CRACK TIP FIELDS FOR RAPIDLY GROWING CRACKS IN VISCOPLASTIC MATERIALS

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A fast propagating crack in a visco-plastic material of Perzyna-type is considered by two different methods. First, the problem of an semi-infinite crack in an infinite plate is examined. An analytical investigation leads to a system of ordinary differential equations, which is solved numerically by a multiple shooting-method. To prove the physical significance of the resulting crack tip fields a mixed Finite Element formulation is introduced in the second part which is applied to the boundary value problem of a running crack in an infinite strip under steady state conditions. Results regarding to the extension of the dominance zone are presented.

INTRODUCTION

If fast crack propagation in conjunction with rapid deformation in inelastic materials is considered, rate effects must be taken into account. Consequently, it is necessary to use viscoplastic material models.

Much work has been done to solve the field equations for running cracks in viscoplastic materials, many of them limited to creep crack growth. One of the most important investigations was published by Hui and Riedel (4) in 1981, who derived a new type of singular field in power law hardening materials. It is characterized by a stress and strain singularity of the form $\sigma_{ij}, \varepsilon_{ij} \sim r^{1/(1-n)}$, where n is the hardening exponent and $n > 3$. In 1983 Lo (6) showed that this result stays in the case of rapid crack growth and consideration of inertia effects. However, differentiation with respect to time is eliminated by a Galilean transformation from a global coordinate system into a local system fixed at the tip. In this study, Lo's work concerning mode III is generalized to problems in plane stress and plane strain for arbitrary material parameters

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and crack tip velocities $\dot{a}$. Therein, a Perzyna-type constitutive equation is used to describe elastic-viscoplastic material behavior. An analytical examination results in a system of nonlinear ordinary differential equations, which is solved by a multiple shooting algorithm. Since the system is stiff and implicit, modified backward differentiation formulas up to fifth order are used to integrate the arising initial value problem. A special Nelder-Mead-Simplex method (NMS) optimizes in combination with Newton's method a residuum to satisfy the boundary conditions of stress free crack faces. Because NMS doesn't require any derivatives, the obtained numerical scheme is very stable and rather efficient. It provides the field quantities, so that further investigations related to the direction of crack propagation, the unloading zone, the viscoplastic strains and the external load in the case of small scale yielding can be performed.

To prove the significance of the crack tip fields the extension of the zone of dominance must be determined by solving boundary value problems. Therefore, a suitable finite element method is developed, which uses a mixed formulation for triangles (Kuessner et al (5)). Since the displacements and their derivatives are treated as independent variables, kinematics and balance equations are only satisfied in weak form. However, the formulation allows a Galilean transformation corresponding to that in the first part. A comparison of the results of both approaches shows a remarkable importance of the crack tip fields, which are derived from the semi-analytical examination.

CRACK TIP FIELDS

Basic Equations

The governing equations needed for the problem formulation are the equation of momentum balance, the kinematics and the material law:

$$\sigma_{ij,j} = \rho\, \ddot{u}_i \tag{1}$$

$$\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \tag{2}$$

$$\dot{\varepsilon}_{ij} = \dot{\varepsilon}^{e}_{ij} + \dot{\varepsilon}^{vp}_{ij} = \dot{\varepsilon}^{e}_{ij} + \frac{1}{\eta}\left\langle \frac{\sqrt{J_2'}}{k} - 1 \right\rangle^{n} \frac{1}{\sqrt{J_2'}}\, \sigma'_{ij} \quad ; \quad n \geq 1 \tag{3}$$

Here, ρ is the mass density, η a viskosity parameter, k the yield stress in shear and n the hardening exponent. σ'_{ij} denotes the deviatoric part of the stress tensor and J_2' its second invariant. As already mentioned, the time derivatives are eliminated by a Galilean transformation (Fig. 1):

$$\frac{\partial}{\partial \bar{x}} = \frac{\partial}{\partial x} \; ; \; \frac{\partial}{\partial \bar{y}} = \frac{\partial}{\partial y} \; ; \; \frac{\partial}{\partial \bar{t}} = -\dot{a}\frac{\partial}{\partial x} \tag{4}$$

A seperation technique yields a system of ordinary differential equations for the angular distributions of stresses and displacements

$$\mathbf{M}\,\frac{\mathrm{d}\mathbf{Y}}{\mathrm{d}\varphi}=\mathbf{f}\,(\mathbf{Y},\varphi)\quad,\quad\varphi\in[0;\pi]\quad. \tag{5}$$

If suitable dimensionless field quantities are introduced, the solution will depend only on the hardening exponent n, Poisson's ratio ν and the Mach number m, which is defined by $m = \dot{a}/c_s$. Therein, $\dot{a}$ is the crack tip speed and c_s the shear wave velocity.

