COMPARISON OF CONSTRAINT MEASURING PARAMETERS IN FRACTURE THEORIES FOR DUCTILE MATERIALS

B.S. Henry and A.R. Luxmoore*

Ductile fracture of metals occurs as a result of nucleation, growth and coalescence of microscopic voids. The main parameters that control void growth are the triaxiality factor, σ_m/σ_v, and the plastic strain. Recently, two-parameter theories (e.g. *J*-Q) have been proposed to quantify crack front constraints and to predict the effects of constraint on macroscopic fracture toughness values. In this paper we use finite element models of low constraint geometries to study the variation of σ_m/σ_v, plastic strain and Q-value with deformation. Our numerical results show that, for a given material, there exists a unique relationship between σ_m/σ_v and Q that is independent of specimen geometry, dimensions, crack depth and deformation level. It can be concluded that σ_m/σ_v and Q are equivalent constraint parameters.

INTRODUCTION

Ductile fracture of metals occurs as a result of nucleation, growth and coalescence of microscopic voids that initiate at inclusions and second phase particles. The main parameters that influence void nucleation and growth, and hence ductile fracture, are the triaxiality factor and the plastic strain. The triaxiality factor is defined as the ratio of the mean hydrostatic stress, σ_m, to the von Mises stress, σ_v.

Recent advances highlight the loss of *J*-dominance in low constraint geometries and the importance of using two-parameter theories (e.g. *J*-Q) to characterise near crack front states of yielded crack geometries. The *J*-Q theory, after O'Dowd and Shih (1), is of particular interest as Q is defined as a triaxiality parameter (1), and it parameterises the strain field when distances ahead of the crack front are normalised by J/σ_{ys} (where σ_{ys} is the yield stress). Following this work (1), the Q-value can be defined as

---

* Department of Civil Engineering, University of Wales, Swansea

$$Q = \frac{\sigma_m - (\sigma_m)_{SSY;T=0}}{\sigma_{ys}} \quad \text{at} \quad \theta = 0,\ r = \frac{2J}{\sigma_{ys}}, \qquad \ldots\ldots\ldots \qquad (1)$$

where r and θ are polar coordinates situated at the crack front, and $(\sigma_m)_{SSY;T=0}$ is termed the reference field and is defined as the standard plane strain small scale yielding (SSY) hydrostatic mean stress solution which is driven by the stress intensity factor, K, alone.

The Q is proposed as a common scale to interpret brittle and ductile fracture, as it quantifies both the hoop and the mean stress relative to a reference stress state (1). In this paper we study the variation of σ_m/σ_v, ε_v and Q with the deformation level using three-dimensional (3-D) elastic-plastic finite element (FE) models of low constraint geometries. The specimens studied are three-point bend (TPB) bars ($0.05 \leq a/w \leq 0.5$, where a is the crack length and w is the specimen width) and centre cracked (CCT) panels ($0.33 \leq a/w \leq 0.77$) in tension with material properties typical of a high strength aluminium alloy. Comparison between σ_m/σ_v, ε_v and Q are made at different distances ahead of the crack front. The main aim of this paper is to study the applicability of the Q as a ductile fracture parameter, and its relationship with the triaxiality factor.

NUMERICAL MODELLING OF THE SPECIMENS

The specimens studied in the present paper were part of a comprehensive experimental and numerical program carried out by the authors and fully detailed in (2) to study low constraint geometries. The 3-D models were meshed using reduced integration twenty-noded isoparameteric brick elements degenerated at the crack front to wedge elements. The nodes at the crack front were coincident but independent. All models shared the same “spider web” mesh surrounding the crack front. The smallest element size was nearly a/1000. There were six element layers through half the model’s thickness. Due to symmetry in boundary conditions and loading, only 1/8 (1/4) of CCT (TPB) specimens were modelled. In applying a remote tensile loading, uniform displacement was imposed on all specimens.

THE USE OF STRESS TRIAXIALITY AS A CONSTRAINT PARAMETER

The σ_m/σ_v field ahead of the crack front at different deformation levels in the shallow TPB specimen No. 4 (a/w=0.05) and deep specimen No. 21 (a/w=0.5) is shown in Figures 1 and 2 respectively. The stress field values were evaluated at the mid-thickness (plane of symmetry) of corresponding specimens. The HRR field was calculated using the tabulated constants given by Shih (3). The stresses are non-dimensionalised by σ_{ys} while the distance, r, of a point ahead of the crack front is non-dimensionalised by J/σ_{ys}.

