FRACTAL ASPECTS OF DAMAGE IN CONCRETE

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Concrete contains as load-bearing skeleton a densely packed aggregate of a wide range of particle sizes. Particularly river aggregate can easily loosen from the cementitious matrix, however, so that debonding a process initiates of structural loosening under increasing loads. A higher magnification allows one to perceive also the smaller particles, and thus the associated bond cracks. Hence, damage is a resolution-dependent phenomenon. Based on geometrical statistical notions and the sieve curve, an estimate is derived for the amount of damage as a function of resolution. In analogy with fracture surfaces, this estimate is interpreted in terms of the fractal concept.

INTRODUCTION

Concrete will gradually loose some of its integrity under increasing, fluctuating or permanently operating loadings. Structural loosening manifests itself in cracking on the various levels of the microstructure, ultimately leading to the development of engineering cracks on the highest level. Various NDT techniques have been used for generating information on this process. The most direct one is by visual observation. Sections of concrete elements can be studied with the aided or naked eye for traces of damage. Altough it is well known that the information provided by different NDT techniques will depend on sensitivity, this is less recognized for visual observations. This paper has the purpose of giving a quantitative basis to this phenomenon.

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MEASURES OF DAMAGE

Damage reveals itself in sections as (approximately) linear traces. The traces are cross-sections of crack surfaces which are scattered through the material body. The appropriate global parameter to measure damage in the section plane is the total crack length per unit of section area, L_A. Its 3-D equivalent is the total crack surface area per unit of volume, S_V. Experimentally, L_A is estimated by the number of intersections per unit length of a superimposed line system and the traces, P_L, or by measuring total projected length of line traces on a line, per unit of area, L''_A. Generally, the trace pattern is conceived as a linear combination of 1-D and 2-D portions, jointly revealing a trace orientation distribution which is sufficiently close to the actual one (Stroeven (1)).

This is a 2-D way of interpreting "damage". There are no additional complications met, however, in analysing it in a 3-D fashion. For randomly distributed cracks it can easily be shown, that $P_L = 0.5S_V$. The "efficiency factor" (0.5) is in case of 2-D and 1-D systems for the optimum direction, respectively, $2/\pi$ and 1. Adopting again the practical approach in which the actual crack system can be replaced in the most general case by a linear combination of 1-D, 2-D and 3-D portions, it is found that

$$S_V = P_L(\frac{\pi}{2})_\perp + (\frac{\pi}{2} - 1)P_L(\frac{\pi}{2})_\| + (2 - \frac{\pi}{2})P_L(0)_\| \qquad (1)$$

Herein, $\perp$ and $\|$ refer to the orientation of the line grid with respect to the orientation plane of the 2-D component, whereas the values, 0 and $\pi/2$, between brackets concern the angle between the line grid and the orientation axis of the 1-D portion. In case of a tensile loading the material damage structure will reveal *partially-planar* orientation. Hence, the 1-D portion will be absent. Alternatively, under direct compressive loadings a ***partially-linear*** damage structure will evolve, so that the planar portion will be missing.

FRACTAL CHARACTER OF DAMAGE

Bond cracks will be slightly curved. The 2-D portion of a partially-planar bond crack system will nevertheless constitute a parallel array of surfaces, with specific surface area S_{V2}. The 3-D component (S_{V3}) mainly originates from virgin cracking and cracking under low stresses due to release of high residual stresses. The cracks are - like the particles - of a very wide range of sizes and distributed as a 'dense random packing'. If it is assumed that at a certain "critical" stage of loading all cracks in the 2-D system have developed up to the same angular extension, 2α, then their size distribution will be identical to that of the particles! Under low magnification only the cracks at the interfaces of the largest particles can be 'detected'. By enhancing the magnification by a factor of 2, the cracks at the interfaces of the particles from the next aggregate fraction are added. When this is repeated for, say, eight fractions of which the particle sizes range from d_0 to d_m, the damage structure will encompass cracks with sizes differing two orders of magnitude (i.e. $d_m/d_0=2^7$).

The surface area, S, of a bond crack between a spherical grain and the matrix is given by

$$S = \pi(h^2 + \frac{1}{4}x^2) \tag{2}$$

where h and x are the height and span, respectively, of the crack surface. For an opening angle 2α, eq (2) yields, $S = \frac{\pi}{2}d^2(1 - \cos\alpha)$. $\overline{S}$ can be calculated at a certain resolution level as an average value for all particles/cracks exceeding the sensitivity level. $\overline{d^2}$ represents the second moment of the particle size distribution function (pdf). The upper and lower limits of the sieve curve area, indicated in the building code (e.g. NEN 3861), are approximated by Stroeven (2) by a straight line and a second order parabola (on the log-scale of particle diameters). It is shown by Stroeven (3), that simple transformation readily yields: $f(d)_u = 2.5d_0^{2.5}/d^{3.5}$ and $f(d)_l = 3d_0^3/d^4$ The first, second and third moments of these psd's are presented in Table 1. d_c is the average size of the grains intersecting a (fracture) plane. Total amount of damage is $\overline{S}.N_V$, in which the particle density, N_V, is given by $N_V = 6V_V/\pi\overline{d^3}$. As an example, substitution for the upper bound yields (Stroeven (2))