Numerical Solution

The system of equations (5) is transformed into a nonlinear optimization problem which is solved numerically by a multiple shooting method. If an approximate value $Y_0 = Y(\varphi = 0)$ is known, the angular distributions of all field quantities can be obtained as the solution of an initial value problem. Evaluation of the residuum

$$R=\sigma^2_{\varphi\varphi}\,(\varphi=\pi)+\tau^2_{r\varphi}\,(\varphi=\pi)+\lambda\,(\pi-\varphi^*)^2\quad\longrightarrow 0\quad, \tag{6}$$

which describes the condition of stress free crack faces, leads in conjunction with an optimization method to an improved approximation Y_0 until $R < \varepsilon_R$. The penalty term in (6) prevents a break in the integration at a certain angle φ^* before the upper limit of the interval $0 \le \varphi \le \pi$ is reached. This might occur, if an insufficient starting vector Y_0 was chosen.

As an example results are presented for the parameters $m = 0.1; 0.4; 0.7$, $\nu = 0.25$ and $n = 9$. Figures 2, 3, 4 show the angular distributions of the circumferential stress, the equivalent stress and the viscoplastic part of the strains in plane stress. Further results are presented in (3).

FINITE ELEMENT INVESTIGATION

The examination is based on a weak formulation of the momentum balance equation (1) and the kinematics (2):

$$\int_\Omega \delta\mathbf{u}^T\nabla^{sym}\boldsymbol{\sigma}\,[\boldsymbol{\varepsilon}]\;d\Omega-\int_\Omega \delta\mathbf{u}^T\mathbf{f}\;d\Omega-\int_{\Gamma_t}\delta\mathbf{u}^T\mathbf{t}\;d\Gamma-\int_\Omega \rho\dot{a}^2\frac{\partial\left(\delta\mathbf{u}^T\right)}{\partial x}\frac{\mathbf{u}}{\partial x}\;d\Omega=0 \tag{7}$$

$$\int_\Omega \delta\boldsymbol{\varepsilon}^T\left(\boldsymbol{\varepsilon}-\nabla^{sym}\mathbf{u}\right)\,d\Omega=0 \tag{8}$$

Here, Ω is the considered domain and Γ_t the part of its boundary, where stress conditions must be satisfied. The body forces are denoted by $\mathbf{f}$ and the traction vector on Γ_t by $\mathbf{t}$. The function $\boldsymbol{\sigma}[\boldsymbol{\varepsilon}]$ is obtained by application of the Galilean transformation (4) to the constitutive equation (3). As can be seen, the inertia terms in (7) are already transformend and written in local coordinates. It should be mentioned, that it is possible to derive the weak formulation from a Hellinger-Reissner variational principle.

The considered domain is discretized by triangular elements to enable a simple division of Ω into circular subdomains around the crack tip. The advantage is, that radial or angular distributions of the field quantities result directly from the FE-calculation. The vectors of variables have the form

$$\mathbf{u} = (u/v)^T \quad , \quad \boldsymbol{\varepsilon} = (u_{,x}/v_{,y}/u_{,y}/v_{,x})^T \quad , \tag{9}$$

where $\boldsymbol{\varepsilon}$ must be the primary variable and $\mathbf{u}$ the constraint one for reasons of stability. Both are approximated by linear shape functions. The integration of the material law is performed by the Euler-backward formula.

To give an example of examination, we consider an infinite strip of fixed width. Due to the symmetry, only the upper half is discretized (Fig. 5). The boundary conditions are $v = 0$ on the ligament and $u = 0$ at the left border to get a well defined problem. The strip is loaded by a prescribed displacement $v = v_0$ on its upper boundary. Figure 6 presents the circumferential distribution of the equivalent stress σ_e for a constant crack tip velocity $m = 0.3$ and linear-elastic material behavior. Therein, the calculated stress has been divided by the well known $1/\sqrt{r}$ singularity, so that the fan-shaped zone around the tip approximately describes the region, where the crack tip field is dominating. The result shows its remarkable extension and significance.

REFERENCES

(1) Chang, T.C., Popelar, C.H. and Staab, G.H., Int. J. Frac., Vol. 33, No. 1, 1987, pp. 31-45.

(2) Deng, X., and Rosakis, A.J., Int. J. Frac., Vol. 58, No. 2, 1992, pp. 137-156.

(3) Heimer, S., Hohe, J. and Gross, D., Arch. Mech., Vol. 47, No. 5, 1995, pp. 899-914.

(4) Hui, C.Y., and Riedel, H., Int. J. Frac., Vol. 17, No. 4, 1981, pp. 409-425.

(5) Kuessner, M., Wriggers, P., Bernspang, L. and Samuelson, A., "Computational Plasticity", Pineridge Press, Swansea, England, 1995.

