APPLICATION OF T*-INTEGRAL TO THE DUCTILE CRACK PROPAGATION ANALYSIS IN THE MEDIA WITH VOIDS

V.I.Kostylev* and B.Z.Margolin*

A computation analysis of crack growth by ductile fracture in the conditions of quasi-static and dynamic loading, based on the T*-integral application, is represented. It has been shown that for the media without voids crack growth may be described by condition $T^*_I(\Delta L)=const=J_{IC}$, if T^*_I-integral is calculated on small integration path moving with the crack tip. To analyse of crack growth in media with voids a new T^*_S-integral with special strip path has been proposed.

INTRODUCTION

To describe crack growth Atluri (1), Brust et al. (2) used the parameter T*:

$$T^* = \lim_{\Delta \to 0} \int_{\Gamma_\Delta} [(U+K)n_1 - t_i \frac{\partial u_i}{\partial x_1}] d\Gamma .$$

The computational and experimental works (1), (2) performed on the compact tension specimens and the edge crack specimens resulted in the following. For the stationary crack under monotonic loading the parameters T* and J-integral (computed by the external path) coincide. As a crack is propagating the J-integral increases continuously, whereas T* grows up to a certain constant level T^*_{const} and does not change with the further increase of ΔL. Note that for different materials and specimens the value of T^*_{const} varies in the range of $(2,5\text{-}10)J_{IC}$.

* Central Research Institute of Structural Materials "Prometey", Saint-Petersburg, Russia.

DUCTILE CRACK GROWTH

To analyse the applicability of T* for the description of crack propagation under monotonic loading some computations was performed by Margolin and Kostylev (3). By means of finite element method the elastic-plastic problem on the crack propagation under the plane strain conditions was solved. We used specimens with the sizes: S=400 mm, 2W=200 mm, 2L=100 mm and properties of material corresponding to the Cr-Mo-V steel: Young's modulus $E=2\cdot10^5$ MPa, Poisson's ratio $\nu=0.3$, $J_{IC}=162$ N/mm. The nonlinear stress vs strain curve was described by dependence $\sigma_{eq}=520+596(\varepsilon^{p}_{eq})^{0,494}$ MPa. The numerical simulation of the ductile crack growth was conducted by providing the self-similarity of the local stress-strain state (SSS) near its tip by means of the appropriate external load selection. The computation of T* was performed by two types of paths (Fig.1a,b) with $\Delta=\Delta_o$ providing the convergence of integrals: $T^*_{\Delta=\Delta o-\delta}=$ $=T^*_{\Delta=\Delta o}$, where $\delta<\Delta_o$. The results obtained (Fig.1c) allow to conclude that $T^*_1(\Delta L)$=const=J_{IC} and hence parameter T^*_1 uniquely controls SSS at the tip of the moving crack; to describe the SSS by means of T^*_2 , the dependence $T^*_2(\Delta L)$ should be used. It is evident that the rise of T^*_2 with the increase of ΔL is related to the material unloading occuring behind the moving crack tip. The unique controlling of the local SSS by the parameter T^*_1 at the crack propagation is caused by the fact that in the small path Γ_Δ (Fig.1a) enveloping only the moving crack tip, mainly the monotonic loading occurs whereas the unloading is practically absent.

It is known that the application of J_R-approach is based on J_R-curves invariance to the type of the loading. At the same time, the variability of experimental data leaves the question of J_R-curves invariance open. Using T*-integral and its property: $T^*_1(\Delta L)$=const, consider the behaviour of J_R-curves under the stable crack growth in the different specimens (center-cracked under tension, single edge notch under tension, three-point bend). J-integral is computed on the integration path passing along the external specimen boundary. Fig.2 shows that the dependences $J(\Delta L)$ obtained for different loading schemes differ essentially, the maximum difference reaching 30% with ΔL=3mm. Thus, the application of J_R-curve obtained for any specimens may lead to considerable errors in the estimate of the stable crack growth for an arbitrary geometry structures.

The dynamic crack growth may be caused both by the unstable crack growth and its growth under the highspeed loading of the structure. It is evident that in the both cases the algorithm of the crack growth computation is the same. The study of the behaviour of the parameter T* at dynamic loading showed that dependences $T^*(\Delta L)$ have the same peculiarities that occur at quasi-static loading. T^*_I behaves most steadily that makes it useful for the numerical simulation of the dynamically growing crack, the rate of the crack growth v being determined from $T^*_I(\Delta L)=const=J_{IC}$. Since T^*_I is the function of v, this non-linear equation is solved only by the iteration method.

