EXPERIMENTAL STRESS ANALYSIS IN THE CRACKED STAGE OF THE REINFORCED BEAMS

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The publication presents the experimental and numerical procedure used to determine the stress state in the photoelastic model of the reinforced beams. The fracture in the reinforced composite elements subjected to bending and their strength are closely related to several processes: matrix cracking, fiber plastic deformation or failure, interfacial debonding between matrix and fibers. The criterion of calculation of the maximum load has been derived out of two processes only: matrix cracking and the deformation of the reinforcement. The strain energy release rate G_c was calculated using the stress intensity factors (K_{II} and K_I) corresponding to the crack propagation of the matrix.

INTRODUCTION

The optical properties of the epoxy resin allow to determine the stresses in the matrix by photoelastic method (1). The stress distribution is characterised by isochromatic patterns. The dimensions of the typical model used in the experiment and material properties are given in Fig. 1. The brittle fracture of the matrix was simulated by introduction of artificially initiated cracks in the tension zone. The fracture mechanics parameters: the stress intensity factor (K_I) and the strain energy release rate (G_c) were determined experimentally. The stress intensity factor K_I was evaluated from load-displacement curve under the ASTM E813-81 standard and using the photoelastic measurement results by employing the Irwin method (2). The data necessery to determine K_I are available in the form of isochromatic fringe loops which occur in the region adjacent to the crack tip. The K_I value can be determined from:

$$K_{IC} = 2\tau_m \sqrt{2\pi \cdot r_m} f(\Theta_m) \quad (1)$$

and

$$f(\Theta_m) = \frac{1}{\sin\Theta}[1 + (\frac{2}{3tg\Theta})^2]^{-\frac{1}{2}} \cdot (1 + \frac{2tg(3\Theta/2)}{3tg\Theta})$$

where: r_m and Θ_m - are the polar coordinates with the origin defined at the crack tip; $2\tau_m = k_\sigma m_i = \sigma_1 - \sigma_2$ is known from the stress optic relation, k_σ - is material- fringe value and m_i - isochromatic fringe order.

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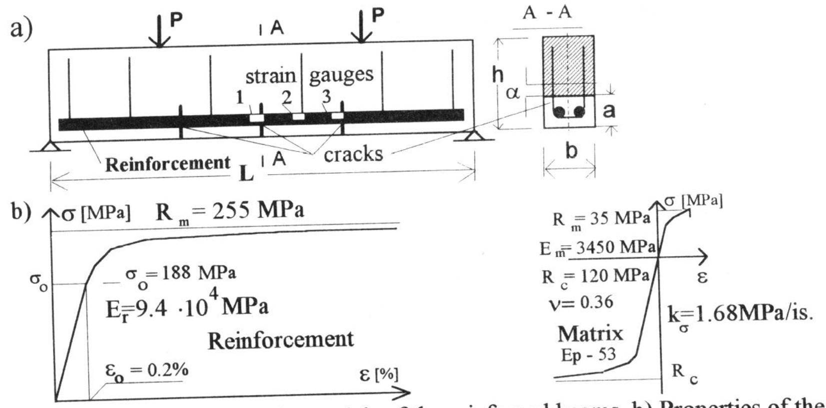
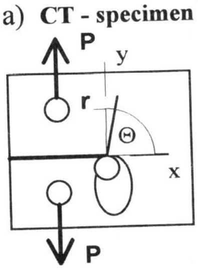
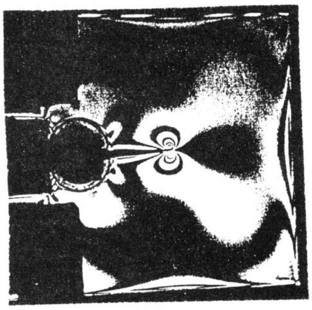


Fig. 1. a) Typical details of the models of the reinforced beams. b) Properties of the reinforcement and the matrix, k_σ - photoelastic constant of the matrix.

