# FRACTURE DUE TO SOLID STRUCTURAL TRANSFORMATION

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A model describing structural transformation of a solid into another solid having different mechanical properties and natural stress-free state is considered. On the base of this model stretching of a rod and longitudinal shear crack propagation are studied.

## INTRODUCTION

Mesostructure of some solids such as polycrystalline metals or rock under action of stress undergoes substantial changes. In some cases when defects of the parental material are numerous the damaged body may be considered as a continuum. In these cases actually phase transformation takes place caused by stress but not temperature. Thermal effects may by neglected while energy consumption must be taken into account.

Solids undergoing substantional changes of structure are usually described by the models of softening material. However existence of softening elasto-plastic material is questionable (Nikitin (1)). A model considered in this paper is alternative to that with softening. On the base of this model

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two problems are considered: *i)* Postcritical dynamical behaviour of a quasistatically stretched rod and *ii)* Propagation of a semi-infinite longitudinal shear crack. Energy consumption in the process of rupture is discussed.

## CONSTITUTIVE MODEL

Consider a solid transforming under action of stress into another solid with different mechanical properties and natural, stress-free state. Thermal effects are neglected while energy consumption on structural transformation will be accounted.

Assume for simplicity that both parental and damaged solids are elastic and undergo infinitesimal strain $\varepsilon_{ij}$. The elastic potential $W_p$ and stress $\sigma_{ij}$ for the parental material may be written in the form

$$W_p = \frac{1}{2} C^p_{ijkl} \varepsilon_{ij} \varepsilon_{kl}, \qquad \sigma_{ij} = C^p_{ijkl} \varepsilon_{kl} \tag{1}$$

Here $C^p_{ijkl}$ is the elasticity tensor of the parental solid. The elastic potential of the damaged body in the same as in (1) reference configuration must involve zero and first oder terms in strain

$$W_d = W_* - C^d_{ijkl} \varepsilon_{ij} \varepsilon^d_{kl} + \frac{1}{2} C^d_{ijkl} \varepsilon_{ij} \varepsilon_{kl}, \qquad \sigma_{ij} = C^d_{ijkl}(\varepsilon_{kl} - \varepsilon^d_{kl}) \tag{2}$$

where $C^d_{ijkl}$ is the elasticity tensor of the damaged solid, $\varepsilon^d_{ij}$ is the kinematic tensor of structural transformation.

Structural transformation takes place when stress state reaches some critical condition, determined in general by a function of the invariants of the stress tensor. In the simplest case as a criterion of structural transformation may be taken the next

$$J_2 = \text{const} \tag{3}$$

where $J_2$ is the second invariant of the stress deviator.

## STRETCHING OF A ROD

Consider quasistatic uniaxial stretching of a rod. The elastic potentials and stresses in the parental and damaged solids are

$$W_p = \frac{1}{2}E_p\varepsilon^2, \qquad \sigma = E_p\varepsilon$$

$$W_d = W_* - E_d\varepsilon_d\varepsilon + \frac{1}{2}E_d\varepsilon^2, \qquad \sigma = E_d(\varepsilon - \varepsilon_d) \tag{4}$$

Here $E_p$ and $E_d$ are Hooke's moduli of the parental and damaged solids, respectively, $\varepsilon_d$ is the kinematic and $W_*$ — energy characteristics of the structural transformation. The process of quasistatic stretching of a softening elasto-plastic rod was proved to be unstable (Nikitin(2)). This proof may be extended to the case under consideration.

