CALCULATION OF MULTIAXIAL FATIGUE FOR AERONAUTICAL APPLICATIONS

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It is now possible to predict fatigue life for most aeronautic applications as, more often than not, they are in a single stress state (such as uni-axial tension). This calculation, however, becomes much more complicated once we are dealing with multiaxial state stresses. In this case, we can call upon simplified calculation methods, for instance by keeping only the maximum main stress... Such methods do not, however, integrate the entire load and cannot correctly predict fatigue life. Moreover, they are not systematically conservative.

The aim of this study is to present a number of models in a form adapted to aeronautic applications. Calculated predictions are compared with the results of bi-axial fatigue tests.

INTRODUCTION

A vast number of criteria are given in the documentation available. A criterion providing a general behaviour model must be consistent with the tendencies observed through simple conventional tests.

It must be independent of the reference system linked to the structure, and, be consistent with Haigh diagrams under tension-compression and torsion.

This study presents the following criteria in compliance with the above points :
- Crossland Criterion (1),
- Sines Criterion (2),
- Dang Van Criterion (3).

For periodical phased loads, the predictions obtained using these criteria comply with the experimental tendencies observed (3).

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MULTIAXIAL CRITERIA

Crossland criterion

The initial equation proposed by Crossland (2) is expressed as a linear combination of the equivalent shear amplitude and the maximum hydrostatic pressure reached during the cycle:

$$T_{eqa} + B_N.P_{max} \leq A_N \quad \text{(Failure : } T_{eqa}+B_N.P_{max} = A_N) \qquad (1)$$

A_N and B_N are positive constants defined for fatigue life N,
T_{eqa} the equivalent shear amplitude, P_{max} maximum hydrostatic pressure.

Definition of P_{max} and T_{eqa}. Hydrostatic pressure p(t) equals a third of the trace of the stress tensor $\Sigma(t)$: $P_{max} = \frac{1}{3}\max_{(t)}[\text{trace}\{\Sigma(t)\}] = \frac{1}{3}\max_{(t)}[\sigma_I(t)+\sigma_{II}(t)+\sigma_{III}(t)]$ with $\sigma_I(t)$, $\sigma_{II}(t)$, $\sigma_{III}(t)$ diagonal values of $\Sigma(t)$.

For any periodical load, the point representing the stress tensor $\Sigma(t)$ describes a closed curve (C_Σ) in the six-dimension stress space that we can call a load trajectory. For radial (or proportional) loads, the load trajectory is a line segment passing through the origin. The eigen axes of the stress tensor $\Sigma(t)$ are fixed during the cycle. In this simple case, and in this case only, the equivalent shear amplitude for a point on the structure is expressed as: $T_{eqa} = \sqrt{J_{2a}}$

where: $J_{2a} = \frac{1}{6}[(\sigma_{Ia}-\sigma_{IIa})^2+(\sigma_{Ia}-\sigma_{IIIa})^2+(\sigma_{IIa}-\sigma_{IIIa})^2]$,

$$\sigma_{Ia} = \frac{\sigma_{Imax}-\sigma_{Imin}}{2}, \quad \sigma_{IIa} = \frac{\sigma_{IImax}-\sigma_{IImin}}{2}, \quad \sigma_{IIIa} = \frac{\sigma_{IIImax}-\sigma_{IIImin}}{2}$$

However, in the most general case of periodical loads, the eigen stress axes vary over time, as the load is not proportional. T_{eqa} is homogeneous at a distance in the hyperplane of $\Sigma(t)$ deflectors. In fact, if we project the load trajectory (C_Σ) onto the deflector hyperplane, we also obtain a closed curve (C_S). T_{eqa} is then expressed as: $T_{eqa} = \frac{1}{2}\frac{D}{\sqrt{2}}$. D is the biggest segment intercepting (C_S). In the deflector hyperplane, D is calculated as follows: $D = \max_{(t1,t2)}\sqrt{\text{trace}\{[S(t1)-S(t2)].[S(t1)-S(t2)]\}}$

where : $S(t) = \Sigma(t)-p(t).Id$, $Id = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Definition of constants A_N and B_N. The two constants can be defined by means of two simple uniaxial tests. The tests selected are generally an alternating tension test (mean stress equals to zero) on an unnotched test specimen and an alternating torsion test on a thin tube.