a) CT m=5.5, r_m=2.2mm,
Θ_m=104°, f(Θ) =1.04
from (1) **K_{IC} =1.2MPa √m**
ASTM - ΔG_C = 0.46 N/mm
K_{IC}≅1.26MPa √m
b) Bending :
m=8.5, r_m=1.2mm,
Θ_m=102°, f(Θ) =1.04
K_{IC} =1.24MPa √m

a) **CT - specimen**





b) **unreinforced specimen**

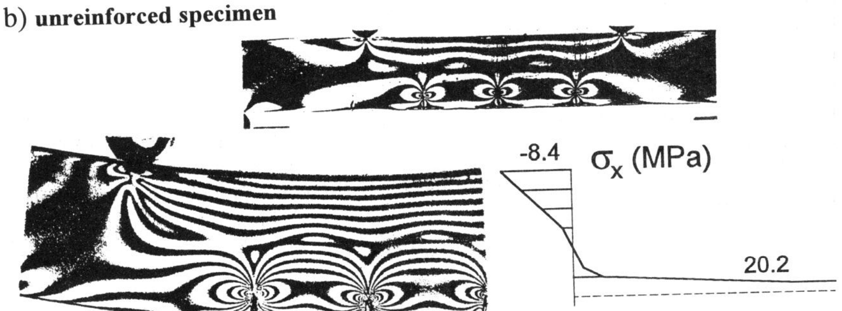


Fig. 2. Fracture mechanics parameters and isochromatic patterns associated with development of cracking. a) CT-specimen according to the ASTM E399 and E813 standards, b) stresses σ_x corresponding to pure bending and typical cracks development of the matrix.

The models were manufactured in the same way as the element of the reinforced concrete using cold casting of epoxy resin "Ep-53" (matrix) and reinforced by the copper bars. The models of the reinforced beams were tested in pure bending especially in the cracked stage up to their collapse. A series of beams was tested to investigate the ultimate limit state corresponding to cracking of the matrix and the plastic deformation of the reinforcement.

CRACKING MECHANISMS AND FAILURE

The stress state in the matrix according to the vertical and horizontal crack propagation was observed by a photoelastic method. The strains in the reinforcement were determined using strain gauges. The critical value of the strain energy release rate (**$G_C=\partial U/\partial A$**) (obtained experimentally) from the relation between the work of the acting forces and the crack surface has been determined. The corresponding displacements of the forces and the propagation of the crack in the matrix allow to determine the stress intensity factor **K_I** and the strain energy release rate G_C from:

$$G_c = \frac{\Delta U}{\Delta A} = \frac{P_i \Delta V_i}{aB} \; ; \quad \text{and} \quad K_I = \sqrt{E_m \cdot G_C} \tag{2}$$

where: ΔU - dissipated energy , **ΔA = a B** - fractured area , **P_i** - force corresponding to the crack propagation, **ΔV_i** - displacement corresponding to the cracks length **(a)**.

For computation of the ultimate moment **M** according to the crack propagation the strain energy release rate G_c was evaluated.

For vertical cracks - perpendicular to beam axis the cracked reinforced beam element is shown in Fig. 3. The strain energy release rate G is equal in this case to the Rice **J** - integral:

$$J_v = \int_s \left(\tfrac{1}{2}\sigma_{ij}\varepsilon_{ij}dx_2 - T_i^n \frac{\partial u_i}{\partial x_1} ds\right) = \frac{1}{E_m}\left[\int_{S1} \sigma_x \tau_{xy} ds_1 + \int_{S2} \sigma_x^2 ds_2\right] \tag{3}$$

The bending moment of crack propagation is given by the superposition principle:

$$M = M_1(K_I) + M_2(Z_a) \tag{4}$$

The bending moment $M_1 = M_1(K_I)$ inducing the stress intensity factor K_I at the crack tip is equal to (3): $K_{IC} = 4.2\frac{M_1}{b}(h)^{-3/2}\sqrt{(1-\frac{a}{h})^{-3} - (1-\frac{a}{h})^3}$

or from (4) $$K_I = \frac{6M_1}{bh^2}\sqrt{\pi a}\,\frac{0.923+0.199(1-\sin\frac{\pi a}{2h})^4}{\cos\frac{\pi a}{2h}}\sqrt{\frac{2h}{\pi a} tg\frac{\pi a}{2h}} \tag{5}$$

and $M_2(Z_a)$ - the moment produced by axial force Z_a in the reinforcement

$$M_1 = \frac{K_{IC} b\sqrt{h^3}}{4.2\sqrt{(1-\frac{a}{h})^{-3} - (1-\frac{a}{h})^3}} \quad \text{and} \quad M_2(Z_a) = Z_a\left(\frac{h+a}{2} - e_o\right) \tag{6}$$

$Z_a = \varepsilon_r E_r F_a$ for $\varepsilon \le \varepsilon_o$ and for $\varepsilon_r \rangle \varepsilon_0$ $Z_a = \sigma(\varepsilon_r)F_a$ and $\sigma(\varepsilon_r) = \sigma(\varepsilon_r/\varepsilon_o)^m$

The ultimate bending moment **M** according to vertical cracking was calculated by means of the critical value of K_{IC} of the matrix and the crack length "a" and the strains of the reinforcement. The ultimate bending moment **M** in function of the crack length "a" according to vertical cracking is presented in Fig. 4.- a).