DUCTILE CRACK PROPAGATION IN MEDIA WITH VOIDS

It should be noted that the correct application of T^*_I-integral is possible in case when material loosenning owing to void evolution under plastic deformation may be neglected. Ductile crack propagation is known to be mainly caused by void nucleation, growth and coalescence that results in the significant loosenning of material. In these cases T^*_I-integral application is very problematical because of the followings. Material loosenning depending on local stress triaxiality and plastic strain appears to be extremely heterogeneous near the crack tip. Therefore different regions near the crack tip are deformed according to different stress-strain curves $\sigma_{eq}(\varepsilon_{eq})$. Then parameter $U=\int\sigma_{ij}d\varepsilon_{ij}$ depends not only on ε_{ij} but on coordinates r and θ, i.e. U is not stress and strain potential. Hence, T^*_I-integral becomes non-invarient to integration path. The latter leads to absence of the unique relation between T^*_I-integral and local SSS.

The following conclusions may be drawn according to studies Margolin and Kostylev (3), Rice and Rosengren (4), McMeeking (5), Rice and Tracey (6). From reference (3)-(5) for elastic-plastic media without voids under loading on I-mode the stress constraint σ_o/σ_{eq} on the line of the crack extent ($\theta=0$) is a function of equivalent plastic strain ε^p_{eq} only and does not depend on coordinate r, i.e. $\sigma_o/\sigma_{eq}=f(\varepsilon^p_{eq})$. Void nucleation may be described equation from reference (3):

$$\rho=\rho_o+\rho_f[1-\exp(-A_r(\varepsilon^p_{eq}-\varepsilon_o))] \qquad (1)$$

where A_r - material constant. Void growth under plastic deformation may be described by equation from reference (6):

$$dR/R=0,28\exp(1,5\sigma_o/\sigma_{eq})d\varepsilon^p_{eq} \qquad (2)$$

So, it may be considered that on the line of the crack extent total area of voids S_v are uniquely determined by ε^p_{eq} value, i.e. $S_v=f_1(\varepsilon^p_{eq})$. For media with voids it may be written by using true stress $\sigma^t_{ij}=\sigma_{ij}/(1-S_v)$: $\sigma^t_{eq}=f_2(\varepsilon^p_{eq})$ and $\sigma_{eq}=f_2(\varepsilon^p_{eq})(1-S_v)$. Taking into account that on the line of the crack extent $S_v=f_1(\varepsilon^p_{eq})$ we have $\sigma_{eq}=f_2(\varepsilon^p_{eq})[1-f_1(\varepsilon^p_{eq})]=f_3(\varepsilon^p_{eq})$. Thus, on this line there exists the unique dependence $\sigma_{eq}=f_3(\varepsilon^p_{eq})$ being invarient to coordinates, i.e. parameter $U=\int\sigma_{ij}d\varepsilon_{ij}$ is potential for ε_{ij} and σ_{ij}. Consequently, T*-integral controls SSS uniquely in the region located near the line of the crack extent if integration is performed on path stretching along this line. Define this path as shown in Fig.3, i.e. as small narrow path stretching along the line of the crack extent and moving with the crack tip. Name such path by strip path and denote it as Γ_S and T*-integral as T^*_S-integral.

To verify the stated propositions numerical analysis of SSS near the crack tip was performed as applied to compact specimen (Fig.3) with the following sizes: W=100 mm, S=120 mm, L=40 mm. According to experimental data the following values of parameters for eqn (1) were taken: ρ_o=20 mm^{-2}, ε_o=0.07, ρ_f=20408 mm^{-2}, A_r=2. The dependences of equivalent stress on equivalent plastic strain $\sigma_{eq}(\varepsilon^p_{eq})$ in various points near the crack tip are shown in Fig.3. Analysis of these results permits to conclude that for points located on the line of the crack extent there exists the unique dependence $\sigma_{eq}(\varepsilon^p_{eq})$, but for points located on the parallel line there not exists such unique dependence. Evidently, T*-integral calculated on integration paths passing through the regions in which the unique dependence $\sigma_{eq}(\varepsilon^p_{eq})$ does not exist has different values. So, T^*_l-integral calculated on path Γ_l depends on the path size Δ in the regions of damaged materials and is invarient only to integration paths passing through the regions of non-damaged materials. At the same time the values of T^*_S-integral does not practically depend on Δ (Fig.4) if T^*_S-integral is calculated on Γ_S path. Hence, T^*_S-integral may be used to predict the crack start in media with voids.

After the crack start T^*_S-integral calculated on Γ_S path does not practically vary if the self-similarity for local SSS is provided. Hence, to simulate crack propagation on ductile failure mechanism with regard for material losenning condition $T^*_S(\Delta L)$=const=J_{IC} must be fulfiled.

CONCLUSIONS

1. It has been shown that crack propagation at quasi-static and dynamic loading may be described by condition $T^*_l(\Delta L)$=const=J_{IC}, if T^*_l-integral is calculated on small integration path moving with the crack tip.