By applying Crossland's equation to both these tests, we can write:

• for the alternating tension test: $\frac{\sigma_{Ia}}{\sqrt{3}} \leq A_N - B_N \frac{(\sigma_{Imoy}+\sigma_{Ia})}{3}$

$\sigma_{Ia} \leq \frac{\sqrt{3}}{1+\frac{1}{\sqrt{3}}B_N}(A_N - \frac{B_N}{3}\sigma_{Imoy})$. On failure: $\sigma_{-1}(N) = \frac{\sqrt{3}}{1+\frac{1}{\sqrt{3}}B_N}A_N$

$\sigma_{-1}(N)$ is the maximum stress corresponding to failure at N cycles for the alternating tension test.

• for the alternating torsion test : $\sigma_{12a} \leq A_N$

On failure: $\tau_{-1}(N) = A_N$. $\tau_{-1}(N)$ is the maximum stress corresponding to failure at N cycles for the alternating torsion test.

hence: $A_N = \tau_{-1}(N)$ and $B_N = 3.(\frac{\tau_{-1}(N)}{\sigma_{-1}(N)} - \frac{1}{\sqrt{3}})$

Positive B_N implies that $\frac{\sigma_{-1}(N)}{\tau_{-1}(N)} < \sqrt{3}$.

Crossland's equation is finally expressed as:

$$T_{eqa} + 3.(\frac{\tau_{-1}(N)}{\sigma_{-1}(N)} - \frac{1}{\sqrt{3}}) \cdot p_{max} \leq \tau_{-1}(N) \quad (2)$$

Constants A_N and B_N are defined for life N. So, for several N, we obtain an array of critical lines. For a pair (P_{max},T_{eqa}), it is therefore possible to determine the corresponding life by means of interpolation. This method, however, implies knowledge of a large number of curves to perfect the interpolation. It does not, moreover, enable us to draw up an analytical relation of the type $N=f(P_{max},T_{eqa})$.

To obtain such a relation, we define constants A_N and B_N not by means of a tension test and a torsion test, but by using two tension tests: alternating and repetitive tension (minimum stress equals to zero).

A_N et B_N are then expressed as:

$$A_N = \frac{\sigma_0(N).\sigma_{-1}(N)}{2.\sqrt{3}.(\sigma_0(N)-\sigma_{-1}(N))} \qquad B_N = \frac{3.(2.\sigma_{-1}(N)-\sigma_0(N))}{2.\sqrt{3}.(\sigma_0(N)-\sigma_{-1}(N))} \quad (3)$$

where $\sigma_0(N)$ is the maximum stress corresponding to failure at N cycles for the repetitive tension test.

For economic reasons (weight and volume gain), calculations in civil aeronautics are generally made with stress levels, corresponding to lives on failure of between 10^4 and 10^7 cycles inclusive. If, for this range of life, we position experimental results, concerning an aluminium alloy, on a graph ($\log(\sigma_{max})$,$\log(N)$), we can observe that these points are approximately alined. From here on, it is possible to model fatigue cycle curves by means of straight lines.