For horizontal cracks - parallel to beam axis the contour along which the J-integral was calculated is shown in Fig. 3-b). The stresses: σ_x, σ_y were determined experimentally and the τ_{xy}, acting along the reinforcement were evaluated from the difference of the forces (Za - Zl) in reinforcement. From (3) and for stresses along the contour S we obtain: In plane stresses case if the material along S is linear, the J-integral becomes (5):

$$J_h = \int_s \left(\tfrac{1}{2}\sigma_{ij}\varepsilon_{ij}dx_2 - T_i^n \frac{\partial u_i}{\partial x_1}ds\right) = \frac{1}{2E_m}\left[\int_{S1}\sigma_x^2 ds_1 - \int_{S3}\sigma_x^2 ds_3\right] + \frac{1}{E_m}\int_{S4}\sigma_x \tau_{xy} ds_4 \qquad (7)$$

$$J_h = \frac{1}{2E_m b^2}\left[\frac{12M_a^2}{(h-a)^3} - \frac{12M_3^2}{(h-e_o)^3} + \frac{Z_a^2}{h-a} - \frac{Z_3^2}{h-e_o}\right] + \frac{2\sigma_{xo}(Z_a - Z_l)}{3E_m b} \qquad (8)$$

where: $M_3 = \dfrac{Mb}{12J_a}(h-e_o)^3$; $\quad Z_3 = \dfrac{Mb}{2J_l}(\beta + e_o)(h-e_o) \quad \sigma_{ox} = \dfrac{M}{J_l}\left(\dfrac{h}{2} - e_o - \beta\right)$

$$Z_a = \frac{M}{J_a}(a - e_o + \alpha)nF_r \qquad Z_l = \frac{M}{J_l}(h/2 - e_o - \beta)nF_r$$

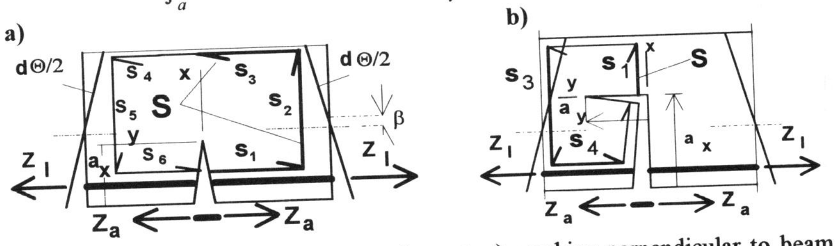


Fig. 3. Cracked reinforced beam element. a) cracking perpendicular to beam axis, b) cracking parallel to the beam axis and contour S considered in the calculation of the J - integral.

The stress intensity factor K_{II} is obtained from J_h -integral using the relation:

$$K_{II} = \sqrt{E_m J_h} \qquad (9)$$

by means of the critical value of $\quad \Delta G_c = \dfrac{1}{E_m}(K_I^{*2} + K_{II}^2) \qquad (10)$

of the matrix for the crack length "a" one obtains the value of the stress intensity factor : $K_I^* = (E_m \Delta G_c - K_{II}^2)^{1/2}$ according to horizontal cracking.
The stress intensity factors K_{II} and K^*_I obtained from (10) are presented in Fig. 4.- b).

EXPERIMENTAL AND NUMERICAL RESULTS

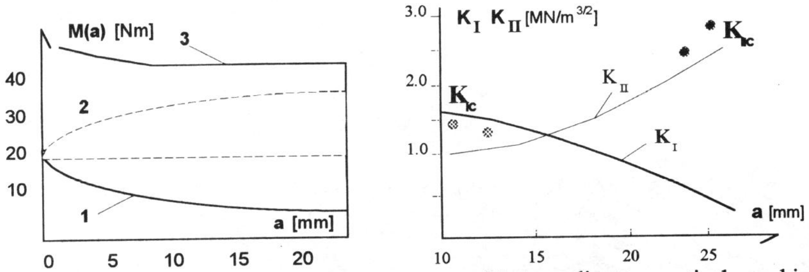


Fig. 4. a) The values of the ultimate moment M according to vertical cracking, b)The values of the K^*_I and K_{II} of the matrix in function of the crack length "a" obtained approximately from (10) according to mixed mode of the fracture.