The results plotted in Figure 1 shows that, for shallow cracked TPB, σ_m/σ_v is a function of r at all finite distances from the crack front. The triaxiality fields fall below the HRR field, with σ_m/σ_v decreasing as the distance from the crack front increases. As the deformation level increases, σ_m/σ_v constraint decreases due to progressive relaxation of the stresses in the crack front region. In contrast, Figure 2 (TPB No. 21) shows that, for deeply cracked TPB, σ_m/σ_v fields ahead of the crack front can be characterised by the HRR. The HRR field is within 10% of the actual triaxiality field at $rJ/\sigma_{ys} = 1$, even at large deformation levels. Those results show that σ_m/σ_v can be used as a measure of constraint. Low constraint geometries show lower σ_m/σ_v than that given by the HRR, with the discrepancy increasing with increasing deformation level and distance ahead of the crack front. In contrast high constraint geometries show σ_m/σ_v values that agree within 10% of the HRR field at $rJ/\sigma_{ys} = 1$, at different deformation levels.

VARIATION OF THE PLASTIC STRAIN AND TRIAXIALITY WITH Q

Figures 3 and 4 show the variation of the equivalent plastic strain (PEEQ) with Q at $rJ/\sigma_{ys} = 2$ for TPB specimens No. 4 and 21 respectively. Figure 3 shows that the plastic strain increases and shifts towards the forward sector as Q becomes more negative. For positive Q-values shown in Figure 4, the plastic strain remained virtually the same over the entire angular function, θ, range. For low constraint geometries (TPB No. 4) the PEEQ value is much higher than high constraint geometries (TPB No. 21). In all cases the strain fields strongly resemble members of the Q-family given by O'Dowd and Shih (4).

Figures 5 and 6 show the variation of σ_m/σ_v with Q at $rJ/\sigma_{ys} = 2$ for the TPB specimens No. 4 and 21 respectively. The σ_m/σ_v factor shows similar variation with θ for all geometries studied. The σ_m/σ_v is a maximum at $\theta=0$ and then decreases steadily. For low constraint geometries, Figure 5, σ_m/σ_v decreases with increasing deformation level. Figure 6 shows that for high constraint geometries, σ_m/σ_v remains virtually constant for the different values of deformation level. The σ_m/σ_v fields presented here closely resemble these of (4). From Figures 3 and 5 we can conclude that for negative Q, σ_m/σ_v decreases while PEEQ increases ahead of the crack front as plasticity develops. In this section we justified the use of Q as a constraint parameter and demonstrated that both PEEQ and σ_m/σ_v are parameterised by Q.

THE UNIQUE RELATION BETWEEN THE TRIAXIALITY FACTOR AND Q

O'Dowd and Shih (1) stated that Q is a measure of triaxiality, but no clear relation between Q and σ_m/σ_v is given in the literature. To examine this relation further, Figure 7 shows a plot the variation of Q with σ_m/σ_v for the different specimens studied in (2). Both Q and σ_m/σ_v were calculated at mid-thickness at a distance of $rJ/\sigma_{ys} = 2$ ahead of the crack front. The relation between Q and σ_m/σ_v is linear, and for a given material, is unique; in other words it is independent of specimen geometry (CCT and TPB), dimensions, crack depth and deformation level. This relation can be presented in the form:

$$\frac{\sigma_m}{\sigma_v} = \left(\frac{\sigma_{ys}}{\sigma_v}\right) Q + \left(\frac{\sigma_{ys}}{\sigma_v}\right)\left(\frac{\sigma_m}{\sigma_{ys}}\right)_{SSY;T=0} . \qquad \text{........} \qquad (2)$$

Equation (2) gives σ_m/σ_v results within 8% of the triaxiality factor values evaluated from the numerical models. Equation (2) is valid within the *J*-Q annulus ($1 \leq 2J/\sigma_{ys} \leq 5$) with the slope remaining virtually constant and the intercept changing by an amount equivalent to the difference in value of the constant reference field, $(\sigma_m/\sigma_{ys})_{SSY;\,T=0}$, at the corresponding distances.

CONCLUSIONS

In this paper we have demonstrated that both PEEQ and σ_m/σ_v are parameterised by Q. A linear relationship between σ_m/σ_v and Q that is unique has been identified. This relationship is independent of specimen geometry, dimensions, crack depth and deformation level. The equation gives results within 8% from those evaluated using the FE and is valid within the *J*-Q annulus. The ability of Q to characterise both PEEQ and σ_m/σ_v and the unique Q-σ_m/σ_v relationship validates the use of Q as a ductile fracture parameter.

REFERENCES

(1) O'Dowd, N.P. and Shih, C.F., "Two-parameter Fracture Mechanics: Theory and Applications", ASTM 24th Symposium on Fr. Mech.., Tennessee, 1992.

(2) Henry, B.S., Luxmoore, A.R. and Sumpter, J.D.G., "Experimental and Numerical Analysis of Low Constraint Geometries", Submitted for Publication.

(3) Shih, C.F., "Tables of HRR Singular Fields Quantities", Materials Research Laboratory, MRL E-147, Brown University, 1983.

(4) O'Dowd, N.P. and Shih, C.F., J. of Mech. and Physics of Solids, Vol. 40, 1992, pp. 939-963.