When stress reaches the critical value $\sigma = \sigma_*$ homogeneous stress and strain states in a rod lose stability, strains are localized at some cross-sections and dynamic process starts. Equation of motion in terms of displacement $u$ is

$$\frac{\partial^2 u}{\partial t^2} = a^2\frac{\partial^2 u}{\partial x^2}, \qquad a^2 = \frac{1}{\rho}\frac{d\sigma}{d\varepsilon} \tag{5}$$

Here $\rho$ is the density of the solid. Velocity of wave propagation $a$ takes value $a = a_p = (E_p/\rho)^{1/2}$ for the parental solid and $a = a_d = (E_d/\rho)^{1/2}$ for the damaged one. Location and number of sections of localization can not be found. The dynamic process in the vicinity of one of them, say at $x = 0$, will be studied. The problem is self-similar since it does not contain any characteristic length or time. Hence the wave fronts radiating from $x = 0$ are straight and stress and particle velocity $v = \partial u/\partial t$ are constant between the fronts. Consider only the region $x > 0$ due to symmetry. The fronts of unloading $x = a_pt$ and structural transformation $x = bt$ start simultaneously. Velocity $b$ of the front and stress drop behind it are to be found. The region $x < a_pt$ is at rest

$$v = 0, \qquad \sigma = \sigma_* \tag{6}$$

In the region $a_pt < x < bt$ between the fronts of elastic unloading and structural transformation both stress $\sigma_1$ and velocity $v_1$ are unknown. Behind the front of structural transformation $v = 0$ due to symmetry while stress $\sigma_2$ is to be found. At the front of unloading the law of momentum conservation gives

$$\sigma_* - \sigma_1 = a_p \rho v_1, \qquad x = a_p t \tag{7}$$

At the front of structural transformation the condition of displacement discontinuity along with laws of momentum and energy conservation give

$$b(\varepsilon_1 - \varepsilon_2) + v_1 = 0, \qquad \sigma_1 - \sigma_2 + b\rho v_1 = 0$$

$$b(W_p - W_d - \frac{1}{2}\rho v_1^2) + \sigma_1 v_1 = 0, \qquad x = bt \tag{8}$$

From (4)

$$\sigma_1 = E_p \varepsilon_1, \qquad \sigma_2 = E_d(\varepsilon_2 - \varepsilon_d) \tag{9}$$

Eqs.(6-9) constitute the system of 6 simultaneous equations for determination of 6 unknowns $\sigma_1$, $\sigma_2$, $v_1$, $\varepsilon_1$, $\varepsilon_2$ and $b$. Solution of this system, for instance for $E_d = \frac{1}{4}E_p$, $\varepsilon_d = \frac{1}{2}\sigma_*/E_p \equiv \frac{1}{2}\varepsilon_*$, $W_* = \frac{1}{2}\sigma_*^2/E_p$, give for the shock front velocity of structural transformation $b \cong 0,3a_p$, for the stress drop behind the front of unloading $\sigma_1 \cong 0,22\sigma_*$ and for strain localization $\varepsilon_2 \cong 2,60\varepsilon_*$.

## PROPAGATION OF LONGITUDINAL SHEAR CRACK

Crack propagation is controlled by energy absorbed near the crack tip. The energy flux into the crack tip for an elastic medium (Broberg(3), Kostrov and Nikitin(4)) along with the fracture criterion give dependence of crack tip velocity on the stress intensity factors. However this dependence was not confirmed experimentally (Knauss and Ravi-Chandar (5)). Descrepancy between the theory and experiment is evidently due to nonelastic behaviour of material near the crack tip. Formation of numerous defects ahead of the crack tip may be modelled as the structural transformation. The elastic potentials and shear stresses $\tau_i$ in the case of antiplane stress state are

$$W_p = \frac{1}{2}\mu_p \gamma_i \gamma_i, \qquad \tau_i = \mu_p \gamma_i$$

$$W_d = W_* - \mu_d \gamma_i^d \gamma_i + \frac{1}{2}\mu_d \gamma_i \gamma_i, \qquad \tau_i = \mu_d(\gamma_i - \gamma_i^d) \tag{10}$$

Here $\gamma_i = \partial w / \partial x_i$ is the shear strain and $w$ is the only nonzero component of displacement.

