ASYMPTOTIC FIELDS OF A MODE III CRACK MEETING AN INCLINED INTERFACE BETWEEN TWO ELASTIC-PLASTIC MATERIALS

J. LI and X.B. ZHANG*

An analytical solution is developed for a crack arbitrarily inclined with respect to the interface between two power-law hardening materials under anti-plane loading. Assuming the hardening exponent $n \to \infty$, the perfectly plastic bimaterial problem is studied. It has been found that, if the crack is in the less stiff material ($\tau_{01} / \tau_{02} \leq 1$), the whole plastic asymptotic fields near the crack tip can be found. On the contrary, if the crack is in the stiffer material ($\tau_{01} / \tau_{02} \geq 1$), the crack tip fields are partially elastic and partially plastic. For power-law hardening materials, the mathematical model can be expressed as a non-linear eigenequation solved numerically. Stress, strain and displacement asymptotic fields are also determined.

INTRODUCTION

The strength of the composite materials is influenced by the existence of defaults such as cracks located near the interfaces between the materials. Most of the experimental and theoretical investigation on this topic has focused on the few special cases of crack orientations such as cracks lying along, or perpendicular to, the interface(1)(2)(3)(4). However, cracks advancing or terminating at arbitrary angles with an interface between two materials may also be found in a variety of engineering structures. In this paper, we present a general study for a crack arbitrarily inclined with respect to the interface between two elastic-plastic materials under an anti-plane loading.

STRESS SINGULARITIES AND ASYMPTOTIC FIELDS

For a pure mode III crack in a homogenous material, the anti-plane shear is characterised by the only non-zero, out-of-plane (along the Z-direction) displacement component w. The only non-zero stresses are τ_x, τ_y in Cartesian coordinates,

*Laboratoire d'Etudes et de Recherches en Mécanique des structures
University B. P. of Clermont-Fd Av. Aristide Briand 03100 Montluçon France

and τ_r ,τ_θ in polar coordinates. The only non-zero strains are γ_x, γ_y and γ_r, γ_θ in the two systems of coordinates. Consider a power-law hardening material showing a Ramburg-Osgood stress-strain relationship:

$$\left(\frac{\tau}{\tau_0}\right)^n = \left(\frac{\gamma}{\gamma_0}\right)$$

in which τ_0 , γ_0 are respectively the yielding shear stress and the yielding shear strain of the material, τ and γ are the effective shear stress and strain, n is the material hardening exponent. Let $\chi(\theta)$ be an angular function such that: $\tau_x = -\tau\sin\chi(\theta)$ and $\tau_y = \tau\cos\chi(\theta)$. the asymptotic fields can be written in polar coordinates (r,θ) as follows:

$$\gamma = \left(\frac{C}{r}\right)^{-n\lambda} \left[(\beta+\alpha)/2-(\beta-\alpha)\cos 2(\alpha\chi+\phi)/2\right]^{n\lambda/2} \tag{1}$$

$$\gamma_r = \gamma\sin(\theta-\chi) \ , \ \gamma_\theta = \gamma\cos(\theta-\chi)$$

$$\tau = \tau_0\left(\frac{\gamma}{\gamma_0}\right)^{1/n}$$

$$\tau_r = \tau\sin(\theta-\chi) \ , \qquad \tau_\theta = \tau\cos(\theta-\chi) \ , \qquad w = (n+1)r\gamma_r \tag{2}$$

where λ is the unknown singularity exponent, C and ϕ are unknown constants which can be determined by boundary conditions, and in which

$$\alpha = \frac{(n\lambda+1)(\lambda+1)}{n\lambda^2} \ , \quad \beta = \frac{n\lambda+1}{n\lambda}$$

The function $\chi(\theta)$ can be determined by means of the following implicit equation:

$$\frac{\sin\theta}{\cos\theta} = \frac{(\beta-\alpha)\sin[(\alpha+1)\chi+1]+(\beta+\alpha)\sin[(\alpha-1)\chi+\phi]}{(\beta-\alpha)\cos[(\alpha+1)\chi+\phi]-(\beta+\alpha)\cos[(\alpha-1)\chi+\phi]} \tag{3}$$