(6) Lo, K.K., J. Mech. Phys. Solids, Vol. 31, No. 4, 1983, pp. 287-305.

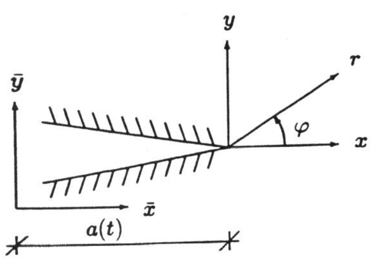

Figure 1
Galilean transformation

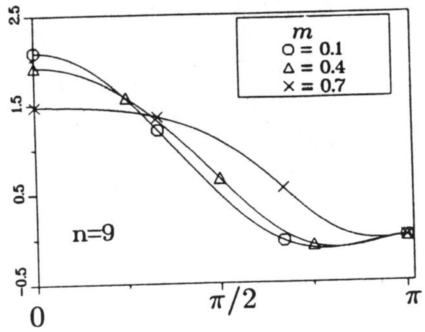

Figure 2
Circumferential stress $\sigma_{\varphi\varphi}$

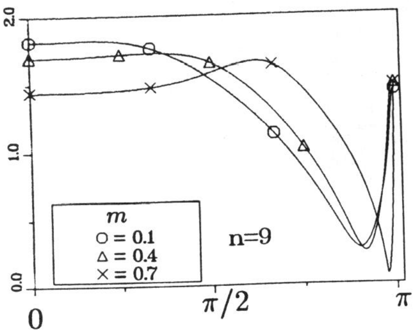

Figure 3
Equivalent stress σ_e

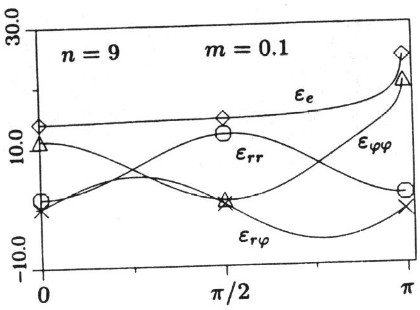

Figure 4
Viscoplastic strains

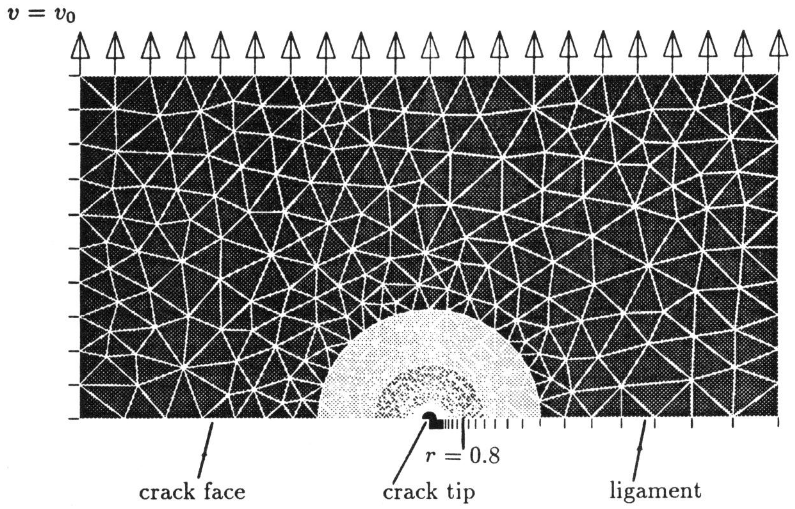

Figure 5
Finite element model

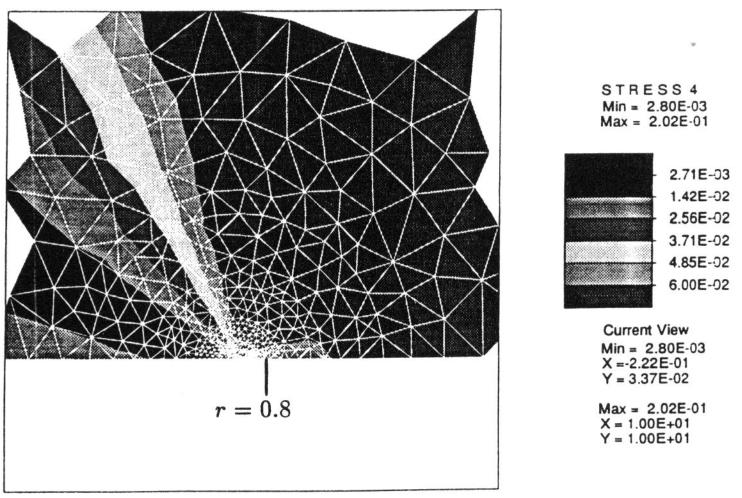

Figure 6
Equivalent stress σ_e times $\sqrt{r}$