Consider stationary propagation of semi-infinite longitudinal shear crack. Refer a crack to the Cartesian system of coordinates $x_1, x_2$ with origin at the moving crack tip and axis $x_1$ directed along a crack. Derivatives with respect to time $t$ turns into derivatives with respect to $x_1$ in stationary motion

$$\frac{\partial}{\partial t} = -v \frac{\partial}{\partial x_1}$$

Equations of motion for the parental and damaged solids take the form

$$\left(1 - \frac{v^2}{b_p^2}\right) \frac{\partial^2 w}{\partial x_1^2} + \frac{\partial^2 w}{\partial x_2^2} = 0, \qquad b_p = \left(\frac{\mu_p}{\rho}\right)^{1/2} \tag{11}$$

$$\left(\frac{v^2}{b_d^2} - 1\right) \frac{\partial^2 w}{\partial x_1^2} = \frac{\partial^2 w}{\partial x_2^2}, \qquad b_d = \left(\frac{\mu_d}{\rho}\right)^{1/2} \tag{12}$$

The upper half of the plane $x_2 = 0$ only may be considered due to the symmetry. The crack banks are assumed to be stress free

$$\tau_2 = 0, \qquad x_2 = 0, \qquad x_1 < 0 \tag{13}$$

The case of trans-sonic velocity will be studied

$$v_d < v < v_p \tag{14}$$

Then Eq.(11) is elliptic and describes the nonsingular state in the elastic forerunner. Consider solutions of Eq.(12) of the form

$$w = c_1 |x_2 + \alpha_d x_1|^{1/2} + c_2 |x_2 - \alpha_d x_1|^{1/2} \tag{15}$$

where $\alpha_d = b_d / (v^2 - b_d^2)^{1/2}$ and $c_1$, $c_2$ are constants. Stress and strain in this case have square root singularity what is needed for finiteness of the energy flux into the crack tip.

Structural transformation takes place along a ray $arctg x_2/x_1 = \varphi = \varphi_*$. Conditions of continuity along $\varphi = \varphi_*$ lead to the trivial solution. Therefore we assume that along the ray $\varphi = \varphi_*$ solution is discontinuous. Continuity of displacement and the law of momentum conservation give along $\varphi = \varphi_*$ the next equations for the case under consideration (Mukhamediev, Nikitin(6))

$$\gamma_1 \cos\varphi_* + \gamma_2 \sin\varphi_* = 0 \tag{16}$$

$$\gamma_1 \sin\varphi_* + \alpha_d^2 \gamma_2 \cos\varphi_* = 0 \tag{17}$$

The energy release $G_\varphi$ along $\varphi = \varphi_*$ is assumed to be finite. Then (Mukhamediev, Nikitin(6))

$$G_\varphi = \frac{v\mu_d \sin\varphi_*}{2\alpha_d}\left(\alpha_d^2\gamma_2^2 - \gamma_1^2\right) = 0 \tag{18}$$

Solution in the region adjoining the crack bank $\pi > \varphi > \varphi_*$ with account of (13) is

$$w = K\left(-x_2 - \alpha_d x_1\right)^{1/2} + K\left(x_2 - \alpha_d x_1\right)^{1/2} \tag{19}$$

Here $K$ plays a role of the stress intensity factor. If $\varphi_*$ lies to the left of the cracacteristic $x_2 = -\alpha_d x_1$ of Eq.(12) the condition (13) can not be met. Hence $\varphi_* < \pi - arctg\alpha_d$ and solution in the region to the right of the characteristic $x_2 = -\alpha_d x_1$ is

$$w = K\left(x_2 - \alpha_d x_1\right)^{1/2} \tag{20}$$

The conditions (17) and (18) lead to

$$\varphi_* = arctg\alpha_d \tag{21}$$

The condition of displacement continuity (16) was not used what seems to be admissible in the case of structural transformation. However this condition appears to be met automatically.

## CONCLUSION

Constitutive description of an internally unstable solid is given. Quasistatic process of loading of such a solid leads to dynamic postcritical process of localization of strain and formation of shock waves of unloading and structural transformation.

Although solutions (19), (20) possess the square root singularity the energy flux into the crack tip calculated on the base of them vanishes. For the first glance the result is dissapointing. However the experimental observations show that the stress intensity factors do not always control crack velocity (Knauss, Ravi-Chandar (5)). Energy release as in elasto-plastic solids is diffused and not concentrated at the crack tip as in elastic solids.

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