ELASTICITY SOLUTION FOR LAMINATED ORTHOTROPIC CYLINDRICAL SHELLS UNDER DYNAMIC PATCH LOAD

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Dynamic response of orthotropic and cross-ply cylindrical shells under local patch load using elasticity approach are studied. The shells are of finite lengths and simply supported at both ends. The highly coupled partial differential equations are reduced to ordinary differential equations with variable coefficients by choosing the solution composed of trigonometric series along the axial and circumferential directions.The resulting ordinary differential equations are solved, using Galerkin method to obtain the finite element model of shell. Finally results are compared with the results obtained from the classical theory.

INTRODUCTION

In recent years, several approaches have been used to study the static and dynamic responses of plates and shells. Essentially these approaches are based on thin shell theory approximation, the shear deformation theories or three-dimensional theory of elasticity. A simple solution has been presented by Suian Li and Xi Wang(1) for crossly laminated shell subjected to axisymmetric loading, by assuming that the ratio of thickness to radius is small and hence can be neglected in respect to unity. This assumption causes the governing differential equations with variable coefficients to be reduced to the ones with constant coefficients.

In this paper solution is presented for dynamic response of orthotropic and cross-ply laminated cylindrical shells of finite lengths. The shell is pinched by diametrically opposite dynamic loads. Each load acts on a small rectangular patch.

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THE PROBLEM FORMULATION

Consider a laminated composite hollow cylindrical shell of length L with M constituent orthotropic laminae. The mean radius and the thickness of layers are denoted by R_k and h_k ,k =1,2,3,..M respectively. The M layers of the shell are oriented such that the material axes of any layer are aligned with the θ-x directions,i.e., the shell is laminated orthotropic. The constitutive equations of a layer are as follows

$$\begin{bmatrix}\sigma_x\\ \sigma_\theta\\ \sigma_r\end{bmatrix}=\begin{bmatrix}C_{11} & C_{12} & C_{13}\\ C_{12} & C_{22} & C_{23}\\ C_{13} & C_{23} & C_{33}\end{bmatrix}\begin{bmatrix}\varepsilon_x\\ \varepsilon_\theta\\ \varepsilon_r\end{bmatrix}\qquad \begin{matrix}\tau_{r\theta}=C_{44}\gamma_{r\theta}\\ \tau_{xr}=C_{55}\gamma_{xr}\\ \tau_{x\theta}=C_{66}\gamma_{x\theta}\end{matrix} \tag{1}$$

The equations of motion in term of displacement components in cylindrical coordination for a material with constitutive Eq.(1) yield

$$C_{11}u_{x,xx} + C_{55}u_{x,rr} + \frac{C_{55}}{r} u_{x,r} + (C_{12}+C_{55}) \frac{1}{r} u_{r,x} +$$
$$(C_{13}+C_{55})u_{r,xr}+C_{66}\frac{1}{r^2} u_{x,\theta\theta}+(C_{12}+C_{66})\frac{1}{r}u_{\theta,\theta x}=\rho\ u_{x,tt}$$
$$(C_{66}+C_{12})\frac{1}{r} u_{x,\theta x}+C_{44}(u_{\theta,rr} + \frac{1}{r} u_{\theta,r}- \frac{2u_\theta}{r^2}) + C_{22}\frac{1}{r^2}$$
$$u_{\theta,\theta\theta}+C_{66}u_{\theta,xx}+\frac{C_{23}+C_{44}}{r} u_{r,\theta r}+(C_{22}+2C_{44})\frac{1}{r^2}u_{r,\theta}=\rho\ u_{\theta,tt}$$
$$(C_{13}+C_{55})u_{x,xr}+(C_{13}-C_{12})\frac{1}{r} u_{x,x}+C_{33}(u_{r,rr}+ \frac{1}{r} u_{r,r})$$
$$-C_{22} \frac{u_r}{r^2} + C_{55}u_{r,xx} + (C_{23} + C_{44}) \frac{1}{r} u_{\theta,\theta r} -(C_{22}+C_{44}-$$
$$C_{23})\frac{1}{r^2} u_{\theta,\theta} + \frac{C_{44}}{r^2} u_{r,\theta\theta} = \rho\ u_{r,tt} \tag{2}$$

For a shell with simply supported at both ends, the boundary data are

$$\sigma_x(0,r)=\sigma_x(L,r)=u_r(0,r)=u_r(L,r)=u_\theta(0,r)=u_\theta(L,r)=0 \tag{3}$$

The boundary conditions on the outer and inner surfaces of the shell are

$$\sigma_r(x,R_0,t) = F(x,t) \quad , \tau_{xr}(x,R_0,t) = \tau_{r\theta}(x,R_0,t) = 0$$
$$\sigma_r(x,R_1,t) = \tau_{xr}(x,R_1,t) = \tau_{r\theta}(x,R_1,t) = 0 \tag{4}$$

