CONCEPTION OF COHESIVE FORCES IN NON-LINEAR FRACTURE MECHANICS

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The attempt to construct a resistance curve for elastic-plastic material theoretically was made in this work. The cohesive forces theory created by Barenblatt is the foundation of this investigation. The algorithm of the solution of non-linear fracture mechanics problem for slow growing crack and the results of calculations coinciding with experimental data satisfactorily are represented.

It is known that crack growth has been begun on condition that $J \geq J_{IC}$ where J is value of J-integral (reference (1)) and J_{IC} is strength constant of material. A domain of stable growth is typical for cracks extending in elastic-plastic continuum. Value of J-integral increases in several times there (reference (2)).

Theoretical finding of dependence $J = J(\Delta a)$ (resistance curve) where Δa is increment of crack length, by traditional methods of fracture mechanics is impossible apparently. The point is that crack tip moving, the singularity of stress and (or) deformation fields moves too. Any correct numerical algorithms simulating this process in conformity to elastic-plastic continuum are not known.

However, this singularity is not property of physical reality but attribute of its mathematical description. Cohesive forces (forces attracting opposite edges of crack) being introduced for consideration, one can construct stable growth crack theory for elastic-plastic materials. The crack tip is not singular point of stress and (or) deformation fields in that theory.

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Therefore the problem of theoretical finding of resistance curve has been introduced to the set of elastic-plastic problems being solved by one of elastic solution method variety.

Fracture mechanics considering action by cohesive forces had been created by Barenblatt (3) (its special case had been considered independently by Leonov and Panasyuk and a little later by Dugdale). He postulated the following properties of that forces:

1. values of cohesive forces and external loads are such that stresses are finite in the crack tip;
2. crack growing, the configuration of crack edges in cohesive zone is invariable (cohesive zone is region where action by cohesive forces is essentially);
3. cohesive zone length is much smaller than crack length.

Barenblatt proved that the value of load with which crack grows in elastic material is independent from cohesive force distribution and is equal to value of load calculated without regard to their action. Thus Barenblatt's theory is physical consistent proof for linear fracture mechanics.

The aim of this work has been to find resistance curves using cohesive forces conception. The problem has been considered in the simplest setting: stable growth of rectilinear semi-infinite normal discontinuity crack in elastic-plastic plane has been investigated (fig. 1). There are cohesive forces **G** on $x \in [-b,0]$ part and external loads **P** in $x = -c$ point at crack edges. Let $c >> b$ and, moreover, let c is much more than the largest linear size of plastic domain around crack tip. In this case J-integral is computed by linear fracture mechanics formulas.

According by experiments (reference (2)), crack tip is blunted at first and is narrowed later when crack grows stable. Therefore it is necessary to refuse the postulate 2 in this case. The relations resolving that problem without detailed description of distribution by cohesive forces cannot be found, apparently. Therefore it is necessary to determine the law of state for those forces. They are expected to have the potential (reference (4)). In this case the simplest form of that law is (reference (5))

$$G = G_M(1 - \Delta/\Delta_M) \ , \qquad \Delta \in [0, \Delta_M]$$
$$G = 0 \ , \qquad \Delta > \Delta_M \qquad (1)$$

where Δ - distance between crack edges, G_M - destroying stress for undefective material (reference (6)) (it is strength material constant complementary to J_{IC}); $\Delta_M = 2J_{IC}/G_M$ - value of Δ in $x = -b$ point.

The simplest setting of the problem predetermines sampling of the simplest rheology model: the material is assumed incompressible, ideal plastic, been in plane strain state. Elastic deformation is subjected to Hooke's law and plastic deformation is subjected to von Mises's law

$$\varepsilon_{mn} = \varepsilon^e_{mn} + \varepsilon^p_{mn} \ ; \quad \sigma_{mn} = \sigma\delta_{mn} + 2E\varepsilon^e_{mn}/3 \ ; \quad \dot{\varepsilon}^p_{mn} = \lambda(\sigma_{mn} - \sigma\delta_{mn}) \ ;$$
$$\lambda \geq 0 \ ; \qquad (\sigma_{mn} - \sigma\delta_{mn})(\sigma_{mn} - \sigma\delta_{mn}) = 2\sigma_Y^2/3 \qquad (2)$$

Here ε_{mn} - deformation (its elastic and plastic components are marked by e and p indexes), σ_{mn} - stress, $\sigma = \sigma_{mm}/3$ - mean stress, δ_{mn} - Kronecker delta, E - Young's module, σ_Y - yield point, λ - indeterminate multiplier yield being found from plastic yield criterion, velocity of some magnitude is marked by point over symbol.

