CALCULATION OF NOTCH ROOT STRESS-STRAIN OF A STEERING ARM

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This paper compares techniques available for the prediction of local elastic/plastic stresses and strains at the notch root of engineering components. A steering arm of a 32 ton truck has been finite element analysed to determine the non-linear response of the component material at a notch root. The elastic stress/strain fields were established and incorporated into both an energy density and Neuber's approach to predict the notch root elastic-plastic strains. The calculated results were then used to plot a number of load-local strain curves for the development of a load history, suitable for fatigue life assessment.

INTRODUCTION

Several techniques are available for establishing the non-linear response of component materials necessary for fatigue life evaluation [1,2]. These include elastic-plastic F.E.A., strain gauge measurement, and approximate solutions based on uni-axial stress-strains, such as Neuber's rule or the energy density method [3,4,5,6]. Although elastic F.E.A. is commonly used to assess the structural integrity of complex components, the application to plasticity, is prohibitively expensive. Post-yield strain gauges can also be used to measure the elastic-plastic strains, however this method relies on prototype component availability. An alternative approach is based upon the extension of uni-axial static yield criteria to obtain an equivalent stress-strain in a complex stress field. When using these techniques the multi-axial stresses are reduced to three principal stresses, the static yield criterion is then applied to give a single equivalent stress, σ_{eq}, or strain, ε_{eq},.

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To find the non-linear response at the notch root, the equivalent stress-strains are used with a numerical method such as Neuber's or the strain energy concept proposed by Glinka.

Neuber's Rule:- Neuber states that the theoretical stress concentration factor, k_t. is the geometric mean of the stress and strain concentration factor, (Fig 1) [2,4].

$$k_t^2 = k_\sigma k_\varepsilon \qquad (1)$$

where: $k_\sigma =$ *Local stress concentration factor*

$k_\varepsilon =$ *Local strain concentration factor*

If the nominal conditions away from the notch are elastic then:

$$\frac{(k_t.s)^2}{E} = \sigma.\varepsilon \qquad (2)$$

The cyclic stress-strain relationship may be defined as:

$$\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{k'}\right)^{\frac{1}{n'}} \qquad (3)$$

where: k' = cyclic strength coefficient and n' = cyclic strain hardening exponent. The notch root stress-strain response may be determined combining equ'ns (2) and (3); Fig 2 shows the graphical solution of the above equations.

Glinka Method:- Glinka postulated that the strain energy density at the notch is k_t times the nominal strain energy density [5] as shown in Fig 3.

The strain energy due to nominal stress may be defined as:

$$W_s = \frac{S^2}{2E} \qquad (4)$$

where: W_s = the elastic strain energy/unit volume due to the nominal stress, S.

The strain energy at the limit of proportionality (yield point) may be defined as:

$$W_\sigma = \frac{\sigma^2}{2E} \qquad (5)$$

where: $W_\sigma =$ strain energy/unit volume due to the notch root yield stress, σ.

If the notch root stress-strain is higher than the limit of proportionality (plastic range), the strain energy corresponds to the area under the stress-strain curve as shown in Fig 3. This area may be defined as:

$$W_s = \int_o^{E_e} E\varepsilon_e . d_{\varepsilon_e} + \int_o^{\varepsilon_p} k'(\varepsilon_p)^{n'}(d\varepsilon_p) \qquad (6)$$

integrating equ'n (8) yields:

$$\frac{(S.k_t)^2}{2E} = \frac{\sigma^2}{2E} + \frac{\sigma}{n'+1}\left(\frac{\sigma}{k'}\right)^{\frac{1}{n'}} \qquad (7)$$

Equ'ns (7) and (3) may be solved using iteraion methods to obtain the notch root stress-strain values for a given value of nominal stress.

Finite Element Analysis

A full 3-D elastic F.E.A. of the steering arm (Fig. 4) was carried out to identify the most critically stressed areas of the component, (Fig 5). A point load of 20 kN was applied to the component., giving nodal surface stress results at the notch root as shown in Fig 6, which were reduced to an equivalent stress and strain (σ_{eq} , ε_{eq}) using a variety of yield criteria. Both Neuber's rule and the energy density approach were used in conjunction with monotonic stress-strain curve to calculate the elastic-plastic strains for load increments of 10 kN from 10 kN to 70 kN; the results are shown in Fig 7.