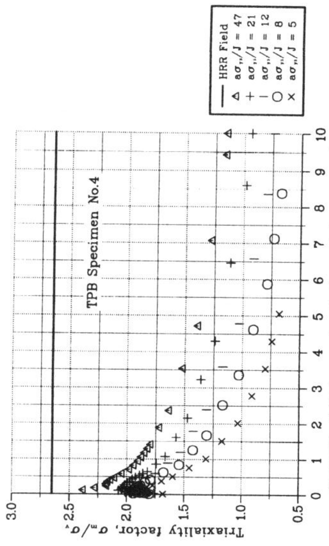

Figure 1 Variation of σ_m/σ_v with rJ/σ_{ys} for TPB No. 4.

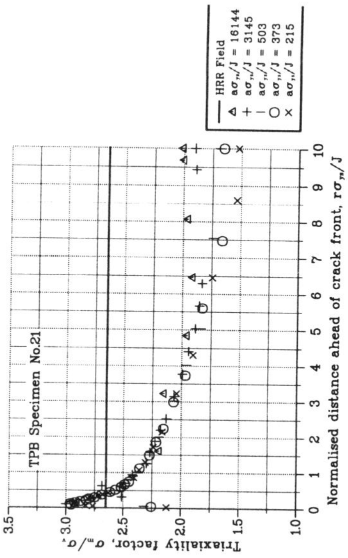

Figure 2 Variation of σ_m/σ_v with rJ/σ_{ys} for TPB No. 21.

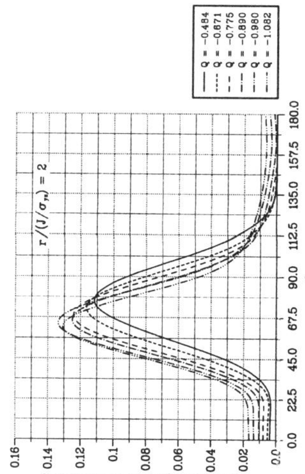

Figure 3 Variation of PEEQ with θ for TPB No. 4.

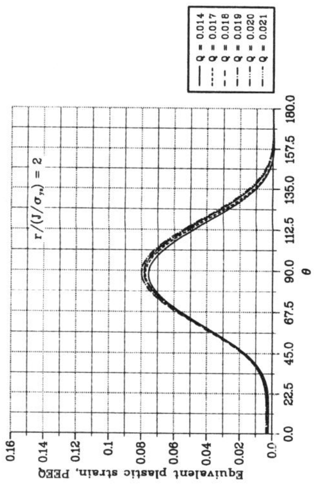

Figure 4 Variation of PEEQ with θ for TPB No. 21.

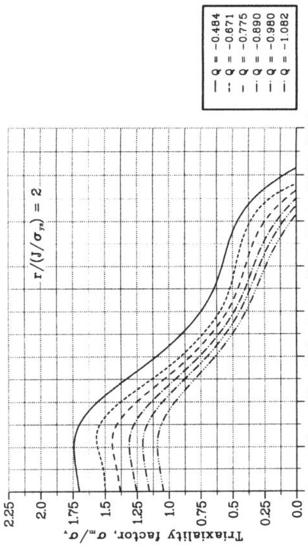

Figure 5 Variation of σ_m/σ_v with θ for TPB No. 4.

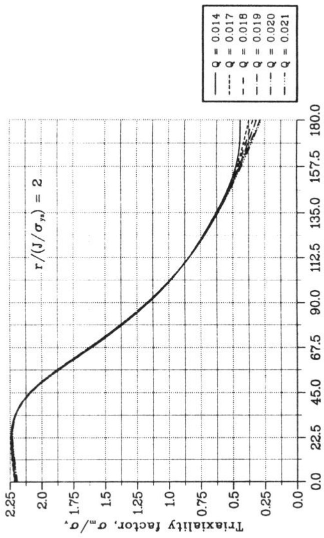

Figure 6 Variation of σ_m/σ_v with θ for TPB No. 21.

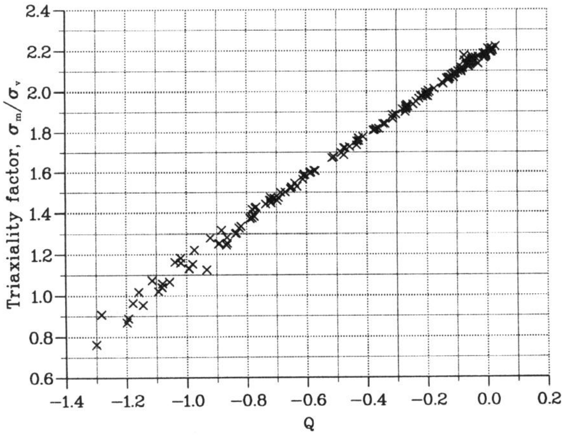

Figure 7 Variation of the triaxiality factor with the Q-value for the different TPB and CCT specimens studied in (2), at different deformation levels, 3-D analyses.