CRACK GROWTH IN PRESSURE SENSITIVE ELASTIC-PLASTIC MATERIALS

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In this paper the dynamic effects are investigated connected with the steady-state stress and deformation fields in the vicinity of a rapidly propagating crack in a pressure sensitive elastic-plastic material. From the standpoint of fracture dynamics the stress and deformation fields are of particular interest in the vicinity of the crack tip. Pressure sensitive materials exhibit a yielding under hydrostatic stress [1-3]. The modelling of the pressure sensitive properties of the material was performed by the Drucker-Prager yield function in order to investigate the crack tip fields. Studies concerning pressure sensitive materials have been performed by Li [4] and Yuan and Lin [5] using the assumptions of the HRR-field theory and by Bigoni and Radi [6,7]. The latter investigated a quasistatic crack growth under mode I-loading conditions.

INTRODUCTION

In the case of a dynamic crack propagation, a large portion of the work of inelastic deformation near the crack tip is dissipated as heat. As a result of the rapid propagation the heat conduction from the crack tip is negligibly small. In this paper, the asymptotic stress, velocity and temperature crack tip fields for fast running cracks in an elastic-plastic, pressure-sensitive material are determined. The asymptotic stress and velocity fields were calculated from a corresponding boundary value problem considering the mathematical consequences of a mode I-loading and associated symmetry effects. Then, the asymptotic temperature field can be calculated directly from the received results of the stress- and velocity fields. Further, for the calculation of the asymptotic crack tip fields the incremental theory of plasticity was applied and stationary crack growth under mode I-loading and plane stress conditions has been adopted.

Crack tip surroundings for a dynamically propagating crack in an elastic-plastic material

Figure 1 shows a physical model of a dynamically propagating crack extending with the constant velocity v in a homogeneous elastic - plastic material under mode I-loading in a plane stress situation. The coordinate system whose origin is attached to the crack tip is shown in Fig. 1. The propagation of the crack takes place in x_1-direction. The crack tip region of the material exhibits an active plastic region in front of the crack tip which is sur-

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rounded by an elastic region. A plastic reloading region exists along the crack surfaces. By considering a mode I-loading situation at the crack tip, then the plastic reloading region is very small compared to the active plastic region. For a mode II-loading situation the crack tip can be surrounded by several elastic and plastic regions [8].

Governing equations

The presented crack problem can be described mathematically by the basic equations of continuum mechanics which consist of the equations of motion

$$\sigma_{ij,j} = \rho\, \ddot{u}_i \tag{1}$$

where ρ is the density of the material, and the incremental constitutive equations read [9]

$$\dot{\varepsilon}_{ij} = \dot{\varepsilon}^{(e)}_{ij} + \dot{\varepsilon}^{(p)}_{ij} = \frac{1}{E}\left\{(1+\nu)\,\dot{\sigma}_{ij} - \nu\,(\mathrm{tr}\,\dot{\sigma}_{ij})\,\delta_{ij}\right\} + \frac{1}{E}\left\{\frac{1}{h}\,Q_{ij}\,(Q_{kl}\,\dot{\sigma}_{kl})\right\}, \tag{2}$$

formulated in cartesian coordinates. In the eqs. (1) and (2) $\dot{(\;)}$ denotes the material time derivative

$$\dot{(.)} \approx -\,v\,\frac{\partial(.)}{\partial x_1} \tag{3}$$

for a steady-state crack extension along the x_1-direction. In eq. (2) $D_{ij}^{(e)}$ describes the elastic portion and $D_{ij}^{(p)}$ the associated incremental plastic portion of the deformation. Finally, Q_{ij} denotes the gradient tensor of the Drucker-Prager yield function

$$\psi(\sigma_{ij}) = \sqrt{J_{2,D}} + \frac{\mu}{3}\,J_{1,S}\;, \tag{4}$$

where $J_{1,S}$ represents the first invariant of the stress tensor and $J_{2,D}$ the second invariant of the deviatoric stress tensor. The yield function (4) reduces to the formulation of von Mises if μ is set to zero. For the plane stress case, the non-vanishing stress and velocity components in the eqs. (1) and (2) are σ_{11}, σ_{12}, σ_{22}, $\dot{u}_1$ and $\dot{u}_2$. This leads to a system of five independent nonlinear differential equations.

