APPLICATION OF THE GASH MODEL FOR ANALYSIS OF DYNAMIC BEHAVIOR OF A ROTOR WITH A CRACK

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Existence of cracks or other defects affects the dynamic characteristics of structures. In this paper is considered the influence of a crack on dynamic behavior of a particular construction - a rotor. For mathematical modeling of the dynamic behavior of a rotor with a crack we used the Gash model of a crack, which, according to latest research perfectly describes the phenomenon of crack "breathing", which is characteristic for rotors. Rotor is discretized by the finite number n of concentrated masses. The obtained system of ordinary differential equations, which describes the behavior of a rotor containing a crack, is being solved numerically. Results point to the conclusion that existence and characteristics of crack in a rotor affect its dynamic behavior.

INTRODUCTION

In recent years the theoretical and experimental investigations were conducted in order to study the oscillatory behavior of a rotor containing a crack. The aim of those investigations is development of a method for detection of the crack position in the rotor as well as its dimensions. There exist many approaches to modeling of cracks in rotors, however, the typical crack behavior, the so called crack "breathing" under the influence of gravity, is completely defined by the simple model proposed by Gash (1). The most recent research of Mueller et al. (2) and Soefker et al. (3) have shown the very good results of the Gash model applications.

Gash model of a crack, shown in Figure 1, is described in the rotating coordinate system $\xi\eta$ in the following way:

$$\begin{bmatrix} \xi \\ \eta \end{bmatrix} = \begin{bmatrix} h + h_a & 0 \\ 0 & h \end{bmatrix} \begin{bmatrix} \mathbf{F}_\xi \\ \mathbf{F}_\eta \end{bmatrix}. \qquad (1)$$

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The deflection h in the crack direction ξ will be augmented for an additional value h_a for the case of the opened crack, where h_a depends on the crack depth, and $\mathbf{F}_\xi$ and $\mathbf{F}_\eta$ are the so called "crack forces". Relative crack deflection $h_r = (h_a/ h)$ was, for different crack depths, determined experimentally. The conditions of the crack opening can be defined by the curvature at the crack site (ξ',ξ''), or approximately by the displacements in the vicinity of the crack $(\xi_{i-e_n},\xi_{i+e_n})$ in the following way:

$$\xi_i > \frac{\xi_{i-e_n} + \xi_{i+e_n}}{2} \quad \text{or} \quad \chi = \chi_i = \xi_i - \frac{\xi_{i-e_n} + \xi_{i+e_n}}{2} > 0, \tag{2}$$

where $e_n = 2$ and χ is the condition for the crack opening. By application of the transformation matrix from the fixed to the rotational system one obtains the stiffness matrix of the element for the discretized model in the internal coordinates in the following form:

$$[\mathbf{K}_e] = \frac{-h_r}{h(1+h_r)} \begin{bmatrix} \sin^2(\Omega t + \beta) & \sin(\Omega t + \beta)\cos(\Omega t + \beta) \\ \sin(\Omega t + \beta)\cos(\Omega t + \beta) & \cos^2(\Omega t + \beta) \end{bmatrix} \tag{3}$$

The matrix depends on conditions of the crack opening (equation (2)), and time, thus the system of the rotor containing a crack in the internal coordinates becomes nonlinear and parametrically excited.

EQUATIONS OF MOTION OF ROTOR WITH A CRACK

The rotor with a crack (or without it) can be considered as the dynamic system with the infinite number of degrees of freedom. For practical calculations the system is being discretized with the finite number of elements.

In Figure 2 is shown the rotor with a crack and imbalance, discretized with the finite number ***n*** of concentrated masses. When the gyroscopic forces are neglected, the equations of motion, for a rotor modeled in this way, in the matrix form can be written as:

$$[\mathbf{M}]\,\{\ddot{\mathbf{q}}\} + [\mathbf{D}]\,\{\dot{\mathbf{q}}\} + [\mathbf{K}]\,\{\mathbf{q}\} = \{\mathbf{f}(t)\} + [\mathbf{N}_n]\,\{\mathbf{h}(\{\mathbf{q}(t)\},t)\} \tag{4}$$

where:

$[\mathbf{M}]$- is the mass matrix or inertial matrix of the order 2n x 2n, Kojić and Mić unović (4) , and for the rotor shown in Figure 2 it has the form:

$$[\mathbf{M}] = \text{diag}\begin{bmatrix} \frac{m}{2} & \frac{m}{2} & \underbrace{m \;\cdots\; m}_{(2n-4)\,\text{times}} & \frac{m}{2} & \frac{m}{2} \end{bmatrix} \tag{5}$$

(here $m = (d^2\pi / 4) \cdot \ell \cdot \rho$ with d as the rotor diameter, which is constant; ℓ as the length of individual element, and ρ as the material density).

