DURABILITY AND FRACTURE CALCULATIONAL MODEL OF SOLIDS UNDER THEIR CONTACT INTERACTION

O.P.Datsyshyn, V.V. Panasyuk*

Basing on the fracture mechanics concepts on crack initiation and propagation in structural materials, the calculational model for prediction of durability of two contacting workpieces, subjected to cyclic loading is formulated in this work. The step-by-step calculation of the fatigue cracks propagation paths is one of the important elements of the model. The crack extension resistance characteristics of material and local fracture criterion under stress-strain state are used for this calculation. This model can be used for investigation of the fracture process and evaluation of durability (by crack extension resistance) of the contacting bodies at such contact types as rolling, sliding, fretting-fatigue, friction fatigue, pulsating contacts etc.

INTRODUCTION

Processes of fracture and wear in contact zones of interacting elements are known to result in the loss of service properties of contact surface leading sometimes to the initiation of the main crack and full loss of structure workability. Theoretical grounds for prediction of durability of such systems has not yet been developed properly. With the use of fracture mechanics conceptions, recently some theoretical approaches to the contacting systems durability evaluation has been formulated for particular types of contact interaction, namely for fretting-fatigue contacts (D.P.Rooke et al. [1], M.C.Dubourg et al. [3], S.Faanes et al. [4], L.A.Dekhovych [2] and other), and for rolling-slip (L.M.Keer et al. [5], A.F.Bower [6] and other). The drawback of all these approaches is that they make the simplified prediction of crack propagation path as if it is rectilinear. But in the contact zone there is always a complex stress state and the crack propagates curvilinearly. Crack propagation path in this process is one of the key elements for durability estimation. In most of the mentioned models the crack initiation period is not accounted too. In this work, as the development of our previous works [7,8], we propose the general

* Karpenko Physico-Mechanical Institute , Lviv, Ukraine

model for durability calculation, overcoming all these drawbacks. This model gives the unified approach to the fracture processes in such common contact forms as rolling, slip, fretting-fatigue, friction fatigue, pulsating contact, etc.

CALCULATIONAL MODEL FOR ASSESSMENT OF DURABILITY

The main statements of the model are the following:

1. One of the contacting solids is supposed to be damaged by cracks. This solid is modelled as an elastic half-plane weakened by a system of curvilinear cracks (Fig.1).

2. The contact influence of the another body (counterbody) is modelled by the action either of stamp or efforts *p(x)* distributed along the boundary half-plane according to certain law (elliptical distribution, concentrated force, constant pressure, etc.) (Fig.2). The function of external loading change (contact or, possibly, some nominal) in contact cycle should be taken into account too. This function depends on λ-parameter (e.g. displacement during rolling, time during fretting-fatigue and pulsating contact), which determines the cycle.

3. The durability is estimated by its two components $N = N_i + N_g$, where N_i is a period (a number of loading cycles) of crack initiation, N_g is a period during which the crack grows from the initial l_0 to critical (permissible) length l_c. This period is named as residual durability.

4. Taking the theoretical approaches (see [9], edited by V.V.Panasyuk) for evaluation the crack initiation period as a basis, a conclusion is made that during contact interaction of the quasibrittle bodies it is advisable to use the formula:

$$N_i = \int_0^{l_0} \upsilon^{-1}\left(\Delta\varepsilon_{II}, A_1, A_2, \ldots, A_m\right) dl, \qquad (1)$$

where υ is the crack growth rate, $\Delta\varepsilon_{II}$ is the range of mode II strain at the crack tip, $A_1, A_2, \ldots, A_m$ are the crack extension resistance characteristics, which are determined experimentally using fatigue fracture diagrams (FFD) in coordinates $(\upsilon,\ \Delta\varepsilon_{II})$; l_0 is experimentally defined material characteristic. Curves $\upsilon(\Delta\varepsilon_{II})$ are the most suitable to achieve the research goal because during the crack initiation (macrocrack formation) under contact loading shear strain is a highly probable parameter which checks the stress-strain state at the crack tips.

5. Propagation of fatigue cracks has been modelled by step-by-step construction of their paths. Let consider two steps: the main, connected with crack tips propagation, the auxiliary connected with the change of λ-parameter in contact cycle. Solving the corresponding contact problem of elasticity theory for half-plane with cracks we determine the stress-strain state at the crack tips, particularly stress intensity factors (SIF) K_I and K_{II} on each step.