In the bimaterial case, consider a semi-infinite crack terminating at an arbitrary angle θ_0, with the interface between two bonded power-law hardening materials (fig. 1). Region 1 and region 3 are occupied by material 1 with the hardening exponent n_1 and the yielding shear stress τ_{01}. Region 2 is occupied by material 2 with the hardening exponent n_2 and the yielding shear stress τ_{02} . By introducing the following boundary conditions: the free surface at the crack lips and the continuity of stresses and displacements at the interface between two materials, a non-linear eigenequation is obtained and the eigenvalue λ can be determined. The asymptotic fields can then be represented as follows :

Case of perfectly plastics materials

Such a case can be considered as the particular case of a power-law material when the hardening exponent $n \to \infty$ .One can distinguish three different cases according to the ratio τ_{01} / τ_{02}.

a). Case when $\tau_{01} / \tau_{02} \leq 1$ Table 1 shows the expressions of the angular function $\chi(\theta)$ and the stress τ_r, τ_θ for each angular interval.

Region	Interval	χ	τ_r	τ_θ
3	$[-\pi, \theta_0 - \pi]$	$-\frac{\pi}{2}$	$\tau_{01}\cos\theta$	$-\tau_{01}\sin\theta$
2	$[\theta_0 - \pi, \chi_2(\theta_0 - \pi)]$	$\theta_0 - \pi + \arccos\left(\frac{\tau_{01}}{\tau_{02}}\sin\theta_0\right)$	$\tau_{02}\sin[\theta - \chi_2(\theta_0 - \pi)]$	$\tau_{02}\cos[\theta - \chi_2(\theta_0 - \pi)]$
	$\left[\chi_2(\theta_0 - \pi), \frac{\chi_2(\theta_0) + \chi_2(\theta_0 - \pi)}{2}\right]$	θ	0	τ_{02}
	$\left[\frac{\chi_2(\theta_0) + \chi_2(\theta_0 - \pi)}{2}, \chi_2(\theta_0)\right]$	θ	0	τ_{02}
	$[\chi_2(\theta_0), \theta_0]$	$\theta_0 - \arccos\frac{\tau_{01}}{\tau_{02}}$	$\tau_{02}\sin[\theta - \chi_2(\theta_0)]$	$\tau_{02}\cos[\theta - \chi_2(\theta_0)]$
1	$\left[\theta_0, \frac{\pi}{2}\right]$	θ	0	τ_{01}
	$\left[\frac{\pi}{2}, \pi\right]$	$\frac{\pi}{2}$	$-\tau_{01}\cos\theta$	$\tau_{01}\sin\theta$

TABLE 1 - Stress fields for the case when $\tau_{01} / \tau_{02} \leq 1$

b). Case when $(1/\sin\theta_0) \geq (\tau_{01}/\tau_{02}) > 1$ In this case, region 1 occupied by material 1 cannot be a plastic zone; but region 3 occupied by the same material is. The asymptotic stress fields for this case are listed in table 2.

Region	Interval	χ	τ_r	τ_θ
3	$[-\pi, \theta_0 - \pi]$	$-\frac{\pi}{2}$	$\tau_{01}\cos\theta$	$-\tau_{01}\sin\theta$
2	$[\theta_0 - \pi, \chi_2(\theta_0 - \pi)]$	$\theta_0 - \pi + \arccos\left(\frac{\tau_{01}}{\tau_{02}}\sin\theta_0\right)$	$\tau_{02}\sin[\theta - \chi_2(\theta_0 - \pi$	$\tau_{02}\cos[\theta - \chi_2(\theta_0 - \pi$
	$\left[\chi_2(\theta_0 - \pi), \frac{\theta_0 + \chi_2(\theta_0 - \pi)}{2}\right]$	θ	0	τ_{02}
	$\left[\frac{\theta_0 + \chi_2(\theta_0 - \pi)}{2}, \theta_0\right]$	θ	0	τ_{02}
1	$\left[\theta_0, \frac{\pi}{2}\right]$	θ	0	τ_{02}
	$\left[\frac{\pi}{2}, \pi\right]$	$\frac{\pi}{2}$	$-\tau_{02}\cos\theta$	$\tau_{02}\sin\theta$