In this case (reference (1))

$$J = 3P^2/(2\pi cE) \quad \text{.......(3)}$$

where value P is not arbitrary: it is calculated according with the postulate 1. J-integral determined by formula (3) can be represented as function of material parameters and crack length increment. The analysis of dimensionalities using, one can result

$$J^* = J^*\left(G_M^*, \sigma_Y^*, \Delta a^*\right) \quad \text{.......(4)}$$

where an asterisk designates dimensionless values:

$$J^* = J/J_{IC} \; ; \qquad G_M^* = G_M/E \; ; \qquad \sigma_Y^* = \sigma_Y/E \; ; \qquad \Delta a^* = \frac{\sigma_Y^2}{EJ_{IC}} \Delta a \quad \text{.........(5)}$$

The calculating algorithm permitting to find the dependence (4) is considered. Elastic-plastic problem is solved by initial stress method. Let some value of load parameter being, there are stresses σ_{mn0} in a body, crack edges displaced from each other on $\Delta_0(x)$ are loaded by cohesive forces $\mathbf{G_0}(x)$ on $x \in [-b,0]$ and external loads $\mathbf{P_0}$ in point $x = -c$. These values get small increments with small increment of load parameter. If increments of stresses are known then further loads $\mathbf{p}(x)$ on crack line are defined by Green's function constructed with the solution of plane Kelvin's problem (reference (7)). It is possible to prove that relations (1) being fulfilled, computation of cohesive forces distribution is reduced to solution of Fredholm's integral equation

$$G^*(\theta) - \chi \int_0^{\psi} G^*(\varphi) \tan\frac{\varphi}{2}\left(1+\tan^2\frac{\varphi}{2}\right) \ln\left|\frac{\sin\frac{\theta+\varphi}{2}}{\sin\frac{\theta-\varphi}{2}}\right| d\varphi = -2\chi q \tan\frac{\theta}{2} + 1 - \Delta_0^*(\theta) +$$

$$+2\chi q_0 \tan\frac{\theta}{2} - \chi \int_0^{\pi} \left[G_0^*(\varphi) + p(\varphi)\right] \tan\frac{\varphi}{2}\left(1+\tan^2\frac{\varphi}{2}\right) \ln\left|\frac{\sin\frac{\theta+\varphi}{2}}{\sin\frac{\theta-\varphi}{2}}\right| d\varphi \quad \text{..........(6)}$$

where following notations are introduced

$$\theta = 2\arctan\sqrt{-x/b} \; ; \qquad G^* = G/G_M \; ; \qquad \chi = \frac{3G_M^2 b}{2\pi EJ_{IC}} \; ; \qquad q = \frac{P}{G_M\sqrt{bc}} \; ;$$

$$q_0 = \frac{P_0}{G_M\sqrt{bc}} \; ; \qquad \Delta_0^* = \frac{G_M}{2J_{IC}} \Delta_0 \quad \text{........................(7)}$$

The values of ψ and q containing in equation (6) are unknown with $G^*(\theta)$ function. They are found by two further conditions. First condition is implication from first Barenblatt's postulate. It is reduced to form

$$\int_0^{\psi} G^*(\varphi)\left(1+\tan^2\frac{\varphi}{2}\right)d\varphi - q = \int_0^{\pi}\left[G_0^*(\varphi)+p(\varphi)\right]\left(1+\tan^2\frac{\varphi}{2}\right)d\varphi - q_0 \quad \text{........(8)}$$

Second equation expresses requirement of becoming zero by cohesive forces in $\theta=\psi$ point

$$G^*(\psi)=0 \quad \text{..(9)}$$

The solution of equation (6) is found easily with use of known expansion

$$\ln\left|\frac{\sin\frac{\theta+\varphi}{2}}{\sin\frac{\theta-\varphi}{2}}\right| = 2\sum_{n=1}^{\infty}\frac{\sin n\theta \sin n\varphi}{n} \quad \text{............................(10)}$$

Its substitution to equation (6) makes possible to represent it with any accuracy as degenerated kernel equation having simple solution method.