$[\mathbf{D}]$ - is the damping matrix of the order 2n x 2n. It can be given as a function of the mass and stiffness matrices as:

$$[\mathbf{D}] = \alpha_{mod}[\mathbf{M}] + \beta_{mod}[\mathbf{K}] \qquad (6)$$

and the following values are proposed in (2) $\alpha_{mod} = 0$ and $\beta_{mod} = 0.00001$.

$[\mathbf{K}]$ - is the stiffness matrix of the order 2n x 2n

$\{\mathbf{f}\}$ - is the imbalance vector

$\{\mathbf{q}\}$ - is the vector of generalized displacements

$\{\dot{\mathbf{q}}\}$, $\{\ddot{\mathbf{q}}\}$ - are the vectors of generalized velocities and accelerations, respectively.

$[\mathbf{N}_n]$ - the matrix of nonlinearities of the order 2 n x 2, which includes nonlinearities caused by the presence of the crack in the rotor. All components of this matrix are equal to zero except: $n_{p,1} = n_{p+1,2} = 1$. Subscript p in this and the following expressions refers to the element that contains the crack.

$\{\mathbf{h}\}$ - is the vector of the crack forces of the order 2 x 1, which has different values for the cases of the opened or the closed crack.

The condition for the crack to be opened, which is in the moving coordinate system defined by the inequality $\chi > 0$ (equation (2)), can be written in the fixed reference frame, by application of the transformation matrix, as:

$$q_{2p}\cos(\Omega t + \beta) + q_{2p-1}\sin(\Omega t + \beta) > \\ > \frac{\left[(q_{2p-2} + q_{2p+2})\cos(\Omega t + \beta) + (q_{2p-3} + q_{2p+1})\sin(\Omega t + \beta)\right]}{2} \qquad (7)$$

For the case of the closed crack, i.e., $\chi \le 0$, the vector of crack forces is:

$$\{\mathbf{h}\} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \qquad (8)$$

For the case of the opened crack, on the other hand, we have:

$$\{\mathbf{h}\} = \begin{Bmatrix} \left[-q_{2p}\cos^2(\Omega t + \beta) - q_{2p-1}\sin(\Omega t + \beta)\cos(\Omega t + \beta)\right]\dfrac{k_2 h_r}{1 + h_r} \\ \left[-q_{2p-1}\sin^2(\Omega t + \beta) - q_{2p}\sin(\Omega t + \beta)\cos(\Omega t + \beta)\right]\dfrac{k_2 h_r}{1 + h_r} \end{Bmatrix} \qquad (9)$$

It is obvious that the components of the vector $\{\mathbf{h}\}$ depend on generalized coordinates and on time, as it was written in the initial equation (4), and that this dependence is, for the case of the opened crack, nonlinear. If we write the matrix equation (4), with application of (9), in the developed form, we shall obtain the system of 2n ordinary differential equations, which is linear for the case of the closed crack, while for the case of the opened crack it is nonlinear and a function of time.

SOLUTION OF THE SYSTEM OF DIFFERENTIAL EQUATIONS

For solving this system of differential equations we used the numerical Runge-Kutta method. It is conceptually very simple, but at the same time pretty reliable, Bakhvalov (5), Mendeš et al. (6). The speed of calculations is increased by application of the adaptive step Δt. The developed computer program consists of the main program and three subprograms for calculations of the functions q_i, their derivatives with respect to time, and the length of the adaptive step.

ANALYSIS OF THE OBTAINED RESULTS

- The written program enables obtaining of displacements and velocities at each point in the two mutually perpendicular directions. For different combinations of the input data different diagrams can be obtained which illustrate relations between different variables.
- By comparison of the corresponding diagrams one can notice that the amplitudes of vibrations are increased due to the presence of the crack, and also that its presence disturbs the harmonic motions of the rotor.
- In Figures 3 and 4 are shown two diagrams out of several hundreds obtained as a result of the application of the described procedure. The first figure shows the phase diagram which clearly illustrates the influence of the presence of the crack on the oscillatory behavior of the rotor, and the latter figure shows the crack force in horizontal direction. The moments of the crack opening and closing can be readily noticed.