TABLE 2 - Stress fields for the case when $(1/\sin\theta_0) \geq (\tau_{01}/\tau_{02}) > 1$
(with $0 \leq \theta_0 \leq \pi/2$)

c). Case when $(\tau_{01} \sin\theta_0 / \tau_{02}) > 1$ In this case, neither region 1 nor region 3 is a plastic zone. The asymptotic stress fields for this case are listed in table 3.

région	Interval	χ	τ_r	τ_θ
3	$[-\pi, \theta_0 - \pi]$	$-\frac{\pi}{2}$	$\frac{\tau_{02}}{\sin\theta_0}\cos\theta$	$-\frac{\tau 2}{\sin\theta_0}\sin\theta$
2	$\left[\theta_0 - \pi, \theta_0 - \frac{\pi}{2}\right]$	θ	0	τ_{02}
	$\left[\theta_0 - \frac{\pi}{2}, \theta_0\right]$	θ	0	τ_{02}
1	$\left[\theta_0, \frac{\pi}{2}\right]$	θ	0	τ_{02}
	$\left[\frac{\pi}{2}, \pi\right]$	$\frac{\pi}{2}$	$-\tau_{02}\cos\theta$	$\tau_{02}\sin\theta$

TABLE 3 - Stress fields for case when $(\tau_{01} \sin\theta_0 / \tau_{02}) > 1$
(with $0 \le \theta_0 \le \pi/2$)

The results of the above cases are illustrated by an example of a mode III crack terminating at a 45° angle with the interface between two perfectly plastic materials. The angular stress distributions are shown in Fig. 2, for τ_{01} / τ_{02} = 1.2 and 0.5 respectively. The values of τ_r and τ_θ are normalised by τ_{02}. This work shows that, for a mode III crack terminating at an inclined interface between two perfectly plastic materials, if the crack is in the less stiff material ($\tau_{01} / \tau_{02} \le 1$), the whole plastic field near the crack tip can be found for any orientation of the crack with respect to the interface. On the contrary, if the crack is in the stiffer material ($\tau_{01} / \tau_{02} > 1$), the plastic solution cannot be found in region 1. Region 3 cannot be a plastic zone if $\tau_{01} \sin\theta / \tau_{02} > 1$. It means that the asymptotic fields near the crack tip are partially elastic and partially plastic. In the case when $\tau_{01} / \tau_{02} > 1$, region 1 becomes an elastic zone and when $(\tau_{01} \sin\theta_0 / \tau_{02}) > 1$, region 3 becomes an elastic zone, too. To obtain the stress fields in the elastic zones, these zones may be considered like the plastic zones with yielding stresses: τ_{02} for region 1 and $\tau_{02} / \sin\theta_0$ for region 3. The yielding stress τ_{01} of material 1 has no influence on the stress distribution in the elastic zones.

Case of power-law hardening materials

According to the hardening exponents of two materials, two different situations can be distinguished : the two materials have different hardening exponents $n_1 \neq n_2$ or the two materials have the same hardening exponent $n_1 = n_2$.

In the first case, Champion and Atkinson (4) have attempted a solution for a mode III crack lying on the interface between two power-law hardening materials by developing more than one term in the displacement expansion. A similar analysis can be carried out for a crack arbitrarily oriented with respect to the interface.

In the second case, $n_1 = n_2 = n$, the eigenvalue can be obtained from equations (1), (2) and the boundary conditions by solving a system of non-linear equations. The eigenvalue λ depends on the inclination of the crack with respect to the interface, on the hardening exponents n and on the ratio of the characteristics of the two materials, G, defined as follows:

$$G = \frac{\tau_{01}}{\tau_{02}}\left(\frac{\gamma_{02}}{\gamma_{01}}\right)^{1/n}$$

A mode III crack terminating at a 45° angle with the interface between two power-law hardening materials is investigated. Two bimaterial configurations (G=5 and G=0.2) are studied. The angular distributions of the stress in both cases for n = 5 are illustrated in fig. 3 and fig. 4 respectively. The stress magnitude is normalised by max($\tau_{\theta 2}$). The convergent evolution of the stress fields can be found by choosing different hardening exponents n, rang from the elastic case to the perfectly plastic case. If the crack is in the stiffer material (G=5 for exp.), this convergence is very rapid. There is practically no difference between the angular distributions of the stress when n=10 and in the perfectly plastic case. If the crack is in the less stiff material (G=0.2 for exp.), this convergence can be also noticed.