6. In contact zone there is a complex stress state and the crack growth in loading cycle is controlled by an amplitude of K-parameter (mixed mode SIF). The latter is connected with the fracture mechanism at the crack tip and determined by correspondingly chosen local fracture criterion on the basis of K_I and K_{II} [9]. In each cycle K-parameter changes having its maximum and minimum values at corresponding λ' and λ''. Crack is assumed to grow only when $\lambda=\lambda'=\lambda^*$, which provides the maximum modules of K-parameter. The crack propagates in direction determined by $\theta_* = F(K_I(\lambda^*)K_{II}(\lambda^*))$ according to chosen local fracture criterion when $\Delta K>\Delta K_{th}$. Here $\Delta K=K_{max}-K_{min}$ is the amplitude of K-parameter in contact cycle; ΔK_{th} is the threshold value of amplitude, the characteristic of the material fatigue crack extension resistance. At each stage of path construction (during corresponding number of contact cycles) values of λ^*, θ_*, ΔK are assumed to be constant.

7. The increments of crack path are constructed in determined θ_* direction at each stage by its approximation as third degree polynomials.

8. Basing on one of the formulae, which describe the kinetical diagram of fatigue fracture of material in (v, ΔK) coordinates, and magnitude of ΔK-parameter at each path construction stage, the rate of crack tip propagation can be determined:

$$v = dl/dN = v(\Delta K(l), C_1, \dots, C_m). \tag{2}$$

Here $C_1,\dots,C_m$ are the characteristics of material cyclic crack extension resistance.

9. While integrating the relation (2) the residual durability is determined

$$N_g = \int_{l_0}^{l_c} \frac{dl}{v(\Delta K(l), C_1, \dots, C_m)} \approx \sum_{k=1}^{j_c} \frac{dl}{v(\Delta K(l), C_1, \dots, C_m)}. \tag{3}$$

Here j_c is the total number of steps of crack path increment, Δl_k and v_k – the crack path increment and the rate at one of crack tips at the k-th calculation step.

10. While constructing of propagation path from several cracks tips simultaneously, the value of their steps should be correlated to their growth rates.

11. To model the possible changes in fracture mechanism during crack propagation the fracture criterion of new mechanism and transition conditions, which account the threshold values of characteristics of cyclic crack extension resistance should be introduced into the algorithm of path calculation.

Note, that the example of the evaluation of residual durability of ШХ15 bearing steel contact surface in the conditions of dry friction is presented in [7]. Examples for the calculation of the path of edge crack propagation during rolling, fretting-fatigue and cyclic indenting are presented in [8].

SINGULAR INTEGRAL EQUATION

As it was stated above, the basic element of the crack propagation model and residual durability estimation is the solution of static contact problem of elasticity theory for half-plane with curvilinear cracks under the action of stamp or contact loading along half-plane boundary. Let consider more general case: the action of stamp. But only principle moments of this problem solution will be touched within frames of this work. Let rigid stamp with arbitrary convex profile of basis presses the boundary of elastic isotropic half-plane (Fig.1) with M curvilinear cuts L_n $(n = \overline{1, M})$. Rather general contact conditions (sticking or Coulomb's law friction) are set between the basis of stamp and the half-plane. The conditions of possible crack edges contact are also general (sticking, slip with friction or smooth contact). Using a singular integral equation (SIE) method elaborated by M.P.Savruk [10] for two-dimensional problems of solids with cracks we present Muskhelishvili complex potentials of the problem under consideration as a sum of functions:

$$\Phi(z) = \Phi_0(z) + \Phi_1(z); \quad \Psi(z) = \Psi_0(z) + \Psi_1(z). \tag{4}$$

Here $\Phi_0(z)$ and $\Psi_0(z)$ characterize the stress state of integral half-plane with boundary loaded by $p_0(x)$ efforts along $x_0 - c \le x \le x_0 + c$ region:

$$\Phi_0(z) = -\frac{1}{2\pi i}\int_{x_0-c}^{x_0+c}\frac{p_0(x)}{x-z}\,\mathrm{d}x; \quad \Psi_0(z) = -\frac{1}{2\pi i}\int_{x_0-c}^{x_0+c}\left[\frac{\overline{p_0(x)}}{x-z} - \frac{x p_0(x)}{(x-z)^2}\right]\mathrm{d}x. \tag{5}$$