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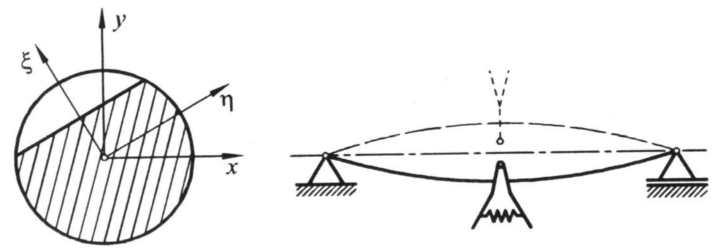


Figure 1 Crack model

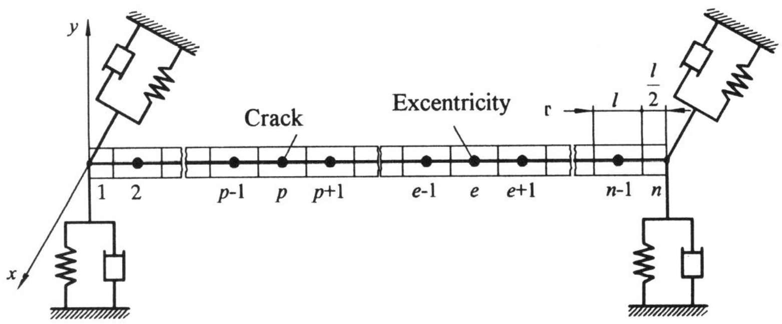
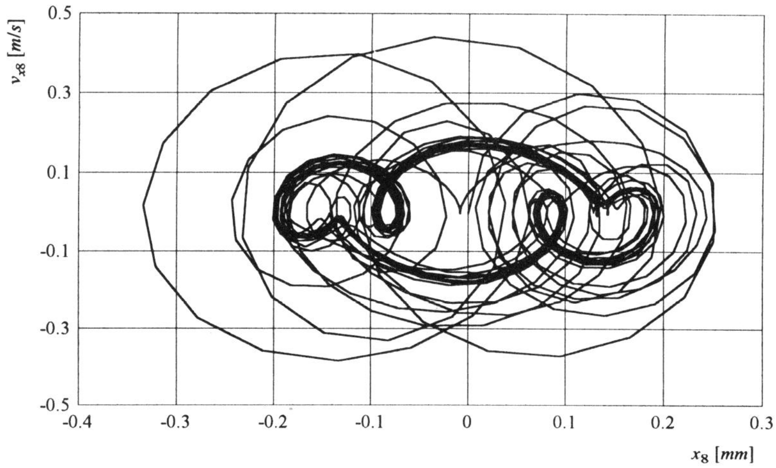
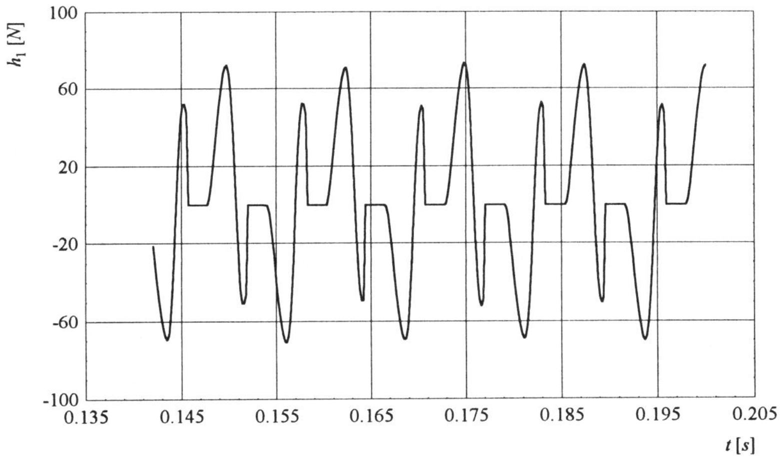


Figure 2 Rotor with a crack modeled by *n* concentrated masses



$h_r = 0{,}1 \quad e_m = 0{,}02 \quad \Omega = 500 \quad \beta_{mod} = 0{,}00001$

Figure 3 Phase diagram for the point 8 in the horizontal direction



$h_r = 0{,}1 \quad e_m = 0{,}02 \quad \Omega = 500 \quad \beta_{mod} = 0{,}00001$

Figure 4 Crack force in the horizontal direction