The conditions of continuity of displacements and stresses between the layers are considered as follows

$$(u_r)_k=(u_r)_{k+1} \qquad (u_\theta)_k=(u_\theta)_{k+1} \qquad (u_x)_k=(u_x)_{k+1}$$
$$(\sigma_r)_k=(\sigma_r)_{k+1} \qquad (\tau_{r\theta})_k=(\tau_{r\theta})_{k+1} \qquad (\tau_{xr}) =(\tau_{xr})_{k+1} \qquad (5)$$

In Eq.(4) F(t) stand for the load per unit area corresponding to the pinching patch load. F(t) is obtained by using a double Fourier summation along θ,x directions as

$$F(t,\theta,x) = \Sigma_m^\infty \Sigma_n^\infty [F_{mn}\sin \frac{m\pi x}{L} \cos n\theta]\ F(t)$$

where m = 1,2,3,.... n = 0,2,4....

For n = 0 $\quad F_{mn} = (\frac{4P_0\theta_P}{\pi^2}\sin (\frac{m\pi L_P}{2L}) \sin (\frac{m\pi}{2}))$

For n≥2 $\quad F_{mn}=(\frac{16P_0}{mn\pi^2})\sin(\frac{m\pi L_P}{2L})\sin(\frac{m\pi}{2})\sin(\frac{n\theta_P}{2})$

$$F(t) = P_0(1-e^{-13100t}) \qquad (6)$$

Finite Element Solution

The solution which satisfies the boundary conditions (3) can be taken as

$$u_r^k = \Sigma_{m=1}^\infty\Sigma_{n=1}^\infty \sin P_m x \cos n\theta\ U_r^k(r,t)$$
$$u_\theta^k = \Sigma_{m=1}^\infty\Sigma_{n=1}^\infty \sin P_m x \sin n\theta\ U_\theta^k(r,t)$$
$$u_x^k = \Sigma_{m=1}^\infty\Sigma_{n=1}^\infty \cos P_m x \cos n\theta\ U_x^k(r,t) \quad , \quad P_m = \frac{m\pi}{L} \qquad (7)$$

After substituating Eq.(7) into Eq.(2) the partial differential equations are reduced to ordinary differential equations. The Galerkin method is used to obtain the finite model of shell. Selection of linear shape functions is based on the prior experience with Galerkin method (2), where the resulting model was as precise as models obtained by higher order elements. Considering linear shape functions for three field variable U_r, U_θ, and U_x as

$$U_r = \langle N_1\rangle\{U_r\},\ U_\theta = \langle N_1\rangle\{U_\theta\},\ U_x = \langle N_1\rangle\{U_x\} \qquad (8)$$

And applying the formal Galerkin method to the governing equations, results into the following dynamic finite element equilibrium equation for each layer

$$[M]_k\{\ddot{X}\}_k + [K]_k\{X\}_k = \{0\} \qquad (9)$$

For nodes which are located at any arbitrary interior kth layer and (k+1)th interface, the continuity conditions are as

$$U^k_{rKI} = U^{k+1}_{rKI+1} \quad U^k_{\theta kI} = U^{k+1}_{\theta kI+1} \ , \ U^k_{xKI} = U^{k+1}_{xKI+1} \qquad (10\text{-a})$$

$$\sigma^k_{rKI} = \sigma^{k+1}_{rKI+1} \ , \ \tau^k_{r\theta kI} = \tau^{k+1}_{r\theta kI+1} \ , \ \tau^k_{xrKI} = \tau^{k+1}_{xrKI} \qquad (10\text{-b})$$

Deriving Eq.(10-b) in term of displacements and expressing the derivations in backward and forward finite difference for kth. and (k+1)th. layers respectively we can obtain U^k_{rKI}, $U^k_{\theta KI}$ and U^k_{xKI} in terms of displacement values of neighboring nodes.
The dynamic finite element equilibrium equation for two neighboring elements at interior kth. and (k+1)th. interfaces become

$$[M]_k\{\ddot{X}\}_k+[K]_k\{X\}_k=\{0\} \qquad [M]_{k+1}\{\ddot{X}\}_{k+1}+[K]_{k+1}\{X\}_{k+1}=\{0\} \qquad (11)$$

Applying traction conditions (4) and expressing the derivatives in backward and forward finite difference for last and first elements respectively, two system of algebraic equations are obtained from which the displacement values on first and last nodes are obtained in term of the neighboring nodes.
Therefore the dynamic equilibrium equations for the last and first elements become

$$[M]_{MI-1}\{\ddot{X}\}_{MI-1}+[K]_{MI-1}\{X\}_{MI-1}=\{F(t)\} \qquad (12\text{-a})$$

$$[M]_1\{\ddot{X}\}_1+[K]_1\{X\}_1=\{0\} \qquad (12\text{-b})$$

By assembling Eqs.(9),(11),(12) the general dynamic finite element equilibrium equation is obtained as

$$[M]\{\ddot{X}\}+[K]\{X\}=\{F(t)\} \qquad (13)$$

Once the finite element equilibrium equation is established, different numerical methods can be employed to solve them in space and time domains. The Newmark direct integration method with suitable time step is used and the equilibrium equation is solved.