Static Measurement of Steering Arm Strains

The static testing of the component was undertaken to verify the accuracy of elastic-plastic strains estimated as above. Two post yield strain gauges were attached to the arm to coincide with the plane of maximum principal strain identified using rosette strain gauges. Monotonic loads were applied in increments of 10 kN up to a maximum load of 70 kN and the strain values were recorded at each load increment, as shown in Fig 7. Plastic collapse of the component occurred when a load of 80 kN was applied.

Comparison of Measured and Estimated Strains

For an applied load of 20 kN the F.E.A. gave a maximum strain value of 1571 micro-strain (node 12). For the same load the measured strain was 1425 micro-strain. Thus, the F.E.A. results were 9% higher than the measured strains in the elastic range Fig 7.

The estimated strains based on both Neuber's rule and energy density method are compared to measured strains in Fig 7. It is apparent that for low values of load, both methods give results which are very close to the measured strains. For high values of load (plastic range) the strain values obtained from both methods deviate from the measured strain values. As the amount of plasticity increases the degree of correlation between measured and Neuber's estimated strains decreases. A range of 6494 (von Mises) and 7077 (Principal stress) is at least 15% higher than the measured strain value of 5668 micro-strain. However, the degree of correlation between the measured and energy density predicted strains remain of the same order of magnitude throughout the elastic and plastic range. A measured value of 5668 micro-strain compares well with the energy method upper and lower values of 5882 and 6070 micro-strain.

Fig 8 shows a plot of measured strain versus estimated strains for both methods. Both Neuber's and the energy based method fall above the 100% correlation line in all cases, showing both methods to overestimate the strain values in both the elastic and plastic range. It must be appreciated that all measured strains are likely to be lower than real peak stresses since measurements are averaged over a small but finite area. Further details of the experimental programme are shown in reference[6].

CONCLUSIONS

i. The predicted strains from linear elastic F.E.A. together with Neuber's rule and the energy method are overestimated in both the elastic and plastic ranges when compared to measured strains.

ii. For low values of nominal stress both methods give results very close to the measured strains. For high nominal stress values, both methods overestimate the local strains, however, better correlation is achieved with the energy approach.

iii. The local plastic strains estimated using energy density method and von Mises equivalent stress exhibit closest correlation to the measured strains. Those estimated using Neuber's equation and equivalent principal stress exhibit least correlation.

Hence it has been shown that for components under complex multi-axial stress state the above equations are an effective and quick method of finding the much needed inelastic strains for durability assessment during the conceptual design stage.

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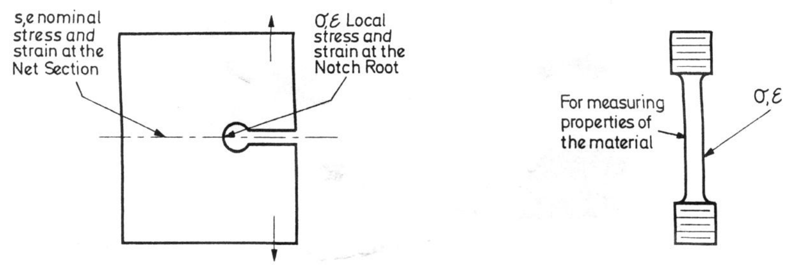

