DAMAGE ACCUMULATION CRITERION FOR MULTIAXIAL HIGH AND LOW CYCLE FATIGUE

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Is this paper a new criterion for damage accumulation and crack initiation under multiaxial low and high cycle fatigue loading is presented. The crack initiation condition for brittle or ductile materials has been formulated using a failure function specifying the critical stress or strain state on a physical plane. The material parameters vary in the course of damage accumulation due to fatigue loading, creep, or environmental degradation.

INTRODUCTION

A major problem in fracture mechanics is associated with formulation of sufficiently simple and accurate condition of damage accumulation and crack initiation for multiaxial stress states. The aim of this paper is to propose damage accumulation and fracture criterion and demonstrate its application to a variety of particular cases, including monotonic and variable loading.

We can distinguish four mechanisms of material rupture, namely: brittle rupture (associated with microplastic effects), ductile rupture (associated with macroplastic deformation), creep rupture (due to viscoplastic deformation) and corrosive rupture. The crack initiation for each case occurs trough a different process and can be specified in terms of critical stress condition for brittle rupture, critical strain condition for ductile or creep rupture.

Consider any arbitrary physical plane Δ in a structural element and the local coordinate system (ξ_1,ξ_2,ξ_3), Fig.1, with its origin at $\mathbf{x}_o(x_{o1},x_{o2},x_{o3})$. The components of the traction vector Σ_i and strain vector E_i on the physical plane Δ are specified by the relations:

$$\Sigma_i(\tau_{n1},\tau_{n2},\sigma_n) = N_{ik}\sigma_{kj}n_j, \qquad E_i(\gamma_{n1},\gamma_{n2},\varepsilon_n) = N_{ik}\varepsilon_{kj}n_j, \tag{1}$$

where $N_{ij} = \cos(\xi_i, x_j)$ and $n_j = \cos(\xi_3, x_j)$. The resulting shear stress and strain in the plane Δ are

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$$\tau_n = \left[\tau_{n1}^{\ 2} + \tau_{n2}^{\ 2}\right]^{1/2}, \qquad \gamma_n = \left[\gamma_{n1}^{\ 2} + \gamma_{n2}^{\ 2}\right]^{1/2}. \tag{2}$$

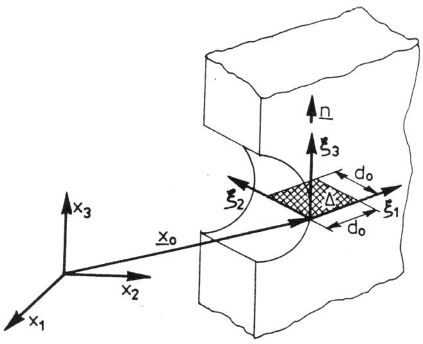

Fig. 1. Physical plane Δ associated with the local coordinate system (ξ_1,ξ_2,ξ_3) and the global system (x_1,x_2,x_3).

STRESS CONDITION OF BRITTLE FAILURE AND EVOLUTION OF MICROPLASTIC DAMAGE

The stress condition is formulated in terms of contact tractions acting on a physical plane Δ. The maximal value of the failure function is reached on a particular plane and the onset of cracking occurs when this value becomes critical, cf. Seweryn and Mróz [2], so that:

$$R_{f\sigma} = \max_{(\mathbf{n},\mathbf{x}_o)} R_\sigma(\sigma_n/\sigma_c, \tau_n/\tau_c) = 1, \tag{3}$$

where $R_{f\sigma}$ is a measure of brittle failure, R_σ denotes the brittle failure function, σ_c and τ_c denote the critical values of stresses for tension and shear, $\mathbf{x}_o$ denotes the positions of a critical point and $\mathbf{n}$ is the unit orientation vector of the critical plane.

The stress brittle failure function $R_\sigma(\sigma_n/\sigma_c,\tau_n/\tau_c)$ is a homogeneous function of contact stresses σ_n and τ_n, for example, the elliptical yield condition combined with the critical shear stress condition in the compression domain, Fig.2.

$$R_\sigma\left(\frac{\sigma_n}{\sigma_c},\frac{\tau_n}{\tau_c}\right) = \left[\left(\frac{\langle\sigma_n\rangle}{\sigma_c}\right)^2 + \left(\frac{\tau_n}{\tau_c}\right)^2\right]^{1/2}. \tag{4}$$

The bracket symbol ⟨ ⟩ denotes: $\langle g\rangle = g$ for $g > 0$ and $\langle g\rangle = 0$ for $g \le 0$.