Construction of the differential equation system for a plane stress state

Due to the plane formulation of the crack problem the near tip fields are dependent of the two coordinates x_1 and x_2. Thus, in general the mathematical description results in a partial differential equation system of the functions $\sigma_{ij}(r,\theta)$ and $\dot{u}_i(r,\theta)$. By introducing characteristic first order asymptotic separable expressions in the coordinates r and θ for the stresses and velocities [10,11], the resulting differential equations based on the eqs. (1) and (2) can be formulated in θ only. The singular behaviour of the fields for $r\to 0$ is covered by the exponent $s<0$ of r in this expressions, where r is the distance with respect to the crack tip. By using the relations

$$\frac{\partial}{\partial x_1} = \cos(\theta)\frac{\partial}{\partial r} - \frac{\sin(\theta)}{r}\frac{\partial}{\partial\theta}\;,\quad \frac{\partial}{\partial x_2} = \sin(\theta)\frac{\partial}{\partial r} + \frac{\cos(\theta)}{r}\frac{\partial}{\partial\theta} \tag{5}$$

the eqs. (1) and (2) can be summarized in a first-order system of nonlinear ordinary differential equations

$$\mathbf{A}\left(\mathbf{x}(\theta),\, s\right)\,\frac{\partial\mathbf{x}(\theta)}{\partial\theta} = \mathbf{B}\left(\mathbf{x}(\theta),\, s\right) \tag{6}$$

where $\mathbf{A}(\mathbf{x}(\theta),s)$ denotes a matrix consisting of nonconstant functions of the solution vector $\mathbf{x}(\theta) := \left[\dot{U}_1(\theta)\,,\dot{U}_2(\theta)\,,\Sigma_{11}(\theta)\,,\Sigma_{12}(\theta)\,,\Sigma_{22}(\theta)\right]^T$.

Due to the strong nonlinearity of the governing equations the solution of this differential equation system is reached by a numerical method only.

The boundary and transition value problem

The elastic and plastic regions of the mathematical model of the crack tip surroundings (Fig. 2) are independent of the variable r. Thus, the transitions between the elastic and plastic regions can be described by the angles θ_{pl} and θ_{el} only. This corresponds with the ordinary differential equation system (6) which is formulated in the variable θ. The mathematical transition conditions

$$Q_{ij}\ \left(Q_{kl}\ \dot{\sigma}_{kl}\right) = 0 \qquad (7)$$

for θ_{pl} and

$$\sigma_e\left(r,\theta_{el}\right) \geq \sigma_e\left(r,\theta_{pl}\right) \qquad (8)$$

for θ_{el} are graphically presented in Figs. 3 and 4 and marked with the points P_1 and P_2, respectively. The transition angles are specified by the crack tip velocity and the material behaviour and are results of the differential equation system (6). The whole crack problem is enclosed by the positions θ=0° and θ=180°. The boundary values are stated by zero stresses on the crack surfaces for θ=180° and the symmetry of the crack planes due to a mode I-loading situation.

The asymptotic crack tip temperature field

In the case of a dynamic crack propagation, a large portion of the work of the inelastic deformation near the crack tip is dissipated as heat. As a result of the high crack tip speed the heat conduction at the crack tip is rather small and nearly negligible [12]. This fact results in a steep increase of the temperature $\tilde{T}$ at the crack tip, which can be calculated from the equation of adiabatic overheating

$$\vartheta\,\sigma_{ij}\,\dot{\varepsilon}^p_{ij} \;=\; \rho\,c\,\dot{\tilde{T}}, \qquad (9)$$

where ϑ is the constant fraction of plastic work converted into heat, c the specific heat and ρ the density of the material. If the asymptotic stress field σ_{ij} and the plastic work rate $\dot{\varepsilon}^p_{ij}$ are known, the asymptotic crack tip temperature field $\tilde{T}(r,\theta)$ can be calculated directly from (9). Due to the behaviour of the stress and velocity fields in the vicinity of the crack tip, the characteristic asymptotic first order function

$$\tilde{T}(r,\theta) = \ K^2\ E\ \ r^{2s}\ T(\theta) \qquad (10)$$

is introduced for the temperature field. Then by using eq. (5) the relation (9) leads to the ordinary differential equation

$$\sin\theta\ \ T'(\theta)\ \ = p\ \vartheta\ g(\theta)\ \ +\ 2\ s\ \cos\theta\ T(\theta) \qquad (11)$$

where p and g(θ) are defined as

$$p := \frac{1}{c\,\rho}\ ,\qquad g(\theta) := f\left(\Sigma_{ij}(\theta)\ ,\ \dot{U}^{(p)}_i(\theta)\ ,\ s\right). \qquad (12)$$

Equation (11) is valid for the plastic regions as well as for the elastic region without consideration of any transition conditions.

Due to the symmetry of the temperature field for $\theta=0°$ the initial values can be calculated from

$$T(\theta = 0°) = -\frac{p\,\vartheta\, g(\theta = 0°)}{2\,s}\ , \quad T'(\theta = 0°) = 0\,. \tag{13}$$

Now the initial boundary value problem (11) is solved numerically using the results for the stresses $\Sigma_{ij}(\theta)$ and velocities $\dot{U}_i(\theta)$ from the boundary value problem (6).