As a result of solution of equations (6) - (9) new value b, increments of displacement of crack edges, increments of cohesive forces, external loads and stresses are found after that next iteration is made and so until convergence conditions will have been fulfilled. After wards next small increment of load parameter is made.

Fixed crack is initial state of growing crack. As fixed crack is in elastic-plastic material, successive loading process transforming fixed crack not having plastic strain in neighbourhood of crack tip, to fixed crack having it is necessary. Pure elastic deformation is got with not large values of σ_Y compared to G_M. Small increments of G_M parameter giving (G_M is parameter of load in that case), it is possible to get cracks having larger and larger plastic zone around of crack tip. Two variants of algorithm construction are possible in that case: to increase G_M with constant value of b (values of J_{IC} and P are calculated) or with constant value of J_{IC} (values of b and P are calculated). The calculations have shown that these variants are equivalent.

After the fixed crack has been obtained the process of its growth is simulated. Δa is load parameter now, G_M and J_{IC} are fixed, b and P are calculated.

Main results of calculations are shown on fig. 2. The calculations has been fulfilled with following input data: $\sigma_Y^* = 2.5\cdot 10^{-3}$ for all variants; value of G_M/σ_Y was changed in the following way: G_M/σ_Y =4 for variant 1 (curve 1 in fig. 2), G_M/σ_Y =5 for variant 2 (curve 2 in fig. 2), G_M/σ_Y =6 for variant 3 (curve 3 in fig. 2).

The graphs represented in fig. 2 are shown that the value of J for fixed crack is not depended from G_M and is equal to J_{IC} . This result agrees with J-integral theory (1) that is correct for fixed cracks. All three curves behave identity: Δa^* growing, J^* grows until maximum. This part of the crack growth is desired resistance curve. The descending branch of curve corresponds to quick propagation of crack and that process is not described by this theory. The obtained curves agree with experimental data (reference (2)).

Crack stable growth process is not bound to continue until maximum value of J^*. The quick propagation of crack can begin previously when appropriate power conditions will have been created. Their consideration falls outside the limits of this investigation.

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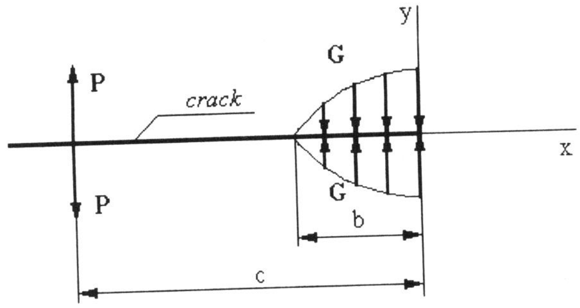

Figure 1 Calculation diagram

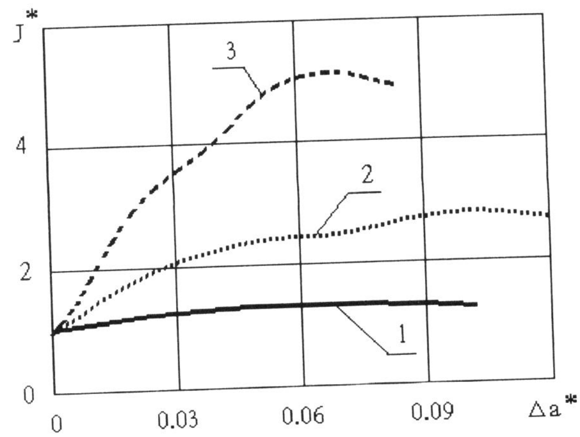

Figure 2 Resistance curves