The use of a specific point (corresponding to $N=10^5$ cycles) and the line gradient (-1/p) result in the following expressions:

$$\sigma_{-1}(N) = \sigma_{-1}(10^5).(\frac{10^5}{N})^{1/p} \quad \text{and} \quad \sigma_0(N) = \sigma_0(10^5).(\frac{10^5}{N})^{1/p} \tag{4}$$

By expressing the Crossland model with $\sigma_{-1}(N)$ and $\sigma_0(N)$, we obtain:

$$A_N = \frac{\sigma_0(10^5).\sigma_{-1}(10^5)}{2\sqrt{3}.(\sigma_0(10^5)-\sigma_{-1}(10^5))}(\frac{10^5}{N})^{1/p} \qquad B_N = \frac{3.(2.\sigma_{-1}(10^5)-\sigma_0(10^5))}{2\sqrt{3}.(\sigma_0(10^5)-\sigma_{-1}(10^5))} \tag{5}$$

finally :

$$N = 10^5.[\frac{\frac{\sigma_0(10^5).\sigma_{-1}(10^5)}{2\sqrt{3}.(\sigma_0(10^5)-\sigma_{-1}(10^5))}}{T_{eqa} + \frac{3.(2.\sigma_{-1}(10^5)-\sigma_0(10^5))}{2\sqrt{3}.(\sigma_0(10^5)-\sigma_{-1}(10^5))} P_{max}}]^P \tag{6}$$

For the 2024 T3 : $N = 10^5.[\frac{220}{T_{eqa} + 1{,}07\ P_{max}}]^5$ (reliability : 50%. Unit : MPa)

with : $\sigma_{-1}(N) = 235.(\frac{10^5}{N})^{1/5}$ and $\sigma_0(N) = 340.(\frac{10^5}{N})^{1/5}$

Sines criterion

The same approach leads to the following expression :

$$N = 10^5.[\frac{\frac{\sigma_{-1}(10^5)}{\sqrt{3}}}{T_{eqa} + \sqrt{3}.(\frac{2.\sigma_{-1}(10^5)}{\sigma_0(10^5)} - 1)\ P_{moy}}]^P \tag{7}$$

P_{moy} : mean hydrostatic pressure

Dang Van criterion

We obtain :

$$N = 10^5.[\frac{\frac{1}{4}\frac{\sigma_0(10^5).\sigma_{-1}(10^5)}{\sigma_0(10^5)-\sigma_{-1}(10^5)}}{\max_{(t)}[\ \tau(t) + \frac{3}{4}\frac{\sigma_0(10^5)-2.\sigma_{-1}(10^5)}{\sigma_{-1}(10^5)-\sigma_0(10^5)}\ p(t)\ ]}]^P \tag{8}$$

$p(t)$: hydrostatic pressure, $\tau(t)$: microscopic shear stress : $\tau(t) = \frac{s_I(t)-s_{III}(t)}{2}$.

$s_I(t)$ and $s_{III}(t)$ are respectively the highest and the smallest eigen value of microscopic S(t) deflectors : $s(t)=S(t)+\rho^*$

with ρ^* : residual microscopic stress. For radial loads : $\rho^* = -\frac{Smax+Smin}{2}$

EXPERIMENTAL VALIDATION

In order to validate this approach, we apply it to predict fatigue life of fit fastener and cold worked holes. The interference process consists in inserting fasteners in bores with negative clearance (the diameter of the fastener is greater than that of the bore). Consequently, the material is pushed back all around the bore circumference. For cold working, the process is the quite the same. It consists in inserting in a hole a mandrel with a greater diameter (2 to 5 % greater are the values usually applied in aircraft structures) and then to pull out the mandrel. Differents cold working process exist with or without split sleeves. Many experimental results reveal that these processes, used to design and manufacture narrow body aircraft, improve the fatigue life of assemblies (see figures 3,4).

Superimposing the external load and the effect of the processes (interference or cold working) produce a biaxial stress state.

The test specimen geometry selected for this study is representative of an assembly used during the manufacture of narrow body civil aircraft (see figure 1).

In 90% of cases with interference process, crack initiation does not take place at the edge of the hole but rather between the bore and the edge of the test specimen. With cold working process, crack initiation only occurs at the edge of the test specimen (see figure 2).

The external load is monotonic. The ratio between the minimum and the maximum loads is set at 0.1.