Fig.1 Nominal and local stress and strain

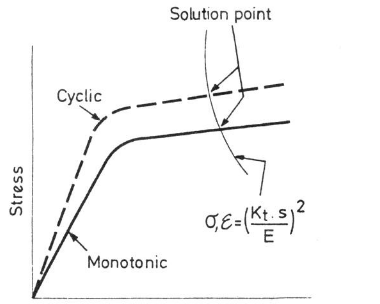

Fig.2 Solving the Neuber Equation

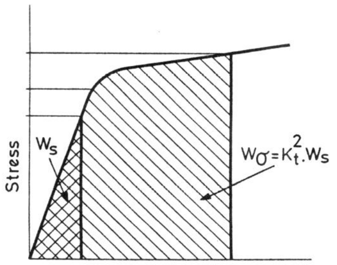

Fig.3 Energy interpretation of the theoretical stress concentration factor

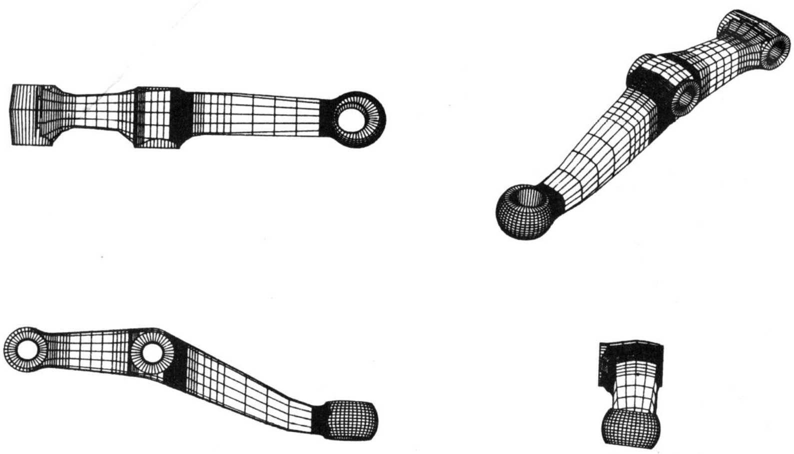

Fig.4 Three dimensional geometric model of the steering arm

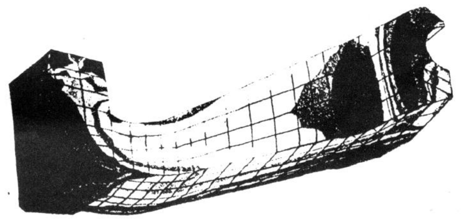

Fig.5 Stress contours of the critical area of the component

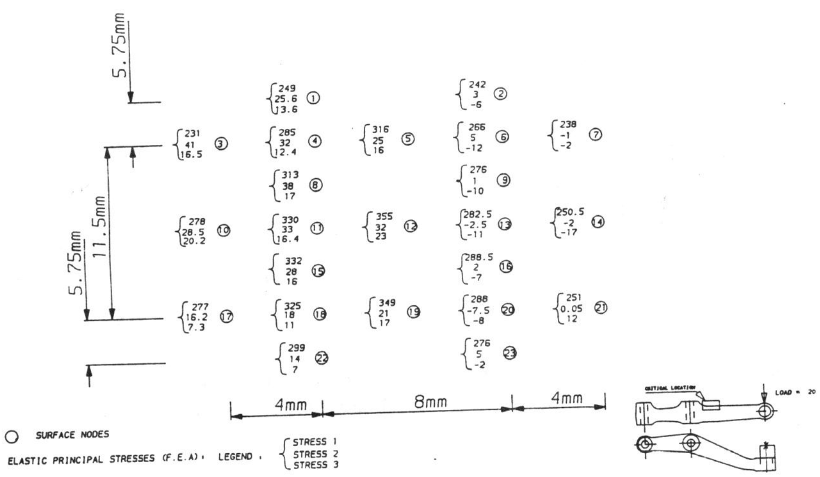

Fig.6 F.E. Calculated maximum principal surface stress at the notch root

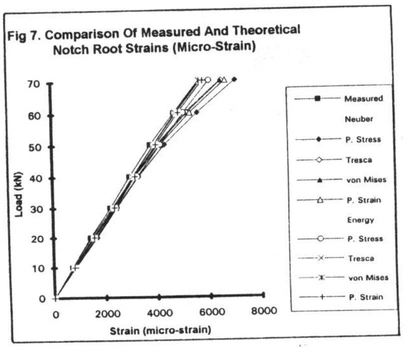

Fig 7. Comparison Of Measured And Theoretical Notch Root Strains (Micro-Strain)

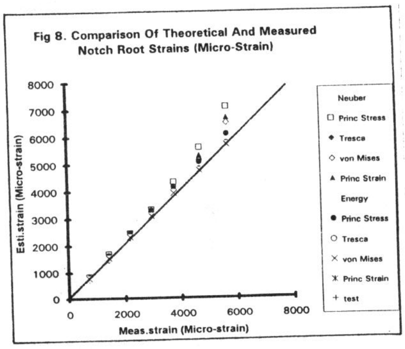

Fig 8. Comparison Of Theoretical And Measured Notch Root Strains (Micro-Strain)