ASYMPTOTIC METHOD IN THE PLANE THEORY PROBLEM FOR THE ORTHOTROPIC STRIP WITH CUTS

I. Andrianov*, M.Matyash* and S. Koblik+

It was suggested the approach for solving of differential equation systems of plane problems of anisotropic and composite materials mechanics. The approach is based on the ideas of asymptotic integration; the ratio of rigidities of material is "small" parameter. In this paper new analytical method has been developed that can predict the stress intensity coefficient (SIC) in the orthotropic strip. All solutions are closed.

INTRODUCTION

Boundary value problems (BVP) of the anysotropic (orthotropic) theory of elasticity is more complicated for solution in comparison with isotropic one if we use some numerical procedure. When coefficients of this system slightly differ from isotropic ones, it is possible to use perturbation procedure with isotropic theory of elasticity relations as first approximation (Lehnitzkiy (1)). In this case we have regular asymptotics. Unfortunately, area of applicability of this approach is not wide.

On the other limit case ratio e of rigidity-to-shear and rigidity-to-tension-compression in one of the directions is small. Then the equilibrium equations of orthotropic body in displacements include small

* Department of Higher Math., Prydneprovie State Academy of Civil Engineering and Architecture.

+ Department of Materials, Clivlend University.

parameter e, and we may used this fact for constructing of simplified BVP. It is worth noting that following asymptotics is singular. Above mentioned procedure was proposed for the first time by Manevich, Pavlenko and Shamrovskiy (2) (see also Manevich, Pavlenko and Koblik(3)), independently by Everstine and Pipkin (4), Spencer (5) and, in the other form, using complex potentials, by Kosmodamyanskiy (6). This approach has wide range of application, provides simple and obvious formulas in many cases and could be succeesfully used while examining BVP for isotropic materials.

Governing relations

We consider strip $|x| \leqslant h/2;\ 0 \leqslant |y| \leq \infty$ tensile by stresses $\sigma_y = P/h$, applied on the infinity. Stresses τ_{xy} on the infinity equal zero. Strip weakened by two kollinear cuts outgoing to the boundaries (see Fig.1).Due symmetry of the problem with respect to axis x we may consider only upper part of the strip. As we plan use complex representation, it is convinient to express throw the function of complex variables not stresses and deformations, but main vectors of loads applied along boundaries (P_x and P_y) and displacements. The we may write:

$P_x = 0$		everywhere on boundaries	
$P_y = P$	for	$x = -h/2$,	$0 < y < \infty$
$P_y = P$	for	$-h/2 < x < a$,	$y = 0$
$V = 0$	for	$a < x < b$,	$y = 0$
$P_y = 0$	for	$b < x < h/2$,	$y = 0$
$P_y = 0$	for	$x = h/2$,	$0 < y < \infty$

Solving of the problem. After splitting governing BVP on small parameter (Manevich, Pavlenko and Koblic (4), Kosmodamyanskiy (8)) for complex function Φ_2 we obtain the following simple formulas

$$P_y = c\ \mathrm{Im}\ \Phi_2(z_2),\quad \sigma_y = d\ \mathrm{Im}\ \Phi_2'(z_2),\quad V = f\ \mathrm{Re}\ \Phi_2(z_2)$$

For function Φ_2 one obtains the BVP

$$
\begin{array}{llll}
\mathrm{Im}\,\varphi_2(z_2) = P/c & \text{for} & x = -h/2, & 0 < y < \infty \\
\mathrm{Im}\,\varphi_2(z_2) = P/c & \text{for} & -h/2 < x < a, & y = 0 \\
\mathrm{Re}\,\varphi_2(z_2) = 0 & \text{for} & a < x < b, & y = 0 \\
\mathrm{Im}\,\varphi_2(z_2) = 0 & \text{for} & b < x < h/2, & y = 0 \\
\mathrm{Im}\,\varphi_2(z_2) = 0 & \text{for} & x = h/2, & 0 < y < \infty
\end{array}
\qquad (1)
$$

For solving BVP (1) we use conformal mapping of halfstrip on the upper halfplane by the function (see also table 1).