$$\overline{S}N_V = S_V = 3(1 - \cos\alpha)\frac{\overline{d^2}}{\overline{d^3}}V_V = \frac{3(1 - \cos\alpha)\sqrt{M}}{d_m}V_V \tag{3}$$

Table 2 presents estimates for 'damage' as a function of resolution, determined by eq (3) assuming $\alpha = 45^0$. The influence of magification (M) is obvious.

FRACTAL PROPERTIES OF CRACKS

The relevant parameter for fractal interpretation of the fracture surface in the model will be the total crack surface area per unit of the dividing surface,

TABLE 1– Moments of the pdf's corresponding to the boundaries for the sieve curves prescribed by the building code.

	$\overline{d}$	$\overline{d^2}$	$\overline{d^3}$	$\overline{d_c}$	$\overline{d_c^2}$
$f(d)_{upper} = \frac{5}{2}\frac{d_0^{2.5}}{d^{3.5}}$	$\frac{5}{3}d_0$	$5d_0^2$	$5d_0^{2.5}d_m^{0.5}$	$3d_0$	$3d_0^{1.5}d_m^{0.5}$
$f(d)_{lower} = 3\frac{d_0^3}{d^4}$	$\frac{3}{2}d_0$	$3d_0^2$	$3d_0^3\ln\frac{d_m}{d_0}$	$2d_0$	$2d_0^2\ln\frac{d_m}{d_0}$

TABLE 2– Estimates for damage at onset of structural loosening for different resolution levels.

Magni-fication	Particle properties Vol. fract.	Size range	Min. size	Max. size	Damage
M	V_V	d_0-d_m	d_0	d_m	S_V
[−]	[−]	$[mm]$	$[mm]$	$[mm]$	$[mm^2/mm^3]$
2	0.1	16–32	16	32	0.004
4	0.2	8–32	8	32	0.011
8	0.3	4–32	4	32	0.023
16	0.4	2–32	2	32	0.044
32	0.5	1–32	1	32	0.078
64	0.6	0.5-32	0.5	32	0.132
128	0.7	0.25-32	0.25	32	0.217

S_A. The increase in surface area is due to particle indentations of the dividing surface. Using the moments of the pdf's presented in Table 1, S_A is determined for the two different sieve curves. Hence,

$$
f(d)_l = 3d_0^3/d^4 \quad \left\{ \begin{array}{rcll}
\overline{x} & = & J_2\overline{d^2}/\overline{d} & = \frac{\pi}{2}d_0 \\
\overline{x^2} & = & J_3\overline{d^3}/\overline{d} & = \frac{4}{3}d_0^2 \ln\frac{d_m}{d_0} \\
\overline{h} & = & \overline{d_c}/4 & = \frac{1}{2}d_0 \\
\overline{h^2} & = & (\overline{d_c^2} - \overline{x^2})/4 & = \frac{1}{6}d_0^2 \ln\frac{d_m}{d_0} \\
\overline{S} & = & \pi(\frac{1}{6} + \frac{1}{4}.\frac{4}{3})d_0^2 \ln\frac{d_m}{d_0} & = \frac{\pi}{2}d_0^2 \ln\frac{d_m}{d_0} \\
N_A & = & (6V_V\overline{d})/(\pi\overline{d^3}) & = \frac{3}{\pi}(V_V/d_0^2 \ln\frac{d_m}{d_0}) \\
\overline{S}N_A & = & S_A & = \frac{3}{2}V_V
\end{array} \right.
$$

$$
f(d)_u = \frac{5}{2}d_0^{2.5}/d^{3.5} \quad \left\{ \begin{array}{rcll}
\overline{x} & = & J_2\overline{d^2}/\overline{d} & = \frac{3\pi}{4}d_0 \\
\overline{x^2} & = & J_3\overline{d^3}/\overline{d} & = 2d_0^{1.5}d_m^{0.5} \\
\overline{h} & = & \overline{d_c}/4 & = \frac{3}{4}d_0 \\
\overline{h^2} & = & (\overline{d_c^2} - \overline{x^2})/4 & = \frac{1}{4}d_0^{1.5}d_m^{0.5} \\
\overline{S} & = & \pi(\frac{1}{4} + 2.\frac{1}{4})d_0^{1.5}d_m^{0.5} & = \frac{3}{4}\pi d_0^{1.5}d_m^{0.5} \\
N_A & = & (6V_V\overline{d})/(\pi\overline{d^3}) & = \frac{2}{\pi}(V_V)/(d_0^{1.5}d_m^{0.5}) \\
\overline{S}N_A & = & S_A & = \frac{3}{2}V_V
\end{array} \right.
$$

in which $J_2 = \int_0^{\pi/2} \sin^2\theta d\theta$ and $J_3 = \int_0^{\pi/2} \sin^3\theta d\theta$ (Stroeven (4)).