Numerical solution and results

Due to the strong nonlinearity of eq. (6) a solution can be obtained in a numerical manner only. Here, a solution is attained by using the multiple shooting method [13]. Figures 5 and 6 give the singularity exponent s in dependence on the normalized crack tip velocity β_t and with the pressure sensitivity μ and the hardening coefficient α as parameters. Finally, the Figs. 7 and 8 show the asymptotic temperature function $T(\theta)$ induced by dynamic crack growth for varying pressure-sensitivities μ, hardening coefficients α, and the normalized crack tip velocity $\beta_t = 0.7$.

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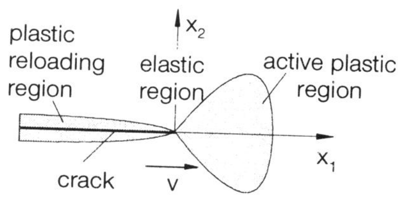

Fig. 1: Crack tip geometry for a rapidly propagating crack

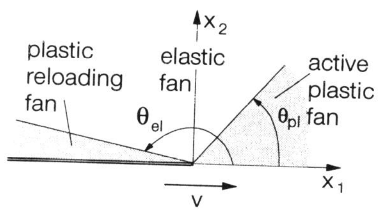

Fig. 2: Mathematical model of the crack tip surroundings for a rapidly propagating crack

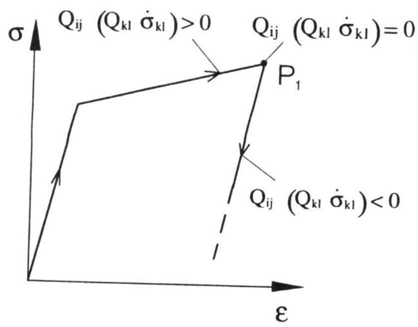

Fig. 3: Transition condition for θ_{pl}

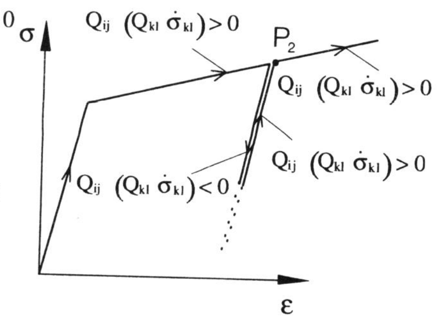

Fig. 4: Transition condition for θ_{el}

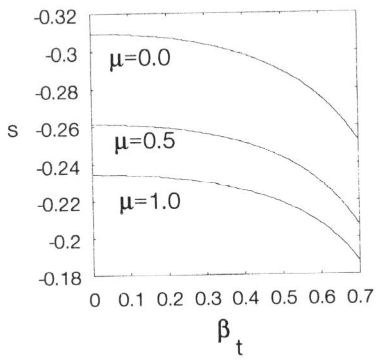

Fig. 5: Singularity exponent s for α=0.2

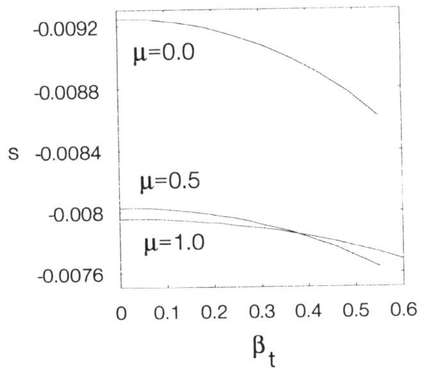

Fig. 6: Singularity exponent s for α=0.0001

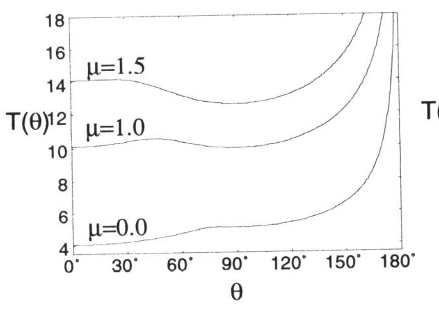

Fig. 7: Temperature T(θ) for α=0.1, β_t=0.7

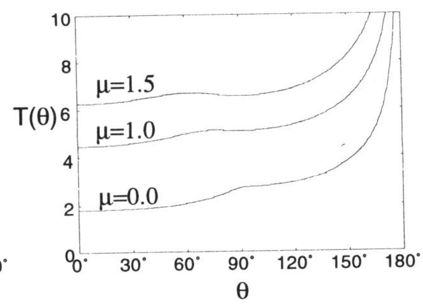

Fig. 8: Temperature T(θ) for α=0.2, β_t=0.7