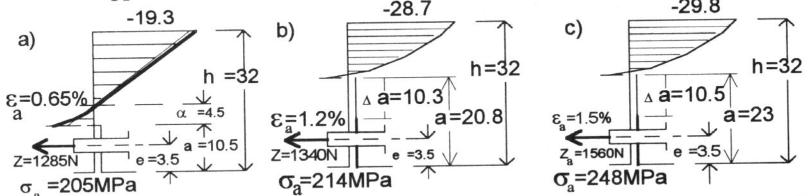


Fig. 5. Distribution of stress σ_x corresponding to crack propagation perpendicular (a, b) and parallel (c) to the beam axis.

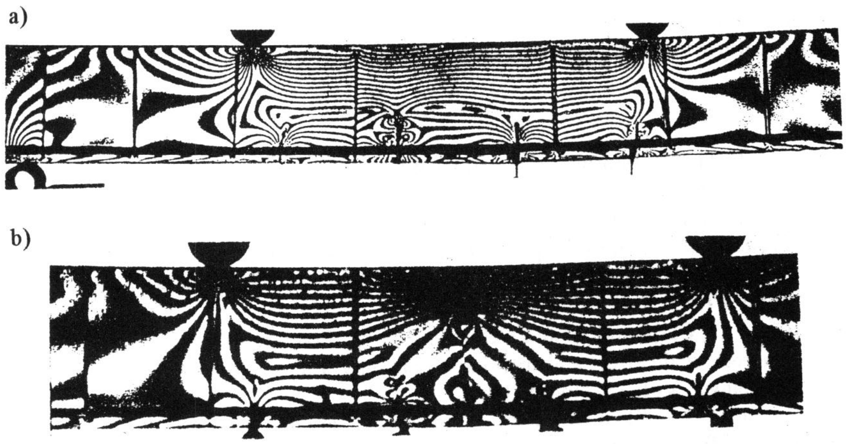


Fig. 6. Photoelastic models of the reinforced beam. The isochromatic patterns $(\sigma_1 - \sigma_2)$ distribution: **a)** before cracking, **b)** typical crack development corresponding to crack propagation perpendicular to the beam axis.

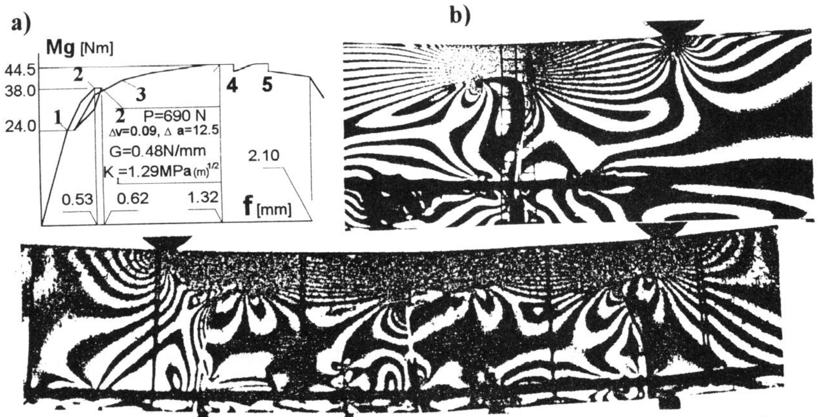


Figure 7. a) Load-displacement curve. b) Isochromatic fringes corresponding to crack propagation parallel to the beam axis.

CONCLUSIONS

The fracture in the reinforced composite elements subjected to bending and their strength are closely related to several processes: matrix cracking, fiber plastic deformation or failure, interfacial debonding between matrix and fibers. The criterion to calculate the maximum load has been derived accounting for processes: matrix cracking and the plastic deformation of the reinforcement. The theoretical ultimate bending moment M was calculated from the stress intensity factor (K_I) corresponding to the crack propagation in the matrix and the plastic deformation of the reinforcement.

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