S_A is the most relevant parameter to define the roughness of major cracks, such as the fracture surface. The planar roughness index, R_S, being the ratio of total fracture surface area and the corresponding area of the dividing plane, is obviously given by

$$R_S = A_{Am} + S_A = 1 + S_A - V_V \tag{4}$$

Basically, this holds only for a 2-D portion of cracks in a partially-planar system in which the orientation plane is parallel to the the dividing plane. Stroeven (2) has indicated how to expand eq (4) to encompass also the 3-D component. It is found that

$$R_S = 1 - V_V + 3V_{V3} + \frac{3}{2}V_{V2} = 1 + 2V_V(1 - \frac{3}{4}\omega) \quad (\omega = \frac{V_{V2}}{V_V}) \tag{5}$$

where the indices 2 and 3 refer to the 2-D and 3-D portions, respectively. The linear roughness index, R_L can be approximated according to Underwood (5) by $R_L = 1 + \pi(R_S - 1)/4$. Taking $\omega = \frac{1}{3}$, substitution of eq (5) in the fractal equation yields (see, e.g., Paumgartner, *et al* (6))

$$\log(1 + \frac{3\pi}{8}V_V(M)) = (D_l - 1)\log M + C \tag{6}$$

D_l is the fractal profile dimension and C a constant (determination of which can be avoided by considering the slope of the curves). Fractal dimensions obtained along this way for fracture surfaces in concrete are close to experimental data obtained by El-Saouma (7). Hence, fracture surfaces and also *the ensemble of surfaces of mesocracks* can be attributed (non-ideal) fractal properties. Cracks through particles will reduce fractal dimension. A lower D_l-value is indeed found for more brittle cementitious composites, like HPC (Prokopski (8), Rawicky and Wojnar (9)).

CONCLUSIONS

Stereological modelling of damage is achieved for concretes containing river gravel aggregate. The development of bond cracks is a major phenomenon in this case. A critical angular extension is assumed at a certain 'critical' stress level , alowing to derive a relationship between total crack surface area per unit of volume and the characteristics of the sieve curve. For that purpose, sieve curves are transformed by simple mathematical manipulations into particle size distribution functions, pdf's. The resolution-dependence is demonstrated, which makes it possible comparing experimental data obtained on cracking in concrete on different resolution levels. The similarity between the morphological properties of fracture surfaces and of an ensemble of mesocracks, renders possible interpreting damage also in fractal terms.

MAIN SYMBOLS USED

P_L	=	number of intersections per unit of line length (mm^{-1})
L_A	=	crack length per unit of area (mm^{-1})
S_V	=	crack surface area, S, per unit of volume (mm^{-1})
M	=	magnification (–)
V_V	=	volume fraction of particles (–)
d	=	particle size, d_0=smallest and d_m=largest particle (mm)
S_A	=	crack surface area per unit area of the dividing plane (–)
R_S	=	planar roughness index (–)
D_l	=	fractal profile dimension (–)
A_{Am}	=	areal fraction of the matrix (–)

REFERENCES

(1) Stroeven, P., J. Microsc., 107, Pt 3, 1976, pp. 313-321.

(2) Stroeven, P., Proc. MRS Symp. Boston, Nov. 27 - Dec. 1, 1995 (to be publ.).

(3) Stroeven, P., J. Composites, 4, 1982, pp. 129-139.

(4) Stroeven, P., "Advanced Studies on Structural Concrete", LNEC, Lisbon, 1994, pp. 125-140.

(5) Underwood, E.E., Acta Stereol., 6/suppl. II, 1987, pp. 170-178.

(6) Paumgartner, D., Losa G. and Weibel, E.E., J. Microsc., 121, Pt1, 1981, pp. 51-63.

(7) El-Saouma, V.E., Barton C.C. and Gamaleldin, N.A., Engr. Fract. Mech., 35, 1, 1990, pp. 47-53.

(8) Prokopski, G., "Brittle Matrix Composites 3", Edited by A.M. Brandt and I.H. Marshall, Elsevier Appl. Sc., London, 1991, pp. 11-20.

(9) Rawicki, Z. and Wojnar, L., Acta Stereol., 11., (2), 1992, pp. 185-189.