RESULTS

A three-layered cross-ply (90/0/90) cylindrical shell composed of graphite-epoxy was considered with the following properties

$E_L/E_T=25$ $G_{LT}/E_T=0.5$ $G_{TT}/E_T=0.2$ $\nu_{LT}=\nu_{TT}=0.25$
The patch size is taken to be $L_p/L = \theta_p/\pi = 1/25$
The nondimensional parameters are used

$$\bar{u}_r = \frac{E_L h u_r}{P_0 L_p R_{max} \theta_p} \qquad \bar{\sigma}_r, \bar{\sigma}_\theta, \bar{\sigma}_x = \frac{h^2}{L_p R_{max} \theta_p}(\sigma_r, \sigma_\theta, \sigma_x)$$

The radial displacement history is shown in Figs.(2) and (3). In these figures. the comparison is made between the present solution and the classical Love-Kirchhoff hypothesis for different radius to thickness ratios R/h · It is clear that the classical theory is applicable only for R/h>100. The main reason for this difference is initial curvature and shear effect.

SYMBOLS USED

C_{ij} = stiffness elastic constants

L = length of cylindrical shell

R_i, R_o= inner and outer radius of cylindrical shell

u_r = radial displacement

u_x = axial displacement

u_θ = circumferential displacement

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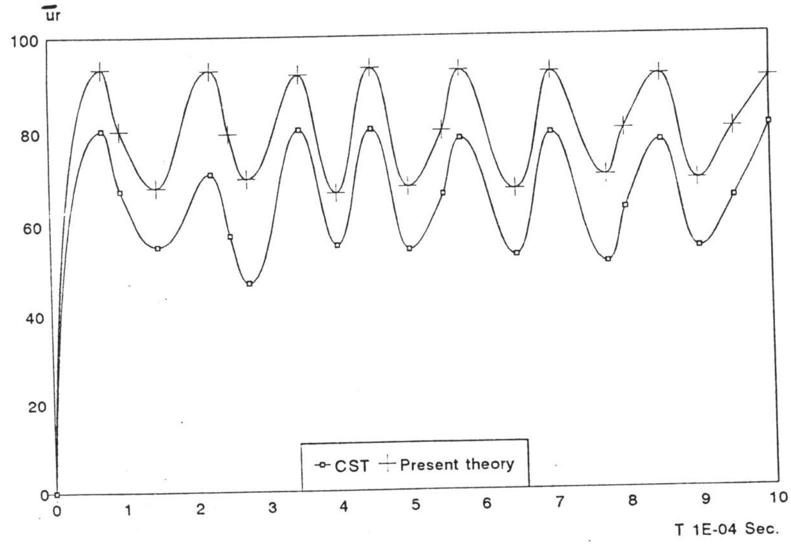

Figure 1 Time history of radial displacement with R/h =10

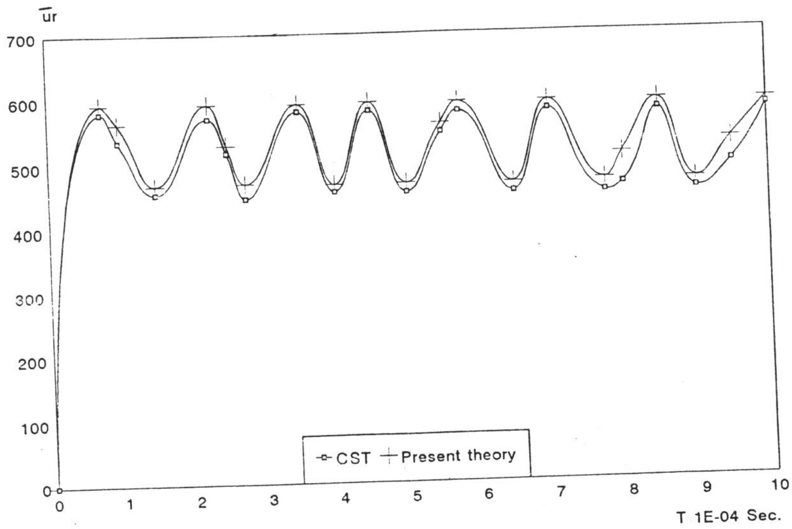

Figure 2 Time history of radial displacement with R/h=100