2. J_R-curves has been shown to depend on loading scheme and type of specimen.

3. A new T^*_S -integral with special strip path of integration has been proposed. It permits to analyse start and growth of crack in material with significant loosenning caused by void nicleation and growth. Condition of stable crack propagation in such material may be written as $T^*_S(\Delta L)$=const=J_{IC}.

SYMBOLS USED

ε_{ij}, ε^{p}_{eq}, ε_{o} = strain: tensor, equivalent plastic, voids nucleation (%)

Γ = integration path enveloping the crack tip

J, T*, J_{IC} = J, T* integrals and critical value of J-integral (N/mm)

K, U = the kinetic and strain energy density (N/mm^2)

L, ΔL = crack length and its increment (mm)

r, θ = polar coordinates (mm) and (rad)

R = void radius (μm)

ρ, ρ_o, ρ_f = concentration: voids, initial voids, inclusion (mm^{-2})

σ_{ij}, σ_o, σ_{eq}, σ^t = stresses: tensor, hydrostatic, equivalent and true (MPa)

S_v = the relative void area (void area per the unit area)

t_i = projection of the force vector on the contour Γ (N/mm^2)

u_i = displacement vector components (mm)

REFERENCES

(1) Atluri, S.N., Eng. Fracture Mech., 16, 1982, pp.341-364.

(2) Brust, F.W., McGowan, J.J., Atluri, S.N., Eng. Fracture Mech, Vol. 23, No. 3, pp.537-550.

(3) Margolin, B.Z. and Kostylev, V.I., Problemy prochnosti, No. 10, 1992, pp.3-14 (in Russian).

(4) Rice, J.R. and Rosengren, G.F., J. Mech. Phys. Solids, Vol. 16, No. 1, pp.1-12.

(5) McMeeking, R.M., J. Mech. Phis. Solids, 25, 1976, pp.357-381.

(6) Rice, J.R. and Tracey, D.M., J. Mech. Phis. Solids, Vol. 17, No. 3, 1969, pp.201-217.

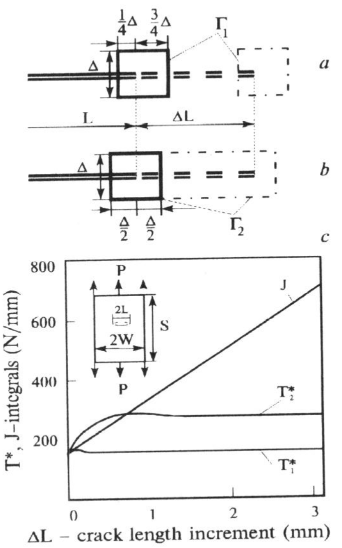

Figure 1. Various integration path /a,b/ and T*, J-integrals vs ΔL /c/.

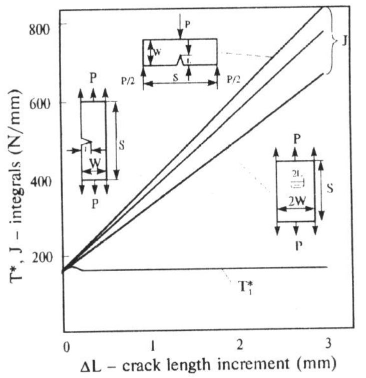

Figure 2. T*, J-integrals vs ΔL for various specimens.

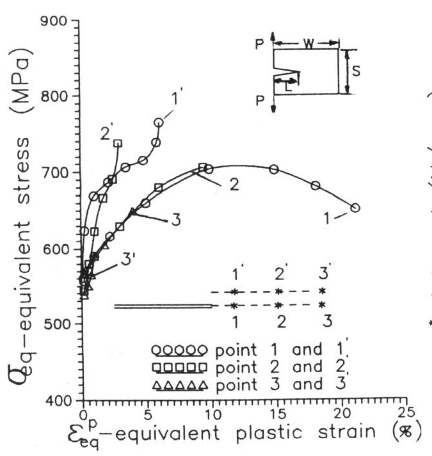

Figure 3. σ_{eq} vs ε^{p}_{eq} for CT specimen with regard for void evolution.

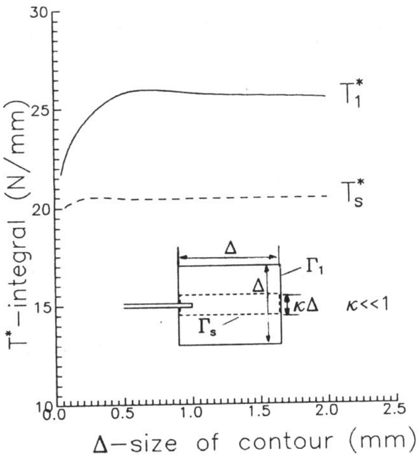

Figure 4. T^*_1 and T^*_s-integrals vs contour sizes .