides again with the edge crack growth. During their extension the edge cracks enter into amplification or shielding zones, alternatively. The σ–ε curve is characterised by a series of snap-back instabilities (figure 5). Their number coincides with the number of the central crack columns. Each instability corresponds to a single column which has a major role in the local amplification or shielding phenomena of the edge cracks.

When the central cracks are very close to the edge ones, the mutual interaction is so strong that the failure evolution involves both. Globally, a ductile behaviour emerges. With reference to figure 6, at first the failure process is controlled by the edge cracks (continuous lines in the diagram). The edge crack tips move and enter into the influence zone of the first central crack column. A local snap-back instability corresponds in the σ–ε curve. At the state ①, the central cracks are in a critical condition, grow and coalesce (dashed line). Finally, at the state ② the edge crack propagation, shielded by the central macrocracks, starts again.

CONCLUSIONS

A numerical model has been developed to simulate the failure evolution of brittle materials modelled as linear elastic homogeneous matrices containing an assigned initial crack distribution. The crack growth, interaction and coalescence are taken into account and result in a global nonlinear stress-strain behaviour. Snap-back instabilities are captured by means of a crack length control scheme suitably modified to account for the presence of many cracks which can interchange the role of *failure controller*.

The fracture evolution of a multi-cracked square plate, cut with two edge cracks and eight symmetrical central cracks, has been numerically simulated. The greater the vertical spacing, more rapidly the influence of the central cracks vanishes; the structure tends to behave as in their absence. On the contrary, if the central cracks are very close to the edge cracks, the fracture process involves the extension of both of them. If the central cracks are neither very far nor very close to the edge cracks, a series of snap-back instabilities is predicted, each related to a local stress amplification followed by a local shielding.

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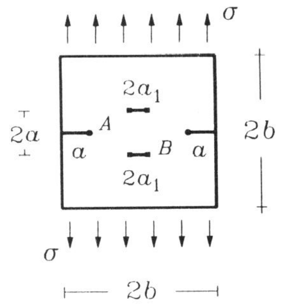

Figure 1: Square plate with two edge cracks and two central cracks.

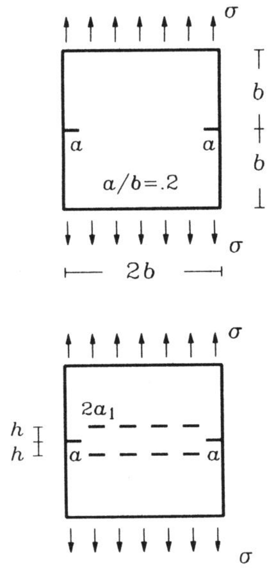

Figure 3: Square plate with two edge cracks and two central crack rows.

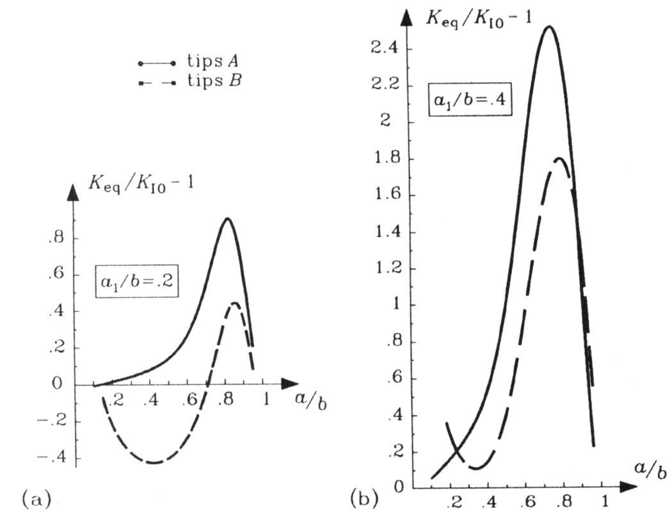

Figure 2: SIFs at the crack tips for the finite plate with two edge cracks and two central cracks: (a) a_1/b=.2; (b) a_1/b=.4.

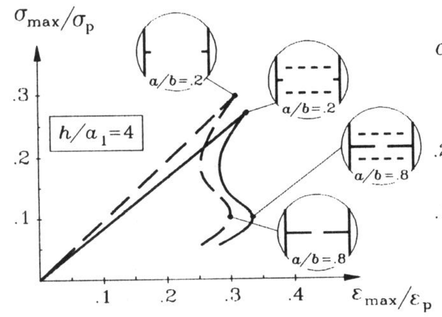

Figure 4: Normalised σ–ε curve for h/a_1=4.

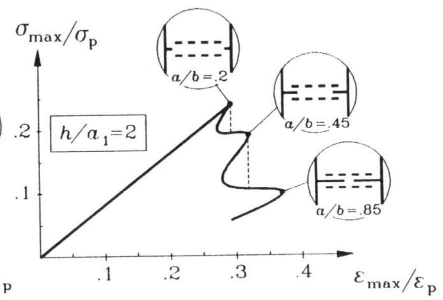

Figure 5: Normalised σ–ε curve for h/a_1=2.

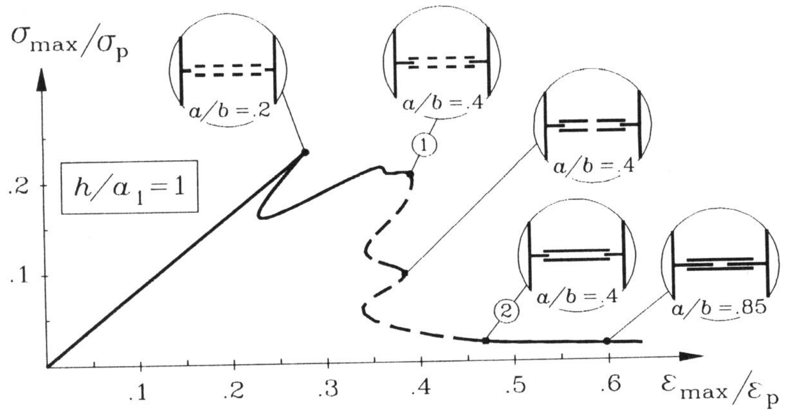

Figure 6: Normalised σ–ε curve for h/a_1=1.