$$\zeta = \xi + i\eta = \sin\,(\pi z_2/h)$$

TABLE 1 – Correspondence of the point of mapping are the following.

z_2	ζ	
$-h/2 + i\infty$	$-\infty + i0$	
$-h/2 + i0$	$-1 + i0$	
$a_j + i0$	$\alpha_j + i0$	$(\alpha_j = \sin(\pi a_j/h))$
$0 + i0$	$0 + i0$	$j = 1, 2$
$h/2 + i0$	$1 + i0$	
$h/2 + i\infty$	$+\infty + i0$	

Then for the function $\varphi_2^*(\zeta) = \varphi_2\,(z_2(\zeta))$ one obtains mixed BVP for the upper halfplane

$$
\begin{array}{llll}
\mathrm{Im}\,\varphi_2^*(\zeta) = P/c & \text{for} & -\infty < \xi < \alpha, & \eta = 0 \\
\mathrm{Re}\,\varphi_2^*(\zeta) = 0 & \text{for} & \alpha < \xi < \beta, & \eta = 0 \\
\mathrm{Im}\,\varphi_2^*(\zeta) = 0 & \text{for} & \beta < \xi < \infty, & \eta = 0
\end{array}
\qquad (2)
$$

Conditions (2) are satisfied by the function

$$\Phi_2^*(\zeta) = \frac{p}{c\pi} \ln \frac{\sqrt{\zeta - \alpha} + \sqrt{\zeta - \beta}}{\sqrt{\zeta - \alpha} - \sqrt{\zeta - \beta}}$$

Asymptotic expression for stress δ_y may be written in the form

$$\sigma_y = d \operatorname{Im} \Phi_2'(z_2) = d \operatorname{Im}\left[\Phi_2^{*\prime}(\zeta) \frac{d\zeta}{dz_2} \right] =$$

$$= d \frac{P}{\pi} \operatorname{Im}\left[\frac{\zeta'_{z_2}}{\sqrt{(\zeta - \alpha_1)(\zeta - \alpha_2)}} \right] = - \frac{P}{h} \operatorname{Im}\left[\frac{\cos \frac{\pi x}{h}}{q_1 q_2} \right]$$

Here $q_i = \sqrt{(-1)^{i+1}\left[\sin \frac{\pi a_i}{h} - \sin \frac{\pi x}{h}\right]}, \quad i = 1,2$

SIC K_i in representation

$$\sigma_y \sim \frac{K_i}{\sqrt{(-1)^i (a_i - x)}} \qquad \text{for} \quad x \to a_i - (-1)^i 0, \quad i=1,2$$

have the following expressions

$$K_i = \frac{P}{\sqrt{\pi h}} \sqrt{\frac{\cos \frac{\pi a_i}{h}}{\cos \frac{\pi a_2}{h} - \cos \frac{\pi a_1}{h}}}, \qquad i=1,2$$

For $h \to \infty$ one obtains formulas for plane with two cuts

$$\sigma_y = \frac{P}{\pi} \frac{1}{\sqrt{(a_2 - x)(x - a_1)}}, \quad K_{a_i} = \frac{P}{\pi} \frac{1}{\sqrt{(a_2 - a_1)}}$$

Comparison of approximate solutions with known solutions shows that error is not large even for the worst (for this approach) isotropic case.

SYMBOLS USED

x, y = axis of coordinates

h = wide of the strip (m)

a_1, a_2 = lengthes of cuts (m)

P = load on the infinity (n/m)

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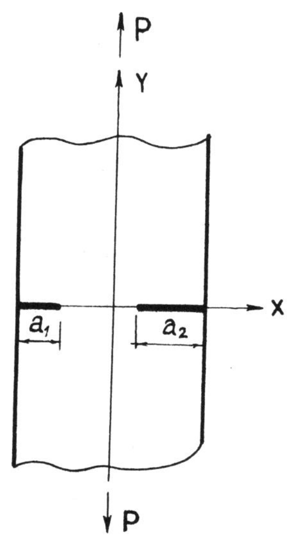

Figure 1 Strip with cuts