CONCLUSION

This work shows that, the elastic-plastic asymptotic fields can be determined for a mode III crack at an arbitrary angle with respect to the interface between two power-law hardening materials. The analysis for perfect plasticity bimaterial can be carried out by taking $n \to \infty$. If the two materials have the same hardening exponent, the classical eigenfunction expansion method can be used. The numerical results indicate that (a) the singularity is stronger as the material containing the crack becomes stiffer, (b) the singularity vanishes as $n \to \infty$. In this case, the angular distribution of stress tends to the perfectly plastic stress field. Further studies should be necessary when the two materials have different hardening exponents.

REFERENCES

(1) Shih C.F. and Asaro R.J., J. of Applied Mechanics, No.55, 1988, pp 299-316
(2) Shih C.F. and Asaro R.J., J. of Applied Mechanics, No 56, 1988, pp 763-779
(3) Chao Y.J. Sutton M.A. and Wu R. Acta Mechanica, No 100, 1993, pp 13-36
(4) Champion C.R. and Atkinson C., Proceding, Royal Society of London, A429,1990, pp 247-257

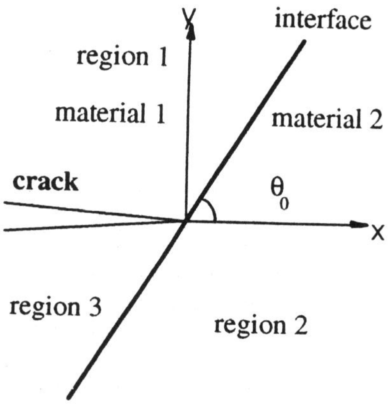

Fig.1 Arbitrarily oriented crack meeting an interface between two elastic-plastic materials

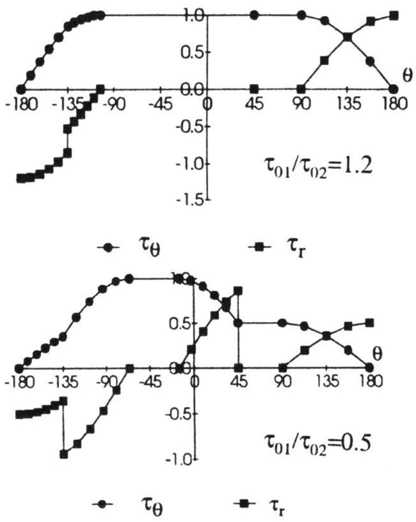

Fig.2 Stress angular distributions for $n \to \infty$ with $\theta_0=45°$

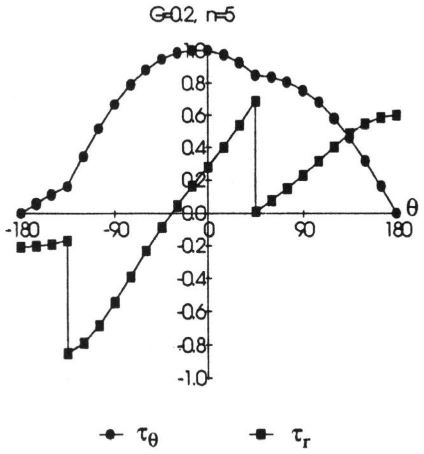

Fig.3 Stress angular distributions for n=5 and G=0.2 with θ_0=45°

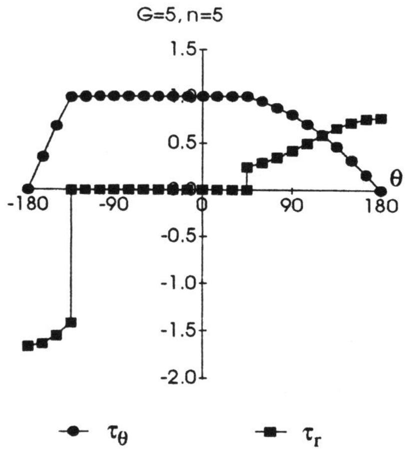

Fig.4 Stress angular distributions for n=5 and G=5 with θ_0=45°