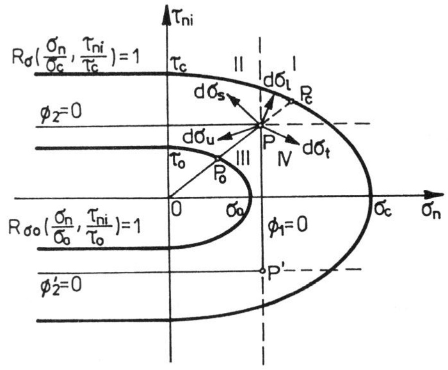

Fig.2. The brittle failure function and the microplastic damage initiation function. The damage accumulation occurs due variation of the stress state.

For large stress gradients occurring in the case of existing cracks or sharp notches, the non-local condition is derived in terms of mean values of σ_n and τ_n on the physical element of size $d_o \times d_o$ (damage zone) [1, 2], Fig.1.

The critical stress values σ_c i τ_c depends on temperature T and on microplastic, macroplastic, viscoplastic and physicochemical damage components ($\omega_{n\sigma}$, ω_{np}, ω_{nv} and ω_{nh}) on the plane Δ [3], thus:

$$\sigma_c = \sigma_{co}(T)(1-\Omega_{n\sigma}), \qquad \tau_c = \tau_{co}(T)(1-\Omega_{n\sigma}), \tag{5}$$

where $\Omega_{n\sigma} = \omega_{n\sigma} + \Phi_\sigma(\omega_{np}, \omega_{nv}, \omega_{nh})$ and σ_{co} and τ_{co} are the critical stress values unaffected by the accumulated damage (rupture stresses for the case of monotonic loading).

The damage accumulation occurs when the stress amplitude exceeds a threshold value associated with the fatigue endurance limit. The fatigue damage develops when the fatigue (or microplastic) damage initiation function exceeds the critical value on a maximal physical plane, cf. Seweryn and Mróz [4, 5], thus:

$$R_{f\sigma o} = \max_{(\underline{n}, \underline{x}_o)} R_{\sigma o}\left(\frac{\sigma_n}{\sigma_o}, \frac{\tau_n}{\tau_o}\right) \rangle 1, \tag{6}$$

where $R_{f\sigma o}$ is the damage accumulation factor, $\sigma_o < \sigma_c$ and $\tau_o < \tau_c$ are the stresses, specifying the fatigue damage initiation for pure tension and shear.

The microplastic damage growth is expressed by the formula proposed by Seweryn and Mróz [5]:

$$d\omega_{n\sigma} = d\omega_{n\sigma}(\Sigma_i, d\Sigma_i, \Omega_{n\sigma}) = \Psi_\sigma(R_\sigma) d\hat{R}_\sigma. \tag{7}$$

The microplastic damage accumulation function $\Psi_\sigma(R_\sigma)$ is determined from experimental investigations. This function can be assumed as a non-linear combination of $R_\sigma - R_{\sigma oc}$, $1 - R_{\sigma oc}$ and $1 - R_\sigma$, where $R_{\sigma oc} = R_\sigma/R_{\sigma o}$, corresponeding to the section PP_o, P_cP_o and P_cP in Fig.2. The function $\Psi_\sigma(R_\sigma)$ can be written in the form:

$$\Psi_\sigma(R_\sigma) = A_\sigma\left(\frac{\langle R_\sigma - R_{\sigma oc}\rangle}{1 - R_{\sigma oc}}\right)^{n_\sigma}\frac{1}{1 - R_{\sigma oc}}, \tag{8}$$

where n_σ and A_σ are material parameters. The increment $d\hat{R}_\sigma$ is specified as the increment of brittle failure function $\left(d\hat{R}_\sigma = \langle dR_\sigma\rangle\right)$ or in terms of increments of stress vector on the physical plane, thus:

$$d\hat{R}_\sigma = \frac{\partial R_\sigma}{\partial \sigma_n}d\hat{\sigma}_n + \frac{\partial R_\sigma}{\partial \tau_{n1}}d\hat{\tau}_{n1} + \frac{\partial R_\sigma}{\partial \tau_{n2}}d\hat{\tau}_{n2}, \tag{9}$$

where the effective stress increments are specified by the relations:

$$d\hat{\sigma}_n = \begin{cases} d\sigma_n & \text{for} \quad d\sigma_n \geq 0 \quad \text{and} \quad \sigma_n \geq 0, \\ 0 & \text{for} \quad d\sigma_n < 0 \quad \text{or} \quad \sigma_n < 0, \end{cases} \tag{10a}$$

and

$$d\hat{\tau}_{ni} = \begin{cases} d\tau_{ni} & \text{for} \quad \tau_{ni}d\tau_{ni} \geq 0, \\ 0 & \text{for} \quad \tau_{ni}d\tau_{ni} < 0. \end{cases} \tag{10b}$$