Functions $\Phi_1(z)$ and $\Psi_1(z)$ describe the stress state of half-plane caused by the derivatives of discontinuities displacements $g'_n(\tau_n)$ on contours of L_n cuts:

$$\Phi_1(z) = \sum_{k=1}^{M}\frac{1}{2\pi}\int_{L_k}\left\{\left[\frac{1}{T_k - z} - \frac{1}{\overline{T}_k - z}\right]e^{i\alpha_k} g'_k(t_k)\,\mathrm{d}t_k + \right.$$

$$\left. + \frac{2i\,\mathrm{Im}\,T_k}{(\overline{T}_k - z)^2}e^{-i\alpha_k}\overline{g'_k(t_k)}\,\mathrm{d}\bar{t}_k\right\}; \quad T_k = t_k e^{i\alpha_k} + z_k^0; \quad z = z_n e^{i\alpha_n} + z_n^0 \tag{6}$$

$$\Psi_1(z) = \sum_{k=1}^{M}\frac{1}{2\pi}\int_{L_k}\left\{\left[\frac{1}{(\overline{T}_k - z)^2} - \frac{1}{(T_k - z)^2}\right]\overline{T}_k e^{i\alpha_k} g'_k(t_k)\,\mathrm{d}t_k + \right.$$

$$\left. + \left[\frac{1}{T_k - z} - \frac{1}{\overline{T}_k - z} - 2i\,\mathrm{Im}\,T_k\frac{\overline{T}_k + z}{(\overline{T}_k - z)^3}\right]e^{-i\alpha_k}\overline{g'_k(t_k)}\,\mathrm{d}\bar{t}_k\right\}.$$

The $p_0(x)$ and $g'_k(t_k)$ complex functions are unknown too. While putting $\Phi(z)$, $\Psi(z)$ functions (4) into corresponding boundary conditions of the problem on cracks edges and on the boundary of half-plane as well as using additional conditions (identical magnitude of displacements on all contours, stamp equilibrium) we receive the system of SIE for $g'(t), p_0(x)$ functions or their real or imagine constituents. The boundaries of contact zones between cracks edges and stamp base with the boundary of the half-plane are unknown. We can determine them by solving integral equations of the problem with use of additional conditions, proceeding from the stress character in these points. SIE can be solved through combination of mechanical quadratures method [10] and iteration procedure. Note that SIE, obtained in [11] correspond to the case when crack edge are in the smooth contact.

Note that, in most interesting cases met in practice the solution of elasticity theory problem for one crack (Fig.2) without considering its edges contact is sufficient to construct the path. Particularly this concerns the edge cracks developing according to normal tear mechanism (mode I). Here we come to SIE

$$\int_{-1}^{1}\left[R(\xi,\eta)\varphi(\xi)+S(\xi,\eta)\overline{\varphi(\xi)}\right]d\xi=\pi P(\xi,\eta),\ |\eta|<1 \tag{7}$$

which kernels and right side for different kinds of contact loading can be found in [10]. This equation was used in all numerical examples presented in [8].

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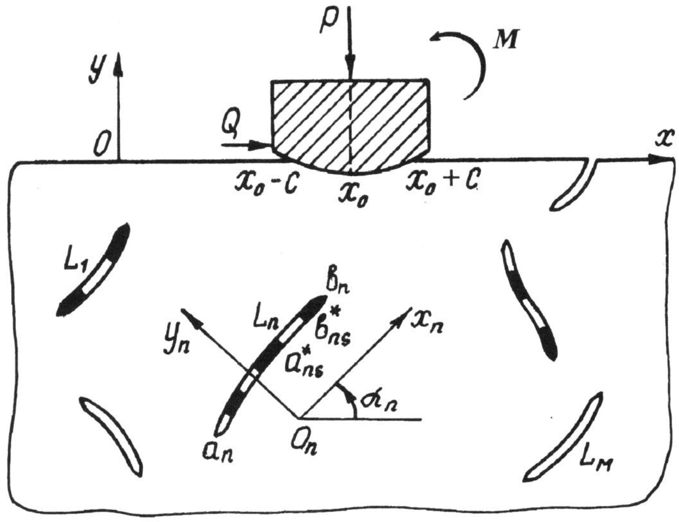

Figure 1 The general scheme of the problem

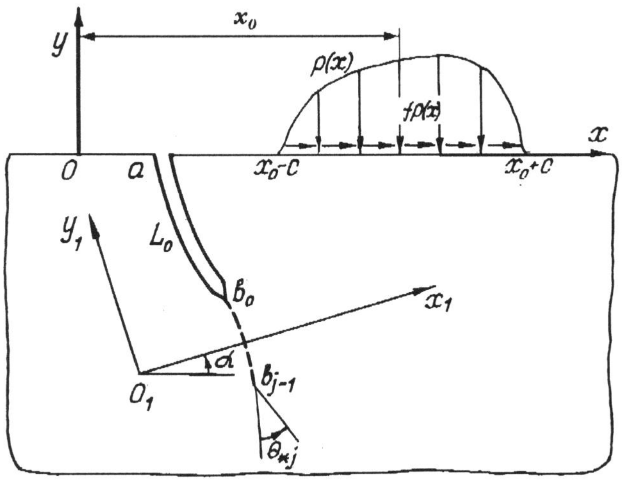

Figure 2 Simplified scheme of the problem