An elastoplastic calculation (use of finite element models) provides stress distribution. We use expressions (6), (7), (8) to calculate fatigue life at any model node. An iso-life mapping indicates the most critical zones (see figure 2)

CONCLUSION

The calculation method presented in this study provides excellent correlation with tests (see figures 3,4). The initiation site is correctly predicted. The models provide approximately the same result. The Dang Van model is the most conservative.

REFERENCES

(1) Crossland B., "Effect of Large Hydrostatic Pressures on the Torsional Fatigue Strength of an Alloy Steel", Proceedings of the International Conference on Fatigue of Metals, Institution of Mechanical Engineers, London 1956.

(2) Sines G., Ohgi G., "Fatigue Criteria under Combined Stresses or Strains", Transactions of the American Society of Mechanical Engineers, Journal of Engineering Materials and Technology, Vol. 103, 1981.

(3) Papadopoulos Y.V., "Fatigue Polycyclique des Métaux : une nouvelle approche", Thèse de l'Ecole Nationale des Ponts et Chaussées, Paris, 1987.

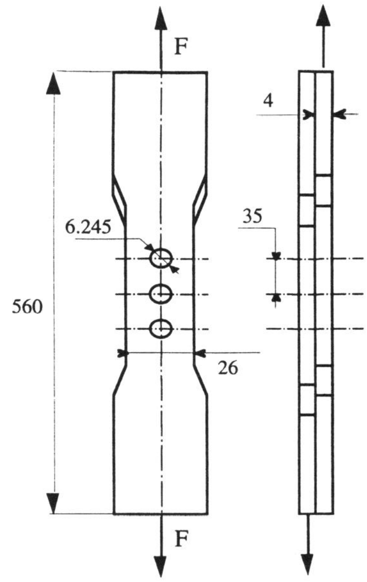

Figure 1 Test specimen

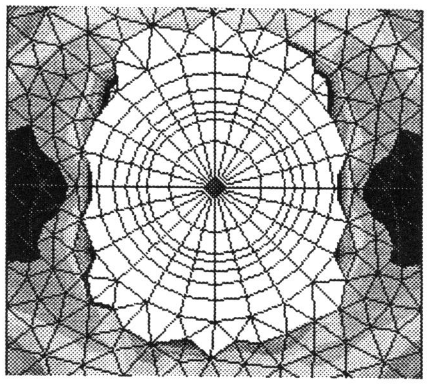

Figure 2 Iso-life mapping

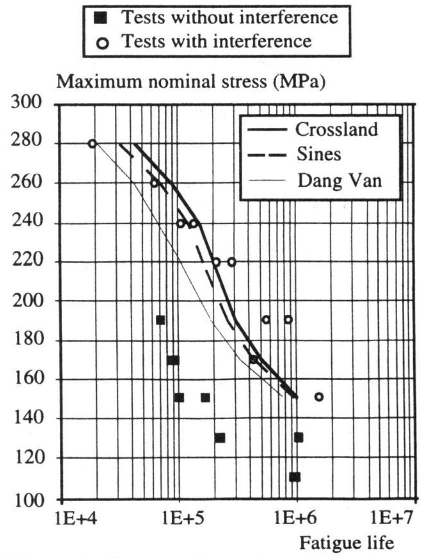

Figure 3 Test results and predictions with interference process

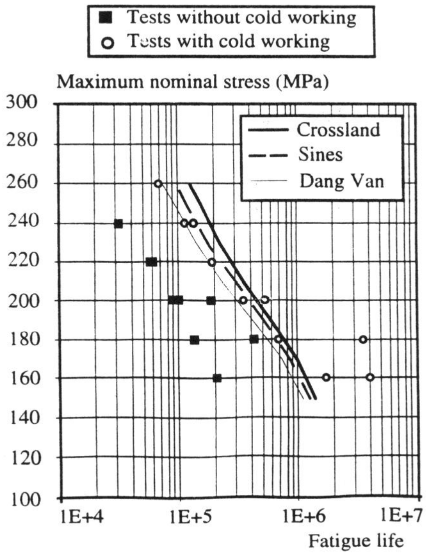

Figure 4 Test results and predictions with cold working process