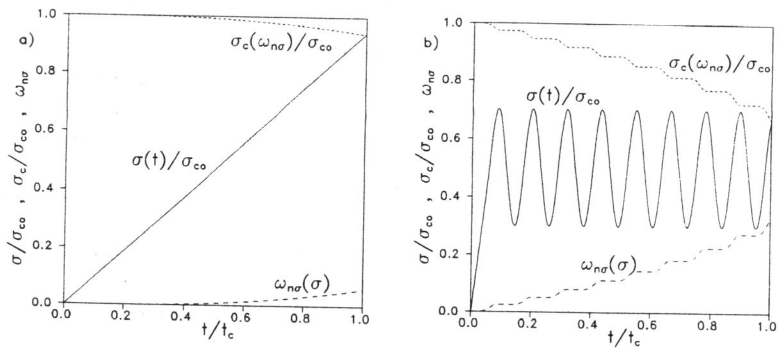

Fig.3. The interaction of a) monotonic and b) fatigue loading with the failure stress evolution due to damage growth.

In the stress plane the effective stress condition provides four domains, namely full active damage growth I, partial damage growth II, IV and unloading domain III. The angular domain of loading-unloading moves with the stress point P in the stress plane, Fig.2.

Consider now the uniaxial tension-compression test, Fig.3a. For increasing stress, the damage accumulation occurs on the critical plane. This induces reduction of failure stress, and for $\sigma(t) = \sigma_c(\omega_{n\sigma})$, the cross-sectional rupture occurs. For cyclic loading, the progressive damage accumulation occurs with resulting decrease of the failure stress, Fig.3b. When in the course of cyclic loading the maximal stress reaches the failure surface, the crack initiation occurs along the critical plane.

DEFORMATION CONDITION OF DUCTILE RUPTURE AND EVOLUTION OF MACROPLASTIC DAMAGE

Similarly as for the stress condition, let us introduce the normal and shear components ε_n, γ_n on the physical plane Δ. The ductile fracture condition is formulated by using the strain failure function $R_p(\varepsilon_n/\varepsilon_c, \gamma_n/\gamma_c)$. The maximal value of this function is reached on particular plane and the onset of cracking occurs when this value becomes critical, thus:

$$R_{fp} = \max_{(\mathbf{n}, \mathbf{x}_0)} R_p(\varepsilon_n/\varepsilon_c, \gamma_n/\gamma_c) = 1, \tag{11}$$

where R_{fp} is a measure of ductile failure, R_p denotes the ductile failure function, ε_n and γ_n denote the normal and shear strains with elastic and plastic components:

$$\varepsilon_n = \varepsilon_n^e + \varepsilon_n^p, \qquad \gamma_n = \gamma_n^e + \gamma_n^p, \tag{12}$$

and ε_c, γ_c, are the critical values of strains, which depend on temperature and the accumulated damage.

Assume, that the macroplastic damage growth on a particular physical plane Δ depends on the value of components of the strain vector E_i and their plastic increments. Using strain failure function the macroplastic damage evolution condition can be written:

$$d\omega_{np} = \Psi_p(R_p)d\hat{R}_p, \tag{13}$$

where:

$$d\hat{R}_p = \left\langle \frac{\partial R_p}{\partial \varepsilon_n} d\varepsilon_n^p + \frac{\partial R_p}{\partial \gamma_{n1}} d\gamma_{n1}^p + \frac{\partial R_p}{\partial \gamma_{n2}} d\gamma_{n2}^p \right\rangle. \tag{14}$$

The macroplastic damage accumulation function $\Psi_p(R_p)$ can be proposed in the form:

$$\Psi_p(R_p) = A_p R_p^{n_p}, \tag{15}$$

where n_p and A_p are material parameters.

Is should be noted that elastic components of strains in the deformation condition of macroplastic evolution correspond to the stresses specifying fatigue endurance limit in the microplastic damage accumulation condition, Eq. (6).

CONCLUDING REMARKS

In the present work, a uniform formulation was presented for damage accumulation and crack initiation or propagation for the case of multiaxial fatigue, brittle failure or ductile failure. The damage accumulation is associated with the physical plane which corresponds to the plane of crack initiation or propagation. The crack initiation and propagation stages can be treated within this formulation. The non-local formulation, applicable to singular or quasi-singular stress distributions provides the possibility to treat singular stress or strain regimes.

Damage accumulation model proposed here enables to consider mutual interaction of all important effects such as microplastic, macroplastic, creep etc., which appear in the fracture process.

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